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Dynamics of an Isolated Vapor Bubble in Saturated Nucleate Boiling*

(Part 3, Effect of Thermal Factors of a Heater)

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One-dimensional model analysis was made to investigate the effect of thermal factors of a heater upon the evaporation rate of a microlayer beneath the bubble. The evaporation rate was found to be markedly influenced by thermal factors such as heat capacity and thermal diffusivity of a heater. Quantitative analysis was also made about the corresponding behaviors of temperature and heat flux on the heater surface.

The effect of heat capacity was really found by a careful inspection of the experimental data, which showed a considerable decrease of the bubble growth rate with a decreasing heat capacity. The thermal factors of a heater were found to influence the bubble growth rate through the mechanism of a microlayer evaporation.

1. Introduction

As previously reported⁽¹⁾, a model analysis was successfully made by considering a microlayer evaporation to explain the experimental data of the growth rate of an isolated vapor bubble on a heated surface. The analysis showed that the role of the microlayer evaporation was not negligible even if the average heat flux through the heated surface was sufficiently small, and that its relative contribution to the bubble growth rate seemed to increase with an increasing heat flux. Under fairly high heat flux conditions where bubbles coalesce on the heated surface, the microlayer evaporation is believed to be the controlling mechanism of the bubble growth⁽²⁾.

To know the behavior of the microlayer beneath the bubble is thus an important problem associated with the bubble growth on a heated surface. Most of studies made hitherto^{(3)~(16)} seem to be directed toward the hydrodynamic investigation of microlayer formation, with a few exceptions of thermal study which includes the process of microlayer evaporation^{(16)~(17)}. In this paper, the result of a quantitative analysis is presented about the influence of thermal factors, especially heat capacity, of the heater

upon the process of microlayer evaporation. And the growth rate of a bubble on the heated surface is also shown to be markedly influenced by such factors.

Notations

- a : thermal diffusivity m^2/hr
- c : specific heat $\text{kcal}/\text{kg}^\circ\text{C}$
- $f(x)$: initial superheat distribution $^\circ\text{C}$
- i : imaginary unit $\sqrt{-1}$
- m : natural integer, or constant
- n : bubble growth exponent
- q : heat flux, or heat load $\text{kcal}/\text{m}^2\text{hr}$
- s : Laplace transformation parameter
- t : time hr
- u : superheat $^\circ\text{C}$
- x : one-dimensional axis m
- D : equivalent bubble diameter m
- J_a : Jakob number $c\rho_l\Delta T/L\rho_v$
- L : latent heat of evaporation kcal/kg
- Q_{ac} : accumulated vapor thickness m
- ΔT : initial wall superheat $^\circ\text{C}$
- α : heat transfer coefficient at the bottom surface $\text{kcal}/\text{m}^2\text{hr}^\circ\text{C}$
- δ : heater thickness m
- ε : microlayer thickness m
- λ : thermal conductivity $\text{kcal}/\text{mhr}^\circ\text{C}$
- ρ : density kg/m^3
- τ : starting time of slowing in bubble growth hr
- $\sim$: Laplace transform

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' : d/dx

Res : residue

Subscripts

1 : heater

2 : microlayer

b : bottom surface of the heater

l : microlayer

v : vapor

lv : between microlayer and vapor

sl : between heater and microlayer

2. Analysis

The thickness of the microlayer will be a function of space and time. However, the thickness is usually very small in comparison with the linear extent of bubble base, and the flow of heat through the microlayer may well be regarded as one-dimensional with a perpendicular axis to the heated surface. Let us consider an approximate model as shown in Fig. 1. A microlayer is assumed to form suddenly on the heater, each thickness being ε and δ . The superheat is initially $f_1(x)$ in the heater and $f_2(x)$ in the liquid. Suddenly at the time $t=0$, the microlayer is formed and accordingly the superheat at the liquid-vapor boundary at $x=\varepsilon$ falls to zero. The dynamic behavior of microlayer formation and associated heat transfer are not taken into account, since the time for the microlayer formation is substantially smaller than its dry-up time in ordinary cases.

A finite amount of heat transfer may be postulated at the bottom surface of the heater. Two limiting cases, however, are considered in the present analysis: one is the adiabatic case, and the other is the fully diabatic case (constant superheat u_b). In the former case, such a finite amount of heat generation should be existent in the heater as is usually encountered in joulean or nuclear direct heating. The former case is one of direct heating, and the latter one of indirect heating.

