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Upward Liquid Flow in Small Tube into which Air Streams*

(2nd Report, Pressure Drop at the Confluence)

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When air streams into a liquid flowing upward in a vertical small tube, two kinds of pressure drops occur; that is, one is the pressure drop along the liquid stream line, and the other is that along the air stream line. After consideration of void fraction, a non-dimensional basic equation is obtained, using a simple model of confluence. For each flow pattern, semi-empirical equations of the former kind of pressure drops are obtained. An empirical equation which covers the whole region of flow pattern is obtained from these equations. An empirical equation of the latter kind of pressure drop which covers the whole region is also obtained.

1. Introduction

In the previous report, experimental apparatus and two-phase flow patterns are described. In case that air streams into the upward liquid flow from the air holes arranged at the wall of the lower part of small vertical tube, pressure drop along the liquid stream line and that along the air stream line occur at the confluence. These are related with the velocity of air and liquid, and void fraction. In this report, void fraction and the pressure drop at the confluence along the liquid and air stream line will be described.

Nomenclature

A : inner cross sectional area of tube
 A_B : total area of air holes
 $A_g: f_g A$
 $A_l = (1 - f_g) A$
 d : inner diameter of tube
 d_B : diameter of air hole
 f_g : void fraction
 f_g' : ratio of cross sectional area of bubble or that of air column in annular flow to A
 $K_g = \frac{2g}{\gamma_l} \frac{(\Delta P_g - \Delta P_\sigma)}{v_{l0}^2}$
 $K_l = \frac{2g}{\gamma_l} \frac{\Delta P_l}{v_{l0}^2}$

$$N = Q_g / Q_l$$

P_{g1} : static pressure of air chamber

P_{l1} : static pressure of liquid just before confluence

P_2 : static pressure of two-phase flow just behind confluence

$$\Delta P_g = P_{g1} - P_2$$

$$\Delta P_l = P_{l1} - P_2$$

ΔP_σ : resistance to emission of air by surface tension

Q_g, Q_l : flow rates of air and liquid

$$R = \gamma_g / \gamma_l$$

$$Re_{g0} = v_{g0} d / \nu_g$$

$$Re_{l0} = v_{l0} d / \nu_l$$

$$S = v_g / v_l$$

$$v_g = v_{g0} / f_g$$

$$v_l = v_{l0} / (1 - f_g)$$

$$v_{g0} = Q_g / A$$

$$v_{l0} = Q_l / A$$

γ_g, γ_l : specific weights of air and liquid

σ : surface tension

ν_g, ν_l : kinematic coefficients of viscosity of air and liquid

2. Pressure drop at the confluence along the liquid stream line

2.1 Definition of pressure drop at the confluence along the liquid stream line

Figure 1 shows an example of measuring results of static pressure of tube taken relative to that of liquid chamber. A in Fig. 1 is the case of liquid alone. In the section of bell-mouth, the pressure drop occurs, too, with

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development of a boundary layer, though it is small compared with that of straight tube. Assuming that the equivalent length of the bell-mouth section is half the axial length of bell-mouth, the author substitutes this equivalent length for the calculated value by Langhaar, and obtains a curve A in Fig. 1. The experimental values coincide well with this curve. The air holes exist about $3.5d$ apart from the entrance, and Re_{10} in the experimental region is from about 200 to 10 000. As the so called entrance length is $0.065 Re$ in laminar flow, and so the entrance length is at least over $13d$ in the limits of these Reynolds numbers, it may be assumed that the velocity profile at the height of air hole is flat.

Even when air begins to stream in from the air holes, the relative pressure of point a in Fig. 1 does not change in ordinary cases, while those of point c and behind it change as shown in B or C. The distance between points b and c is about $3.5d$. The pressure P_2 is obtained by the straight line drawn linking the measured values of point c and behind it. In case of froth flow, the line drawn linking the measured values makes often a curve. At that time, the pressure P_2 is obtained by the straight line drawn through the upper stream half section of the part from point c to tube end. It is defined that the pressure drop at the confluence along liquid stream line is $P_{11} - P_2$, where P_{11} is the

Fig. 1 An example of measured result of static pressure

Fig. 2 Model of two-phase flow

pressure of point b and equal to that in case of no air.

