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Response Analysis of Piping Systems to Multi-Inputs* (Response Estimation and its Fluctuation)

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For the safety of nuclear power plants, piping systems of the primary loop should be designed as earthquake-resistant structures. Piping, supported at both ends by some supporting structures, are the most fundamental configuration in a nuclear power plant.

The dynamic response of a piping system to two random inputs is analyzed theoretically and the characteristics of response spectrum of such a system are discussed. And also the way of superposing its amplification factor from those of single-input models, which were already developed, is discussed. For design, an approximate formula of estimating amplification factor is given based on modal analysis method for pipings.

Experimental studies are done by an analog computer. One hundred modified white noise signals are used as pseudo-earthquakes to avoid the effect of sampling or the locality of a particular earthquake.

Quantitative discussions of response spectrum of this model are made.

1. Introduction

Aseismic design of equipment and piping systems in nuclear power plants is very important, to ensure the safety of them and to prevent the radiation hazard to the public during and after a violent earthquake.

Fundamental procedure on these subjects has been reported by the author's group⁽¹⁾. After that, the main practice^{(2) (3)} has been established by the two successive sub-committees for Aseismic Design of Nuclear Facilities (ADNF) in JSME. The theory supporting this practice was reported in the references (4) and (5) precisely. Response calculus of mechanical components has been analyzed as a two-degrees-of-freedom system (one freedom is building and the other is mechanical component) by one of the authors and Sato⁽⁶⁾. In many cases, a machine structure (mechanical components including the piping system) is far smaller than the building structure in weight. So, in this case, the reaction force from the machine structure to the building is assumed to be negligible in the analysis of the whole system.

Here, the aim of this paper is to establish a practical procedure for the response analysis of a

machine structure, which is supported at two or more supporting points, such as a piping system, by using the result obtained through the method employing a two-degrees-of-freedom model, i.e., superposing response values in a scheme from the response spectrum table of a two-degrees-of-freedom system. Fundamental approach to a two inputs problem was partly discussed in some papers⁽⁷⁾, and the basic idea in this paper is similar to them. But, as described above, the aim of this paper is to establish the practical schemes of calculi for response analysis.

Furthermore, intrinsic errors of transient response analysis are also discussed by means of response calculi to many pseudo-earthquake inputs using an analog computer. Fluctuations of amplification factors to pseudo-earthquakes in an ensemble increase as a system multiplex. These fluctuations give an essential influence on response calculi, but a theoretical study on the fluctuations of amplification factor will be discussed in next report.

2. Fundamental equations

In this chapter, fundamental equations are derived in order to analyze the vibration characteristics of a bridged system, i.e. a model which consists of a primary cooling piping and two supporting buildings in a nuclear power plant.

The following assumptions are made in the

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analysis:

1) Vibration analysis of piping system is based on the theory of linear beams arbitrarily shaped in a space. Therefore, the piping system can be approximated by a lumped mass model.

2) Force transmission from building to piping is one-directional.

3) Vibration of buildings is shear-type one.

4) Damping ratio of piping for each mode of vibration is sufficiently small.

These assumptions are satisfied in many practical cases. Hereafter, two approaches are adopted in parallel. One is direct analysis of a linear beam and the other is that of a lumped mass system corresponding to the linear beam. For the building system, vibration characteristic is based on the assumption of 3). The model discussed in this paper consists of a piping and two supporting buildings (in Figs. 1 and 2).

Equation of motion for building i is

$$\ddot{y}_{bi}(t) + 2\zeta_{bi}\omega_{bi}\dot{y}_{bi}(t) + \omega_{bi}^2 y_{bi}(t) = -\alpha(t) \quad \dots\dots\dots (1)$$

where ζ_{bi} and ω_{bi} are the damping ratio and undamped natural frequency of the building i : $y_{bi}(t)$ and $\alpha(t)$ are the relative displacement of the building i to the ground motion and the input

Fig. 1 Model of continuous system

Fig. 2 Model of multi-masses-springs system

acceleration to the whole system.

2.1 Analysis by linear beam theory (continuous case)

When ξ and $y(\xi, t)$ are the curvilinear coordinate along the members of piping from building 1 and the relative displacement of piping to a ground motion, the equation of motion for pipings is

$$\frac{\partial^2 y(\xi, t)}{\partial t^2} + D_\xi y(\xi, t) = -\alpha(t) \quad \dots\dots\dots (2)$$

where, partial differential operator $D_\xi = a^2 \frac{\partial^4}{\partial \xi^4}$ for

bending or $D_\xi = -b^2 \frac{\partial^2}{\partial \xi^2}$ for torsional and longitudinal vibration. The constant a^2 or b^2 is $EI/\rho A$, $GI_p/\rho I_p$, $EA/\rho A$ for bending, torsional, longitudinal vibration; and EI , GI_p , EA , ρA and ρI_p are bending, torsional, tensile rigidity, line density of pipings (beams) and moment of gyration respectively.

Boundary conditions for bending vibration are

$$\left. \begin{aligned} \frac{\partial y(0, t)}{\partial \xi} &= 0, \quad \frac{\partial y(L, t)}{\partial \xi} = 0 \\ y(0, t) &= y_{b1}(t), \quad y(L, t) = y_{b2}(t) \end{aligned} \right\} \quad \dots\dots\dots (3)$$

Displacements for torsional and longitudinal vibrations at $\xi=0$ and L are all none, where L is the total length of a piping system.