Let a denote thermal diffusivity, λ : thermal

(a) Case of direct heating

(b) Case of indirect heating

Fig. 1 One-dimensional model of transient heat conduction in composite layers

conductivity, α : heat transfer coefficient at the bottom surface, q : heat load, c : specific heat, ρ : density, u : superheat, and let subscripts 1 and 2 correspond respectively to the heater and the microlayer. Then the following equations hold.

Diffusion equations:

$$\frac{\partial u_1}{\partial t} = a_1 \frac{\partial^2 u_1}{\partial x^2} + E \quad (1)$$

$$(E = q/c_1\rho_1\delta, \text{ or } E=0)$$

$$\frac{\partial u_2}{\partial t} = a_2 \frac{\partial^2 u_2}{\partial x^2} \quad (2)$$

Boundary conditions:

at $x=-\delta$

$$\frac{\partial u_1}{\partial x} - \alpha(u_1 - u_b) = 0 \quad (3)$$

$$(\alpha=0, \text{ or } \alpha \rightarrow \infty)$$

at $x=0$

$$u_1 = u_2 \quad (4)$$

$$\lambda_1 \frac{\partial u_1}{\partial x} = \lambda_2 \frac{\partial u_2}{\partial x} \quad (5)$$

at $x=\varepsilon$

$$u=0 \quad (6)$$

Initial conditions:

at $t=0$

$$u_1 = f_1(x), \quad u_2 = f_2(x) \quad (7)$$

Equations (1) and (2) are reformed by Laplace transformation into

$$\frac{d^2 \tilde{u}_1}{dx^2} - \omega_1^2 \tilde{u}_1 = -\omega_1^2 F_1(s, x) \quad (8)$$

$$\frac{d^2 \tilde{u}_2}{dx^2} - \omega_2^2 \tilde{u}_2 = -\omega_2^2 F_2(s, x) \quad (9)$$

where the symbol $\sim$ is meant as the Laplace transform, and

$$\omega_1^2 \equiv s/a_1, \quad \omega_2^2 \equiv s/a_2 \quad (10)$$

$$F_1(s, x) \equiv [f_1(x) + \tilde{E}]/s \quad (11)$$

$$F_2(s, x) \equiv f_2(x)/s \quad (12)$$

are used. The change of ε with time may be neglected in the first approximation for the discussion of rapid thermal transient behaviors, and ε is assumed to be constant in the following analysis.

The general solutions of Eqs. (8) and (9) are immediately obtained as in the forms

$$\tilde{u}_1 = C_1 \sinh(\omega_1 x) + D_1 \cosh(\omega_1 x) + F_1(s, x) \quad (13)$$

$$\tilde{u}_2 = C_2 \sinh(\omega_2 x) + D_2 \cosh(\omega_2 x) + F_2(s, x) \quad (14)$$

From the boundary conditions, the coefficients are all determined to give

$$\tilde{u}_1 = \frac{G(s)[\sigma \cosh X_2 \sinh Y_1 - \sinh X_2 \cosh Y_1] - \sigma F(s)[R_s \sinh Y_1 + R_c \cosh Y_1]}{R_s \sinh X_2 + \sigma R_c \cosh X_2} + F_1(s, x) \dots (15)$$

$$\begin{aligned} \tilde{u}_2 = & \frac{G(s)[\cosh X_2 \sinh Y_2 - \sinh X_2 \cosh Y_2] - F(s)[R_s \sinh Y_2 + \sigma R_c \cosh Y_2]}{R_s \sinh X_2 + \sigma R_c \cosh X_2} \\ & + \{F_1'(s, 0)/\sigma\omega_1 - F_2'(s, 0)\} \sinh Y_2 + \{F_1(s, 0) - F_2(s, 0)\} \cosh Y_2 + F_2(s, x) \dots (16) \end{aligned}$$

where the following notations are substituted.