2.2 Basic equation

Practical phenomenon at confluence is a very complex flow of three dimensions. This complex flow is simplified to the model shown in Fig. 2. The momentum principle is applied to the two cross sections 2 and 1 of top and bottom shown as broken line in Fig. 2. The assumptions are made as follows.

(a) Q_g which streams in from both sides has not a vertical component of velocity.

(b) The distance between the two cross sections is very small and the effects of liquid head and of the tube friction are negligible.

In some of flow patterns, a large difference of velocity between parts of liquid is found. Liquid is divided into the center part (e.g. liquid drops as shown in Fig. 2) and the side part (e.g. liquid film as shown in Fig. 2). The momentum, flow rate, average velocity of the center part passing through 2 are symbolized as M_{lc} , Q_{lc} and v_{lc} , and those of the side part are symbolized as M_{ls} , Q_{ls} and v_{ls} . Those of air passing through cross section 2 are symbolized as M_g , Q_g and v_g . Those of liquid passing through the bottom section 1 are symbolized as M_{l0} , Q_l and v_{l0} . The difference of momentum between cross sections of top and bottom is equal to the pressure difference. Then,

$$M_{lc} + M_{ls} + M_g - M_{l0} = A \Delta P_l \quad (1)$$

As ΔP_l is obtained from the pressure of the part beyond $3.5d$ apart from the confluence, the state of flow of cross section 2 is substantially the same as that of section c. If the hydraulic mean depth after confluence is large, the velocity profile at cross section 2 may be flat. However, the hydraulic mean depth of liquid film such as shown in Fig. 2 is small, and the so called entrance length of the film is shorter as a result of turbulent flow. Accordingly, the velocity profile at cross section 2 is not always flat. The definition of hydraulic mean depth m is as follows.

$$m = A_l / \pi d$$

The thickness of liquid film h is far smaller than d , then it may be said that $m \approx h$. The depth of liquid film in fish-scale type slug flow and annular flow is about $0.1d$, and accordingly m is also about $0.1d$. The distance of $3.5d$ which is that from the confluence to point c is 35 times as long as that of hydraulic mean depth, and so, the velocity profile of point c can't be flat, even in case of about 1000 in Reynolds number ($= 4mv_l/v_l$). In this case, with

the coefficient of velocity profile c_1 , next equation is made.

$$M_{ls} = \int 2\pi(\gamma_l/g)v_{ls}r^2 dr = c_1(\gamma_l/g)Q_{ls}v_{ls} \dots (2)$$

where v_{ls} : liquid velocity at the point of distance r from the tube center line.

It may be assumed that the velocity profiles other than v_{ls} are flat because of the large hydraulic mean depths.

Accordingly

$$\left. \begin{aligned} M_{lc} &= (\gamma_l/g)Q_{lc}v_{lc} \\ M_g &= (\gamma_g/g)Q_gv_g \\ M_{lo} &= (\gamma_l/g)Q_{lo}v_{lo} \end{aligned} \right\} \dots (2')$$

Meanwhile, next relations exist.

$$\frac{Q_{lc}}{v_{lc}} + \frac{Q_{ls}}{v_{ls}} + \frac{Q_g}{v_g} = A \dots (3)$$

$$Q_{lc} + Q_{ls} = Q_l \dots (4)$$

Then, from Eq. (1),

$$K_l = 2 \left\{ \left(\frac{Q_{lc}}{Q_l} + c_1 \frac{Q_{ls}S_c}{Q_lS_s} + RNS_c \right) \left(\frac{Q_{lc}}{Q_l} + \frac{S_sQ_{ls}}{S_cQ_l} + \frac{N}{S_c} \right) - 1 \right\} \dots (5)$$

where

$$v_g/v_{lc} = S_c, \quad v_g/v_{ls} = S_s$$

If there is no difference of velocity in each part of liquid, v_{lc} is equal to v_{ls} . When the velocity profiles of liquid and air are flat, Eq. (5) becomes as follows

$$K_l = 2N\{1/S + R(S+N)\} \dots (6)$$

Velocity ratio S is related with void fraction f_g as follows,

$$S = N(1/f_g - 1) \dots (7)$$

2.3 Void fraction

About the void fraction of air and liquid two-phase flow, there are studies by R.W. Rockhart and R.C. Martinelli⁽¹⁾ (simplified as L.M.), Yagi and Kato⁽²⁾, S.G. Bankoff⁽³⁾, Akagawa and Sakaguchi^{(4)~(6)}, Aoki and Inoue and others^{(7)~(8)}, and Ueda^{(9)~(10)}. All of them are about the fully developed flow, and except L.M., they concern only a vertical tube. The tube in the paper (2) is 8 mm diameter, one in the paper (8) are 9 and 5 mm diameter, and one in the paper (9) is 9 mm of diameter.

Yagi and Kato have made an experiment of annular and mist flow of water and oil in such condition as $d=10.25$ mm, $G_l=4.5\sim 250$ kg m⁻² sec⁻¹, $G_g=7\sim 40$ kg m⁻² sec⁻¹ (where G_l, G_g : weight flow rates of liquid and air), and have derived the following empirical equation, which is not nondimensional.

$$1/f_g - 1 = 1110 \left(\frac{v_{lo}}{v_{go}} \right)^{0.88} G_l^{-0.6} \mu_l^{0.2} \dots (8)$$

where

$$G_l: \text{kg m}^{-2} \text{ sec}^{-1}$$

$$\mu_l: \text{coefficient of viscosity of liquid, kg sec m}^{-2}$$

Equation (8) is shown as dotted line in Fig. 3(a) for temperature of 20°C.

The L.M. method is said to have a good accuracy specially in turbulent region ($Re_{lo} > 2000$, $Re_{go} > 2000$). In case that both air and liquid flow rates are those of turbulent region,

$$X_{lt} = (\gamma_l/\gamma_g)^{0.5} N^{-0.9} (\nu_l/\nu_g)^{0.1} \dots (9)$$

has been used as the abscissa. Applying water and air to Eq. (9), the author calculates $1-f_g$ for the function of N as shown in broken line of Fig. 3(a).

Aoki and others⁽⁸⁾ have derived the following equations. In case of bubble flow and $v_{lo} \geq 1$ m sec⁻¹, it is

$$1/f_g = 1 + 1/N \dots (10)$$

In case of slug and annular flow, it is

$$1/f_g + f_g = 2 + 1/N \dots (11)$$

These are shown in a one-dot-dash line of Fig. 3(a). Ueda⁽¹⁰⁾, referring to the above studies by Aoki and others, have obtained the following equation.

$$\left. \begin{aligned} 1/f_g &= 1.20(1 + 1/N) + v_b/v_{go} \\ v_b &= 2.40 \times 10^{-8} \frac{(\gamma_l - \gamma_g)}{\mu_l} d^2 \left(\frac{\mu_l}{\mu_g} \right)^{5/3} \end{aligned} \right\} \dots (12)$$

Fig. 3 Void fraction

In condition of writer's experiments, v_b is about 7×10 m/sec. Therefore, it is negligible. Then, Eq. (12) becomes

$$1/f_g = 1.20(1+1/N) \quad (13)$$

that is,

$$1-f_g = \frac{N+6}{6(N+1)} \quad (14)$$

From Eq. (7),

$$S = 0.2N + 1.2 \quad (15)$$

Equation (14) is shown in a solid line of Fig. 3(a). It is found by comparing these curves with each other that there are some differences between the results of those studies.

Now, the author's experiments will be described. Rapid shut valves which have been used universally for measuring the void fraction, can't be used in this experiment, because the flow is just behind the confluence. The section from $3.5d$ to $15d$ after confluence is the object of this experiment and photographs are taken by stroboscope and high speed movie camera. Photographs by stroboscope are taken for measuring the diameter of bubble or air column, but, because of the effects of convex lens on tube, the measured values need some corrections. Two kinds of methods for these corrections are employed; one is that of obtaining a calibration curve by inserting into the tube round bars whose diameters have been measured beforehand, the other is that of getting the calibration by calculation using measured values of refractive index of methylmethacrylate resin and liquid (c.f. 1st Report, Table 2). Accordingly, the flow patterns whose void fraction can be obtained by photograph are those of fish-scale type and annular flow, because, in fish-scale type slug flow, the liquid films between bubbles are very thin. With high speed movie camera, the velocity of bubble can be measured, and therefore, the void fractions of bubble, piston type, simple and fish-scale type, and long slug flow are obtained.