$$\begin{aligned} k &\equiv \omega_2/\omega_1 \equiv \sqrt{a_1/a_2}, \quad \sigma \equiv k\lambda_2/\lambda_1, \quad X_1 \equiv \omega_1\delta, \quad X_2 \equiv \omega_2\varepsilon, \quad Y_1 \equiv \omega_1x, \quad Y_2 \equiv \omega_2x \\ R_s &\equiv \omega_1 \sinh X_1 + \alpha \cosh X_1, \quad R_c \equiv \omega_1 \cosh X_1 + \alpha \sinh X_1 \\ F(s) &\equiv \{F_1'(s, 0)/\sigma\omega_1 - F_2'(s, 0)/\omega_2\} \sinh X_2 + \{F_1(s, 0) - F_2(s, 0)\} \cosh X_2 + F_2(s, \varepsilon) \\ G(s) &\equiv \alpha F_1(s, -\delta) - F_1'(s, -\delta) - \alpha u_0/s \end{aligned}$$

There are two kinds of heat flux to be considered, namely, the flux between heater and microlayer (q_{sl}) and the flux between microlayer and vapor (q_{lv}). The former is related to the change of the heater temperature, and the latter controls the growth rate of a bubble. The Laplace transforms of these kinds of flux are easily determined to be

$$\tilde{q}_{sl} = -\lambda_2 \left(\frac{d\tilde{u}_2}{dx} \right)_{x=0} = -\lambda_2 \left[\frac{\omega_2 \{G(s) \cosh X_2 - F(s) R_s\}}{R_s \sinh X_2 + \sigma R_c \cosh X_2} + \frac{\omega_2 F_1'(s, 0)}{\sigma\omega_1} \right] \dots (17)$$

$$\begin{aligned} \tilde{q}_{lv} = & -\lambda_2 \left(\frac{d\tilde{u}_2}{dx} \right)_{x=\varepsilon} = -\lambda_2 \left[\frac{\omega_2 G(s) \cosh^2 X_2 - \omega_2 R_s \cosh X_2 F(s) - \omega_2 G(s) \sinh^2 X_2 - \omega_2 \sigma R_c \sinh X_2 F(s)}{R_s \sinh X_2 + \sigma R_c \cosh X_2} \right. \\ & \left. + \cosh X_2 \{\omega_2 F_1'(s, 0)/\sigma\omega_1 - F_2'(s, 0)\} + \omega \sinh X_2 \{F_1(s, 0) - F_2(s, 0)\} + F_2'(s, \varepsilon) \right] \dots (18) \end{aligned}$$

By using the formula⁽¹⁹⁾

$$\phi(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Phi(s) e^{st} ds = \sum \text{Res} \{\Phi(s) e^{st}\} \dots (19)$$

which holds for an arbitrary function $\phi(t)$ and its Laplace transform $\Phi(s)$, and by introducing a new parameter γ which is defined by

$$s = -a_1 \gamma^2 \dots (20)$$

the inverse transforms of Eqs. (15)~(18) are obtained as in the following, where $u_1(x=0)$ is presented instead of u_1 and u_2 .

(Case of direct heating)

$$u_1(x=0) = \frac{q\varepsilon}{\lambda_2} + \sum_{m=1}^{\infty} \{u_1\}_m \dots (21)$$

$$\{u_1\}_m \equiv \frac{2\sigma(\Delta T - q\varepsilon/\lambda_2) \cos(\gamma_m \delta) \exp(-a_1 \gamma_m^2 t)}{\gamma_m \{(k\varepsilon + \sigma\delta) \sin(\gamma_m \delta) \cos(k\gamma_m \varepsilon) + (\delta + \sigma k\varepsilon) \cos(\gamma_m \delta) \sin(k\gamma_m \varepsilon)\}}$$

$$q_{sl} = q + \sum_{m=1}^{\infty} \{q_{sl}\}_m \dots (22)$$

$$\{q_{sl}\}_m \equiv \frac{2\lambda_2 k(\Delta T - q\varepsilon/\lambda_2) \sin(\gamma_m \delta) \exp(-a_1 \gamma_m^2 t)}{(k\varepsilon + \sigma\delta) \sin(\gamma_m \delta) \cos(k\gamma_m \varepsilon) + (\delta + \sigma k\varepsilon) \cos(\gamma_m \delta) \sin(k\gamma_m \varepsilon)}$$