The ratio of cross section of bubble or air column to that of tube f_g' is obtained by calculation from the liquid thickness of side film measured with photograph in fish-scale type and annular flow. Assuming that f_g' is equal to f_g , experimental values f_g are shown in Fig. 3 (a), (b). Figure 3(a) shows the values of 6mm tubes. The experimental values of f_g coincide with Eq. (10), (11) by Aoki and Inoue or Eq. (14) by Ueda as shown in Fig. 3(a), (b), so, in this paper, Eq. (14) will be used for the equation of void fraction.

In Fig. 3(a), (b), the data on $N > 10$ are those

of annular flow or nearly such flow. The measuring errors of these regions are about ± 0.04 in the ordinate. In the region of large N and v_{10} , entrainment (drops contained in air column) must be increased. If the value of entrainment is roughly estimated from the experimental results by Anderson et al.⁽¹¹⁾ with the condition of 10 mm of tube diameter and air water two-phase upward flow, it is about 10~20% of total flow rate of liquid in that region.

Figure 3(b) indicates the case of tube No. 3 which contains gasoline. Data on water coincide with Eq. (14), but data on gasoline are less than Eq. (14) in the region of $N > 100$. The boundary velocity of air which generates liquid drops from the liquid surface is proportional to $\sqrt{\sigma/\gamma_g}$ according to M.N. Kemelman⁽¹²⁾ or proportional to $\sqrt{\sigma/\gamma_g}$ according to N.A. Moscharoff⁽¹²⁾. Therefore, it may be easy to assume that entrainment of gasoline is more than that of water. In an annular flow of tubes Nos. 20 and 21, $1-f_g$ is slightly more than Eq. (14), but indication of this is omitted.

The values of liquid thickness of the side part in annular or fish-scale type slug flow and the values of bubble velocity measured by high speed camera are the mean values in the section from $3.5d$ to $15d$ after confluence. No significant difference is found between the upper stream part and the lower.

Fig. 4 Bubble flow

In spite of the fact that those studies referred to above are about the fully developed flow and all of them are not of small tubes, the differences between the author's results and the results by others are not larger than the differences between them.

2.4 Flow patterns and pressure drop at the confluence

(1) Bubble flow The experimental values of K_l in bubble flow of tap water are shown in Fig. 4. Figure 4(a) shows tubes Nos. 3 and 20 (both of them, $d_B=0.5d$), and Fig. 4(b) shows tubes Nos. 7 and 21 (both of them, $d_B=0.25d$). Assuming that bubble velocity in the same cross section is uniform, that is $c_1=1$, $S_c=S_s=S$, we have Eq. (6). Because N is small in bubble flow, and $R \ll 1$,

$$K_l \doteq 2N/S \quad (16)$$

Aoki and others⁽⁸⁾ have presumed that the relative velocity of liquid and gas can be neglected at $v_{l0} > 1$ m/sec. In that case, $S=1$ is admitted, and next equation is derived from Eq. (16).

$$K_l \doteq 2N \quad (17)$$

In case of small v_{l0} , the relative velocity of liquid and air can't be neglected. The relative velocity is affected by the diameters of bubble and tube, and the analysis of it is not simple. Now, the author will apply Eq. (14) here.

Substituting Eq. (14) into Eq. (16), then

$$K_l = 10N/(N+6) \quad (18)$$

Equations (17), (18), are shown in Fig. 4. It is found that experimental values exist between these two curves.

(2) Slug flow In this section, piston type, simple slug, long piston type and fish-scale type slug flow will be discussed. Let the piston type flow to represent the former three kinds of flow, and the author will discuss the piston type flow and fish-scale type slug flow. The model of piston type flow is shown in Fig. 5. There are four kinds of momentum as follows. The signs in parentheses show the flow rate and

Fig. 5 Model of piston flow

velocity of liquid or air.

M_{lc} : liquid slug part; center part (Q_{lc} , v_{lc})

M_{ls} : liquid part between air slug and the wall of tube; side part (Q_{ls} , v_{ls})

M_{gB} : large bubble (Q_{gB} , v_{gB})

M_{gs} : small bubble in liquid slug (Q_{gs} , v_{gs})

Because Q_{gs} is very small and v_{gs} is not large, M_{gs} can be neglected. Then,

$$v_{gB} = v_{g0}/f_g \quad (19)$$

$$v_{lc} = v_{l0} + v_{g0} \quad (20)$$

From continuity equation,

$$v_{ls} = (v_{lc} - f_g' v_{gB}) / (1 - f_g') \quad (21)$$

As $1 - f_g'$ is about 0.35⁽⁵⁾, Q_{lc} is small if the air slug is not so long. As v_{lc} is less than v_{lc} ,

(a), (b) Slug flow (part 1)

(c) Slug flow (part 2)

Fig. 6

M_{ls} is considerably less than M_{lc} . Accordingly, considering only M_{lc} for momentum of liquid, next equation is obtained.

$$M_{lc} = (\gamma_l/g) Q_{lc} v_{lc} = (\gamma_l/g) Q_l v_{lc} \quad \dots\dots\dots(22)$$

Also, because N is small, M_{gB} can be neglected. Then, from Eq. (1), next equation is obtained.

$$K_l = 2 \left(\frac{Q_l}{A} \frac{v_{lc}}{v_{l0}^2} - 1 \right) \quad \dots\dots\dots(23)$$

Substituting Eq. (20) into Eq. (23), we get

$$K_l = 2N \quad \dots\dots\dots(24)$$

This is equivalent to the flow of $S=1$. Solid lines in Fig. 6(a), (b), (c) indicate Eq. (24).

In case of fish-scale type flow, the liquid which flows at center of tube is only a thin film of liquid between bubbles, and that velocity is equal to v_{gB} . Accordingly, it can be assumed that $Q_{lc} = 0$, $S_c = 1$, $Q_{ls} = Q_l$, and next equation is derived from Eq. (5).

$$K_l = 2 \{ (c_1/S_s + RN)(S_s + N) - 1 \} \quad \dots\dots\dots(25)$$

Assuming the velocity profile as follows

$$v_{lsr} = c_2(d/2 - r),$$

then we get

$$Q_{ls} = \int_{r_i}^{d/2} 2\pi r v_{lsr} dr = 2\pi c_2 h \left(\frac{d}{4} - \frac{h}{3} \right)$$

where r_i : radius of bubble

$$h = d/2 - r_i$$

also

$$Q_{ls} = \pi(d^2/4 - r_i^2) v_{ls} = \pi h(d - h) v_{ls}$$

from Eq. (2),

$$c_1 Q_{ls} v_{ls} = 2\pi \int_{r_i}^{d/2} v_{lsr}^2 r dr = \pi c_2^2 h^3 \left(\frac{d}{3} - \frac{h}{2} \right)$$

When c_2 is canceled from the above equations,

$$c_1 = \frac{4}{3} \frac{(1 - h/d) \{1 - (3/2)(h/d)\}}{\{1 - (4/3)(h/d)\}^2}$$

Because $h/d \ll 1$, the terms of not less than second power of h/d can be neglected, then

$$c_1 \doteq \frac{4}{3} \left(1 + \frac{1}{6} \frac{h}{d} \right)$$

indicating more roughly, we can state $c_1 \doteq 1.33$.

Because the shearing force at the side interface in fish-scale type slug flow is weak, and so, the velocity profile may be similar to parabola, it can be said that c_1 is less than 1.33, even if it is that of a fully developed flow.

Now, putting $S=S_s$ and substituting Eq. (15) into Eq. (25), we get

$$K_l = 2 \left\{ \frac{5c_1 N}{N+6} + c_1 - 1 + \frac{6}{5} RN(N+1) \right\} \quad \dots\dots\dots(26)$$

Broken lines in Fig. 6(a), (b), (c) are those of $c_1 = 1.33, 1.15, 1.0$. The experimental values of tubes Nos. 3 and 20 are shown in Fig. 6(a), those of Nos. 7 and 21 shown in Fig. 6(b), and

that of No. 20 with liquid other than tap water shown in Fig. 6(c). In the region of $v_{l0} \geq 0.35$ m/sec, the flow pattern is almost simple or fish-scale type slug flow, and the experimental values in Fig. 6(a)~(c) fall into the curve of Eq. (26) of $c_1 = 1.15$. Accordingly, in the region of $v_{l0} \geq 0.35$ m/sec, Eq. (26) of $c_1 = 1.15$ is the approximative equation. In the region of $v_{l0} < 0.1$ m/sec, the greater part of the flow is a long slug flow, and some part is a piston type flow. Experimental values of this region lie between Eq. (24) and Eq. (26).