$$q_{lv} = q + \sum_{m=1}^{\infty} \{q_{lv}\}_m \dots (23)$$

$$\{q_{lv}\}_m \equiv \frac{2\lambda_2 k(\Delta T - q\varepsilon/\lambda_2) \{\sin(\gamma_m \delta) \cos(k\gamma_m \varepsilon) + \sigma \cos(\gamma_m \delta) \sin(k\gamma_m \varepsilon)\} \exp(-a_1 \gamma_m^2 t)}{(k\varepsilon + \sigma\delta) \sin(\gamma_m \delta) \cos(k\gamma_m \varepsilon) + (\delta + \sigma k\varepsilon) \cos(\gamma_m \delta) \sin(k\gamma_m \varepsilon)}$$

where γ_m is the m -th root of the equation

$$\sin(\gamma_m \delta) \sin(k\gamma_m \varepsilon) - \sigma \cos(\gamma_m \delta) \cos(k\gamma_m \varepsilon) = 0 \dots (24)$$

(Case of indirect heating)

$$u_1(x=0) = \frac{\Delta T + q\delta/\lambda_1}{1 + \sigma\delta/k\varepsilon} + \sum_{m=1}^{\infty} \{u_1\}_m \dots (25)$$

$$\{u_1\}_m \equiv \frac{-2\sigma(\Delta T - q\varepsilon/\lambda_2) \sin(\gamma_m \delta) \exp(-a_1 \gamma_m^2 t)}{\gamma_m \{(k\varepsilon + \sigma\delta) \cos(\gamma_m \delta) \cos(k\gamma_m \varepsilon) - (\delta + \sigma k\varepsilon) \sin(\gamma_m \delta) \sin(k\gamma_m \varepsilon)\}}$$

$$q_{sl} = \frac{\lambda_2 k(\Delta T + q\delta/\lambda_1)}{k\varepsilon + \sigma\delta} + \sum_{m=1}^{\infty} \{q_{sl}\}_m \dots (26)$$

$$\{q_{sl}\}_m \equiv \frac{2\lambda_2 k(\Delta T - q\varepsilon/\lambda_2) \cos(\gamma_m \delta) \exp(-a_1 \gamma_m^2 t)}{(k\varepsilon + \sigma\delta) \cos(\gamma_m \delta) \cos(k\gamma_m \varepsilon) - (\delta + \sigma k\varepsilon) \sin(\gamma_m \delta) \sin(k\gamma_m \varepsilon)}$$

$$q_{lv} = \frac{\lambda_2 k(\Delta T + q\delta/\lambda_1)}{k\varepsilon + \sigma\delta} + \sum_{m=1}^{\infty} \{q_{lv}\}_m \dots (27)$$

$$(q_{iv})_m \equiv \frac{2\lambda_2 k (\Delta T - q\varepsilon/\lambda_2) \{\cos(\gamma_m \delta) \cos(k\gamma_m \varepsilon) - \sigma \sin(\gamma_m \delta) \sin(k\gamma_m \varepsilon)\} \exp(-a_1 \gamma_m^2 t)}{(k\varepsilon + \sigma\delta) \cos(\gamma_m \delta) \cos(k\gamma_m \varepsilon) - (\delta + \sigma k\varepsilon) \sin(\gamma_m \delta) \sin(k\gamma_m \varepsilon)}$$

where γ_m is the m -th root of the equation

$$\cos(\gamma_m \delta) \sin(k\gamma_m \varepsilon) + \sigma \sin(\gamma_m \delta) \cos(k\gamma_m \varepsilon) = 0 \quad (28)$$

In deriving the above solutions, the functions of initial superheat are assumed to take the following forms:

$$f_1(x) = \begin{cases} A_{10} + A_{11}x + A_{12}x^2 & \text{(direct heating)} \\ A_{20} + A_{21}x & \text{(indirect heating)} \end{cases} \quad f_2(x) = \begin{cases} B_{10} + B_{11}x & \text{(direct heating)} \\ B_{20} + B_{21}x & \text{(indirect heating)} \end{cases}$$

where the coefficients $A_{10}, \dots, B_{21}$ are as determined as to satisfy the initial conditions having ΔT as the superheat of the heated surface and q as the initial equilibrated heat flux (to be equal to the heat load).

Direct heating (stainless steel and water under the atmospheric pressure, $q = 10^5$ kcal/m² hr)

Indirect heating (stainless steel and water under the atmospheric pressure, $q = 10^5$ kcal/m² hr)

Fig. 2 Calculated results of superheat, heat flux, and accumulated vapor thickness vs. time

The accumulated thickness of the evaporated vapor from the microlayer is calculated from the equation

$$Q_{ac} = \frac{\int_0^t q_{lv} dt}{L\rho_v} \dots\dots\dots (29)$$

Before starting to calculate the solutions, an appropriate set of parameters such as δ , ε , q , ΔT , and the physical properties of materials should be chosen, and the necessary roots of γ_m should be determined by Eq. (24) or Eq. (28).