(3) Froth flow In froth flow, the turbulent force is larger than the surface tension while the shearing force at the interface between liquid and air is not so strong as that in annular flow, accordingly, it can't sustain the liquid film upward flowing on the wall of tube. So, the upward flowing air is obstructed with the liquid film falling down the air column, and the upward velocity drops suddenly. The pressure of air increases so much that the air pushes aside the liquid and streams out. This process is repeated. It frequently happens that, observed with the high speed camera, the instantaneous velocity of the apex of distorted bubble is less than half the value of v_{g0} . Froth flow is one of strong turbulence and variation.

Defining M_g and M_l as those momentums averaged by time which passes through the cross section 2 in Fig. 2, then

$$A \Delta P_l = M_l + M_g - M_{l0} \quad \dots\dots\dots(27)$$

Considering the velocity profile of air and liquid, and the variation with time, correction factors c_3' and c_3'' are used. That is,

$$M_g = c_3' A \frac{\gamma_g}{g} \frac{v_{g0}^2}{f_g}$$

$$M_l = c_3'' A \frac{\gamma_l}{g} \frac{v_{l0}^2}{1 - f_g}$$

If $c_3' = c_3'' = c_3$ is assumed,

$$M_g + M_l = c_3 A \left(\frac{\gamma_g}{g} \frac{v_{g0}^2}{f_g} + \frac{\gamma_l}{g} \frac{v_{l0}^2}{1 - f_g} \right) \quad \dots\dots\dots(28)$$

From (2)', (27) and (28),

$$K_l = 2 \left\{ c_3 \left(\frac{1}{1 - f_g} + R \frac{N^2}{f_g} \right) - 1 \right\} \quad \dots\dots\dots(29)$$

If Eq. (14) can be applied here, Eq. (29) becomes as follows.

$$K_l = 2 [c_3 \{ 5N/(N+6) + 1 + 1.2RN(N+1) \} - 1] \quad \dots\dots\dots(30)$$

Solid lines of Fig. 7(a), (b), (c) indicate Eq. (30) of $c_3 = 1.0, 1.2, 1.5$. Figure 7(a) shows tubes Nos. 3 and 20, and Fig. 7(b) shows tubes Nos. 7 and 21. The cases of the liquid other than tap

water in tube No. 20 are shown in Fig. 7(c), and have no significant difference from others. All of Fig. 7(a), (b), (c) indicate that experimental values lie between $c_3=1$ and 1.5.

(4) Annular flow In annular flow, the momentum of Q_g is large, and so, the effect of the velocity profile of Q_{ls} becomes small. Then, the author assumes that $c_1=1.2$, here. Q_{ls} is the entrainment (that is, drops of liquid in the

air column), and if $S_e=1$ is assumed, next equation is obtained from Eq. (5).

$$K_l = 2 \left\{ \left(\frac{Q_{lc}}{Q_l} + \frac{1.2}{S_e} \frac{Q_{ls}}{Q_l} + RN \right) \times \left(\frac{Q_{lc}}{Q_l} + S_e \frac{Q_{ls}}{Q_l} + N \right) - 1 \right\} \dots (31)$$

Now

$$\frac{Q_{ls}}{A(1-f_g')} = v_{ls}$$

Because the volume of entrainment is remarkably smaller than air volume

$$Q_g / A f_g' \approx v_g$$

When

$$Q_{lc} / Q_l = E,$$

$$S_e = \frac{v_g}{v_{ls}} = \frac{1}{1-E} \frac{1-f_g'}{f_g'} N \dots (32)$$

If Eq. (14) is used for f_g' , from Eq. (32),

$$S_e = 0.2(N+6)/(1-E) \dots (33)$$

From Eq. (31)

$$K_l = 2 \left[\frac{E+6(1-E)^2}{(N+6)} + RN \right] \times \{E+1.2(N+1)\} - 1$$

Because E is small, terms of not less than second power of E can be neglected. As N is large, and R is small, E/N can be neglected, too.

$$K_l \approx 2.4N \left[\frac{5.16+1/N+(N-5)E}{(N+6)} + R(N+1) \right] \dots (34)$$

Equation (34) of $E=0, 0.1, 0.2$ is represented by the solid lines of Fig. 8(a), (b). Experimental values lie between two lines of $E=0$ and $E=0.1$.

Fig. 8 Annular flow

2.5 Equation of pressure drop covering the whole region

In this section, pressure drops of whole region of all tubes are summarily discussed. Empirical equations which cover the whole region are by all means necessary to the design of carburetor. Figure 9(a) is a collection of the experimental values of tube No. 3. Solid lines in Fig. 9(a) indicate Eq. (6) with constant parameter S . If S were an independent variable, next equation would be obtained from Eq. (6).

$$\frac{\partial K_l}{\partial S} = 2N \left(-\frac{1}{S^2} + R \right)$$

$$\frac{\partial^2 K_l}{\partial S^2} = 4N/S^3 > 0$$

Accordingly, when $S=1/\sqrt{R} \doteq 28$, K_l becomes the smallest value, and the solid line exists at extreme right side of this figure. Practically, it is easily seen in Fig. 9(a) that S is a function of N , except in the case of very small v_{l0} . Summarily speaking, experimental values lie between $S=1$ and $S=28$.

Referring to the semi-empirical equations discussed in above sections, the author obtains the next empirical equation which covers the whole region of N .

$$K_l = 2.4N[(4.8 + 0.04N)/(N+6) + 1.1RN] \quad \dots\dots\dots(35)$$

A dotted line in Fig. 9(a) shows Eq. (35). Flow pattern map with $v_{l0} \sim N$ is shown as a reference below this figure.

Figure 9(b) shows the case of tubes Nos. 1, 2, 4, 5 which use tap water in the region of $v_{l0}=0.089 \sim 0.35$ m/sec, and tube No. 3 which uses gasoline. The solid line and dotted line are obtained respectively for $R=1.22 \times 10^3$ (water) and $R=1.82 \times 10^3$ (gasoline), and these curves generally agree with the experimental values. The case of $v_{l0}=0.35 \sim 2.1$ m/sec which is not shown is also similar to Fig. 9(b). The cases of tubes Nos. 6, 8, 9, 12 are identical. Figure 9(c) shows tube No. 22 (2 mm diameter) of ordinary temperature and 40°C. The case of No. 23 is also similar to No. 22. The effect of temperature is found to be insignificant, in the experiments of tubes Nos. 3, 20, 22 in 10°C and 40°C.

Summing up these results, it can be said that Eq. (35) can be applied to the case of $d=6 \sim 2$ mm, $v_{l0}=0.2 \sim 4$ m/sec, temperature 10°C $\sim$ 40°C with water and the case of ordinary temperature with gasoline.

Now, let's discuss about the case of $v_{l0} < 0.2$ m/sec in which K_l is larger than that of Eq. (35). The first reason for this peculiarity may lie in the phenomena of piston type flow and long slug flow which are discussed in Eq. (24). But, the following reason may also be mentioned. As long as v_{g0} is not very great and v_{l0} is small, the flow pattern is piston type or long slug flow, and the static pressures along the passage are activity caused by buoyancy as shown in curve C of Fig. 1. However, just after passing through the air holes, large bubble can't completely leave the air holes on account of adhesion, and accordingly, buoyancy doesn't occur there. This phenomenon is notable in the flow in annuli. It

(a) Pressure drop at the confluence along liquid stream line

(b), (c) Pressure drop at the confluence along liquid stream line

Fig. 9

is possible more or less in case of such a tube as used in this experiment. The pressure P_2 is obtained from the pressures of the points after c, and so, when the pressure curve C is acclivity, $P_{11}-P_2$ may be larger than true pressure drop, on account of the length of dead buoyancy. When v_{10} is small, $P_{11}-P_2$ is also small. Accordingly, the error of $P_{11}-P_2$ can't be neglected and K_l becomes larger than that of large v_{10} . It may be for this reason that K_l of smaller tube becomes much larger as shown in Fig. 9(c).

3. Pressure drop at the confluence along air stream line

Defining P_{g1} as the total pressure of air chamber (c.f. Fig. 2) and ΔP_g as $P_{g1}-P_2$, the author examines ΔP_g . ΔP_g consists of three kinds of pressure drops; the first is the resistance caused by the adhesion and surface tension of the bubble, the second is the pressure drop in flowing through air holes, and the third is that due to confluence. When ΔP_g is expressed non-dimensionally as above described K_l , the effects due to the adhesion and surface tension are not negligible, especially at small v_{10} and small d_B . When, at constant Q_l and no Q_g , the pressure of air chamber is increased slowly, finally air begins to stream out from the air holes.