The solutions are presented in Fig. 2 for the case where the heater is made of stainless-steel which is immersed in water under the atmospheric pressure. The initial superheat of the heated surface could be assumed by the established correlation equations for ΔT and q . In these examples, $\Delta T = 20^\circ\text{C}$ is assumed against $q = 10^5 \text{ kcal/m}^2\text{hr}$.

In the case of direct heating, $u_1(x=0)$ decreases sharply and monotonically with time, provided ε and δ are sufficiently small. In contrast, q_{lv} once reaches the peak, and then decreases to approach a constant value. The time to the peak is longer for larger ε and δ . The evaporation heat flux q_{lv} decreases more rapidly for smaller δ .

The comparison of both the cases of heating in Fig. 2 shows that there is a considerable difference between the two results. However, the case of indirect heating that has a constant bottom surface temperature is an extreme idealization having a heat source on the bottom surface. Hence, the reduction of heat capacity

$$u_1 = \frac{C}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\exp\{tz - (z/a_1)^{1/2}x\} dz}{z \{\cosh(z/a_2)^{1/2}\varepsilon + \nu \sinh(z/a_2)^{1/2}\varepsilon\}} \dots\dots\dots (38)$$

By considering that $\exp\{-(z/a_1)^{1/2}x\}$ and $\sinh(z/a_2)^{1/2}\varepsilon$ are both double-value functions, Eq. (38) is rewritten to

$$u_1 = C \left[1 - \frac{1}{2\pi i} \int_0^\infty e^{-t/r} \left\{ \frac{\exp\{-x(r/a_1)^{1/2}e^{-i\pi/2}\}}{\cosh\{(r/a_2)^{1/2}e^{-i2/\pi}\varepsilon\} + \nu \sinh\{(r/a_2)^{1/2}e^{-i\pi/2}\varepsilon\}} + \frac{\exp\{-x(r/a_1)^{1/2}e^{i\pi/2}\}}{\cosh\{(r/a_2)^{1/2}e^{i\pi/2}\varepsilon\} + \nu \sinh\{(r/a_2)^{1/2}e^{i\pi/2}\varepsilon\}} \right\} \frac{dr}{r} \right] \dots\dots\dots (39)$$

The final form of Eq. (39) may be expressed as

$$u_1 = C \left[1 - \frac{2}{\pi} \int_0^\infty \frac{\{\cos(\xi\varepsilon) \sin(\eta\xi x) + \nu \sin(\xi\varepsilon) \cos(\eta\xi x)\} \exp(-a_2\xi^2 t) d\xi}{\xi \{\cos^2(\xi\varepsilon) + \nu^2 \sin^2(\xi\varepsilon)\}} \right] \dots\dots\dots (40)$$

where

$$\xi \equiv (r/a_2)^{1/2}, \quad \eta \equiv (a_2/a_1)^{1/2} \equiv k^{-1}$$

are used. Similarly, the equation

$$u_2 = C \left[1 - \frac{2\nu}{\pi} \int_0^\infty \frac{\sin \xi(x+\varepsilon) \exp(-a_2\xi^2 t) d\xi}{\xi \{\cos^2(\xi\varepsilon) + \nu^2 \sin^2(\xi\varepsilon)\}} \right] \dots\dots\dots (41)$$

is found for u_2 .

The correct solutions are then obtained by rewriting $C-u$ as u again. The final forms are

$$u_1 = \frac{2C}{\pi} \int_0^\infty \frac{\{\cos(\xi\varepsilon) \sin(\eta\xi x) + \nu \sin(\xi\varepsilon) \cos(\eta\xi x)\} \exp(-a_2\xi^2 t) d\xi}{\xi \{\cos^2(\xi\varepsilon) + \nu^2 \sin^2(\xi\varepsilon)\}} \dots\dots\dots (42)$$

does play inversely for each case. If a finite heat transfer is assumed at the bottom surface, the same trend may appear in both cases.