The relations between ΔP_g just before the streaming out of air and the diameter of air hole are shown in Fig. 10. The solid line in Fig. 10 indicates the calculated pressure value ΔP_{g0} which is needed for a bubble to leave the hole. This pressure ΔP_{g0} is also calculated from the next equation.

Fig. 10 Pressure needed for bubble to leave the hole

$$\Delta P_{g0} = 4\sigma/d_B \quad \dots\dots\dots (36)$$

Experimental values lie below this solid line. The reason for this and the reason for dispersion may be very small variation of air chamber pressure, roughness of the surface of air holes, and the error of value of properties of the materials. The tubes of 6 mm and 3 mm diameter are slightly worse in roughness than that of 2 mm caused by the difficulty in machining of methylmethacrylate resin, and maximum height of surface roughness of the holes of these tubes is about one hundredth of mm.

The pressure difference ΔP_{g0} is found in the limits of these experiments not to be affected by the water velocity. Even after air begins to blow out, in bubble flow or slug flow, air is blowing out discontinuously, and so the effect of surface tension still exists. Therefore, considering the difference between the experimental value and that of Eq. (36), the author sets ΔP_g in the state of blowing out equal to 90% of ΔP_{g0} . In annular flow, ΔP_g is large, and therefore ΔP_g can be neglected.

Now, non-dimensional pressure drop K_g will be discussed. We set

$$K_g = \frac{2g}{\gamma l} \frac{\Delta P_g - \Delta P_g}{v_{10}^2}$$

where

$$\Delta P_g = 3.6\sigma/d_B$$

Fig. 11 Pressure drop at the confluence along air stream line

Values of K_g of tube No. 20 are shown in Fig. 11(a). These values are similar to K_l of tube No. 20, and no effect of v_{l0} is seen in this figure. The total areas of air holes of tube No. 20 are equal to the cross area of tube (c.f. Table 1 in 1st Report). Figure 11(b) indicates the effects of the total areas of air holes in 6 mm diameter tubes. In case of large air holes such as those of No. 6, K_g is nearly equal to Eq. (35) of K_l , but in case of small air holes such as those of No. 9, K_g becomes much larger. When the air holes are as large as those of No. 6, the pressure of air chamber P_{g1} is only ΔP_g larger than P_{l1} .

Now, we set next equation.

$$\Delta P_g - \Delta P_g = \Delta P_l + c_4 \frac{\gamma_g}{2g} v_B^2$$

where

$$v_B = Q_g / A_B$$

$$c_4 = \text{const.}$$

Dividing both sides by $\gamma_l v_{l0}^2 / (2g)$, we get next equation.

$$K_g = K_l + cR \left(\frac{A}{A_B} \right)^2 N^2 \dots\dots\dots (37)$$

For $c=2.2$, substituting the total air holes areas of each tube into Eq. (37), we get the curve shown in Fig. 11(b). Therefore, next equation is an empirical equation of K_g .

$$K_g = K_l + 2.2R \left(\frac{A}{A_B} \right)^2 N^2 \dots\dots\dots (38)$$

4. Summary and conclusions

When air is streaming into the liquid flowing upward in a small vertical tube, pressure drop occurs at the confluence. This pressure drop and the void fraction are discussed. The conclusions are as follows.

(1) Void fraction in slug flow and annular flow just behind the confluence can be indicated

by Eq. (14), which has been applied to a fully developed flow.

(2) A non-dimensional basic equation (5) which indicates the pressure drop at the confluence along liquid stream line is obtained.

(3) Semi-empirical equations of this pressure drop of each flow pattern are obtained. Equation (17) or (18) for bubble flow, Eq. (24) for piston type, simple slug and long slug flow, Eq. (26) for fish-scale type slug flow, Eq. (30) for froth flow, Eq. (34) for annular flow are obtained.

(4) Empirical equation (35) of this pressure drop, which covers the whole region of N , is obtained.

(5) Empirical equation (38) of the pressure drop at the confluence along air stream line, which covers the whole region of N , is obtained.

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