To investigate the effect of thermal diffusivity of the heater, let us consider two kinds of heaters having large heat capacities and different values of thermal diffusivity. In Fig. 1, such modifications as $x \rightarrow -x$, $|\delta| \rightarrow \infty$, and $f_1(x) = f_2(x) = C(\text{constant}) > 0$ are enough, if the initial superheat is assumed to be uniform throughout the heater and the microlayer. For convenience $C-u$ is rewritten to u to find the following equations in Laplace transformed field.

Diffusion equations:

$$\frac{d^2 \bar{u}_1}{dx^2} - \omega_1^2 \bar{u}_1 = 0 \dots\dots\dots (30)$$

$$\frac{d^2 \bar{u}_2}{dx^2} - \omega_2^2 \bar{u}_2 = 0 \dots\dots\dots (31)$$

Boundary conditions:

$$\bar{u}_1(s, 0) = \bar{u}_2(s, 0) \dots\dots\dots (32)$$

$$\lambda_1 \bar{u}_1'(s, 0) = \lambda_2 \bar{u}_2'(s, 0) \dots\dots\dots (33)$$

$$\lim_{x \rightarrow \infty} \bar{u}_1(s, x) \rightarrow 0 \dots\dots\dots (34)$$

$$\bar{u}_2(s, -\varepsilon) = C/s \dots\dots\dots (35)$$

The solutions for $\bar{u}_1$ and $\bar{u}_2$ are obtained to be

$$\bar{u}_1 = \frac{C \exp(-\omega_1 x)}{s \{\cosh(\omega_2 \varepsilon) + \nu \sinh(\omega_2 \varepsilon)\}} \dots\dots\dots (36)$$

$$\bar{u}_2 = \frac{C \{\cosh(\omega_2 x) - \nu \sinh(\omega_2 x)\}}{s \{\cosh(\omega_2 \varepsilon) + \nu \sinh(\omega_2 \varepsilon)\}} \dots\dots\dots (37)$$

where

$$\nu \equiv \frac{\lambda_1}{\lambda_2} \sqrt{\frac{a_2}{a_1}} \equiv \frac{1}{\sigma}$$

is substituted. The inverse transform of $\bar{u}_1$ is written to be

$$u_2 = \frac{2C\nu}{\pi} \int_0^\infty \frac{\sin \xi(x+\varepsilon) \exp(-a_2 \xi^2 t) d\xi}{\xi \{\cos^2(\xi \varepsilon) + \nu^2 \sin^2(\xi \varepsilon)\}} \dots (43)$$

The accumulated thickness Q_{ac} is found subsequently to be

$$\begin{aligned} Q_{ac} &= \frac{1}{L\rho_v} \int_0^t \lambda_2 \left(\frac{\partial u_2}{\partial x} \right)_{x=-\varepsilon} dt \\ &= \frac{2\lambda_2 C\nu}{\pi L\rho_v} \int_0^\infty \frac{\{1 - \exp(-a_2 \xi^2 t)\} d\xi}{a_2 \xi^2 \{\cos^2(\xi \varepsilon) + \nu^2 \sin^2(\xi \varepsilon)\}} \dots (44) \end{aligned}$$

With water as the microlayer and with copper, nickel or stainless steel as the heater, the numerical results of Eq. (44) are shown in Fig. 3. These results indicate that the growth rate of a bubble is influenced by thermal diffusivity of the heater even if the heater has sufficient heat capacity. When the microlayer consists of organic liquids having low conductivity and small latent heat of evaporation, the influence of the heater thermal diffusivity is usually small and negligible in comparison with the influence of heat capacity.

3. Discussions

Ogawa⁽²⁰⁾ measured the growth rate of bubbles on heaters of different thicknesses. A parallelepiped bakelite box, a plane of which was replaced by a stainless steel plate, was immersed in stagnant water at saturation temperature under various pressures. The heated surface was set horizontally upward, and was heated directly by the direct current. The heat load was given as high as possible in natural convection condition so as to form the developed thermal boundary layer. Then the heat load was increased stepwise up to about 10^5 kcal/m²hr to initiate the bubble growth. The first bubble was measured in each experiment by means of

Fig. 3 Effect of the thermal conductivity of the heater on the microlayer evaporation (calculated results)

a high speed camera operated at about 5000 frames per second. The surface temperature was calculated from the change of electrical resistance of the heater.

Generally, the equivalent diameter D of a bubble and the time t after growth commenced are approximately in the relation $D=Kt^n$. As is commonly known, $D=C_0 J_a \sqrt{at}$ holds for spherical bubble growth in a uniformly superheated liquid⁽²¹⁾. Similarly, $D=C_a J_a^n a^{1/2} t^n$ holds for bubble growth on a heated surface in a non-uniformly superheated liquid⁽²⁰⁾. Since the coefficient K includes J_a and hence ΔT , it is necessary to divide K by a function of ΔT in order to obtain a ΔT -independent parameter. In the first approximation a parameter $K/\Delta T$, i.e. the bubble growth constant, may be introduced. Strictly speaking, $K/\Delta T$ is ΔT -independent for uniformly superheated case^{(21),(23)~(25)}, but it slightly decreases with increase of ΔT for non-uniformly superheated case. The data on $K/\Delta T$ vs. δ obtained by Ogawa⁽²⁰⁾ are plotted in Fig. 4. Marked change of $K/\Delta T$ with δ clearly observed in this figure should be a strong support of the above-mentioned analysis. (The symbol $\bar{\delta}$ indicates the extent of data scatterings.)

For the spherical bubble growth in a uniformly superheated liquid, the growth equation

$$D = \frac{4\sqrt{3}}{\sqrt{\pi}} \frac{c\rho_l \sqrt{a}}{L\rho_v} \Delta T t^{1/2} \dots (45)$$

holds except in the starting period of growth⁽²¹⁾, and the bubble growth constant is calculated from the equation

$$\frac{K}{\Delta T} = \frac{4\sqrt{3}}{\sqrt{\pi}} \frac{c\rho_l \sqrt{a}}{L\rho_v} \dots (46)$$

The numerical values obtained from Eq. (46) are also indicated in Fig. 4. As indicated previously⁽¹⁾, the growth constant in non-uniform field could be larger than that in uniform field as a result of microlayer evaporation.

Fig. 4 Effect of the heat capacity of the heater on the bubble growth constant in saturated nucleate boiling (experimental results⁽²⁰⁾)

Fig. 5 Effect of the heat capacity of the heater on the starting time of slowing in bubble growth in saturated nucleate boiling (experimental results⁽²⁰⁾)

As δ approaches zero, it is natural to expect that $K/\Delta T$ approaches the value where the bubble growth is wholly controlled by the evaporation of the surrounding bulk liquid. In Fig. 4, the asymptotic value is seen to be considerably smaller than the value in uniform field, which is attributed to the fact that the effective superheat is smaller than the wall superheat $\Delta T^{(1)}$. For the combination of stainless steel and water, the contribution of the microlayer evaporation to the bubble growth is experimentally well negligible for δ smaller than 0.01 mm, and the asymptotic value of $K/\Delta T$ is found to fall to about 0.1.

In the later stage of growth in non-uniform field, the data appear to deviate from the relation $D=Kt^n$ owing to the reduction of average superheat of the surrounding liquid. The time when 5% deviation of D occurs is provisionally called the starting time of slowing in bubble growth and denoted by τ . Figure 5 shows the experimental data on τ vs. δ , which indicate that the driving force of bubble growth is kept longer for the thicker heater. This fact also substantiates the role of microlayer evaporation. From the author's experiment using 10 mm thick stainless steel and water under the atmospheric pressure, τ is 3~6 msec for the heat load between $3 \times 10^4 \sim 2 \times 10^5$ kcal/m²hr. The curve for 1 atm in Fig. 5 may thus increase up to several milliseconds.

In contrast, the bubble growth exponent n has no appreciable correlation with δ as shown in Fig. 6, where a salient effect of pressure as was previously indicated⁽²²⁾ can be observed.

4. Conclusions

(1) The thermal behavior of the composite system of a microlayer and a heater was made clear by a simplified model analysis.

(2) Thermal diffusivity of the heater was found to have an effect on the bubble growth. The effect was, however, generally small for liquids having low thermal conductivity and small latent heat of evaporation.

(3) That the bubble growth could be in-

Fig. 6 Effect of the heat capacity of the heater on the bubble growth exponent in saturated nucleate boiling (experimental results⁽²⁰⁾)

fluenced by the heat capacity of the heater was substantiated by an investigation of the experimental data obtained for heaters of different thicknesses.

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