When the coordinates are adopted as in Fig. 3, the equations of motion are modified as follows. In this figure, $z(\xi, t)$, $u(\xi, t)$ and $v(\xi, t)$ are the absolute displacement of a beam, the ground displacement and the quasi-static deflection of a beam respectively, and among these, there are the following relations,

$$\left. \begin{aligned} z(\xi, t) &= y(\xi, t) + u(\xi, t) \\ y(\xi, t) &= w(\xi, t) + v(\xi, t) \end{aligned} \right\} \quad \dots\dots\dots (4)$$

Here, $w(\xi, t)$ is the dynamic deflection of a beam and equals to the solution of forced vibration for both fixed end points, and $v(\xi, t)$ has the following characteristic

$$D_\xi v(\xi, t) = 0 \quad \dots\dots\dots (5)$$

in its own definition. From Eqs. (4) and (5), Eq. (2) becomes

Fig. 3 Coordinates of continuous system

$$\frac{\partial^2 w(\xi, t)}{\partial t^2} + D_i w(\xi, t)$$

$$= -\alpha(t) - g_1(\xi) \ddot{y}_{b1}(t) - g_2(\xi) \ddot{y}_{b2}(t) \dots (6)$$

where $g_i(\xi)$ is a statical deformation function of the point ξ on the beam applied with a unit displacement at support i and fixed at the other one. From $g_i(\xi)$, $v(\xi, t)$ can be written by the relative displacements of the buildings,

$$v(\xi, t) = g_1(\xi) y_{b1}(t) + g_2(\xi) y_{b2}(t) \dots (7)$$

Here, Eq. (7) was already used in deriving Eq. (6). Equation (7) can be solved for relative piping movement by means of modal analysis method, that is,

$$w(\xi, t) = \sum_{j=1}^{\infty} X_j(\xi) \varphi_j(t) \dots (8)$$

Then the equation of motion of j -th mode becomes

$$\ddot{\varphi}_j(t) + 2\zeta_j \nu_j \dot{\varphi}_j(t) + \nu_j^2 \varphi_j(t) = -\{\alpha_{0j} \alpha(t) + \beta_{0j} \ddot{y}_{b1}(t) + \gamma_{0j} \ddot{y}_{b2}(t)\} \dots (9)$$

where a damping term is added to the original equation, and it is also assumed that the damping ratio ζ_j is small. Furthermore, α_{0j} , β_{0j} , γ_{0j} are exciting coefficients referring to the j -th mode to external uniform acceleration and effects of motions of supporting buildings 1 and 2. Equation (6) is consistent with Eq. (18) for a multi-degrees-of-freedom system in its form. It is recognized that the vibration behavior of a beam can be described by means of accelerations of the ground and both supporting buildings (on the other hand, the equations of motion of a beam can also be described by means of displacements only). The ν_j is the j -th natural frequency of the beam and α_{0j} , β_{0j} , γ_{0j} are

$$\left. \begin{aligned} \alpha_{0j} &= \frac{1}{L} \int_0^L [X_j(\xi)]_{acc} d\xi \\ \beta_{0j} &= \frac{1}{L} \int_0^L X_j(\xi) g_1(\xi) d\xi \\ \gamma_{0j} &= \frac{1}{L} \int_0^L X_j(\xi) g_2(\xi) d\xi \end{aligned} \right\} \dots (10)$$

where $[\]_{acc}$ means the component of a direction excited. Equation

$$\ddot{\varphi}_j(t) + 2\zeta_j \nu_j \dot{\varphi}_j(t) + \nu_j^2 \varphi_j(t) = -\{\beta_{0j} \ddot{z}_{b1}(t) + \gamma_{0j} \ddot{z}_{b2}(t)\} \dots (9)'$$

is also obtained from Eq. (9) by transferring the right hand side of it, where $\ddot{z}_{bi}(t)$ is the absolute acceleration of the building i . The following relation

$$g_1(\xi) + g_2(\xi) = 1 \dots (11)$$

or

$$\beta_{0j} + \gamma_{0j} = \alpha_{0j} \dots (12)$$

for j -th mode was used from Eq. (9) to Eq. (9)'. From Eq. (9)', the absolute acceleration of the beam $\ddot{z}(\xi, t)$ is obtained as

$$\ddot{z}(\xi, t) = \begin{cases} \sum_{j=1}^{\infty} X_j(\xi) \left[\frac{\nu_j^2}{\nu_j'} \int_0^t \{\beta_{0j} \ddot{z}_{b1}(\tau) + \gamma_{0j} \ddot{z}_{b2}(\tau)\} \times e^{-\zeta_j \nu_j (t-\tau)} \sin \{\nu_j' (t-\tau) + 2\varphi_j\} d\tau \right] & : 0 < \xi < L \\ \ddot{z}_{b1}(t) & : \xi = 0 \\ \ddot{z}_{b2}(t) & : \xi = L \end{cases} \dots (13)$$

where ν_j' and φ_j are the j -th damped natural frequency and the phase angle of the beam vibration, and these are

$$\nu_j' = \nu_j \sqrt{1 - \zeta_j^2}, \quad \varphi_j = \tan^{-1}(\zeta_j / \sqrt{1 - \zeta_j^2}) \dots (14)$$

$X_j(\xi)$, ν_j , $g_1(\xi)$, etc. in Eqs. (8), (9), (10) can be generally obtained by means of numerical computations of matrix method or of model experiments.

2.2 Analysis by lumped mass theory (multi-degrees-of-freedom case)

The authors treat the so-called "shear spring model" as a multi-masses-springs system. In this case, the main procedure for derivation of fundamental equations and their solutions are almost similar to those of continuous case. For simplicity, it is assumed that only the building 2 moves (see Fig. 4). Then, equations of motion of i -th mass are

$$\begin{cases} m_i \ddot{z}_i(t) + k\{2y_i(t) - y_{i-1}(t) - y_{i+1}(t)\} = 0 \\ k\{2v_i(t) - v_{i-1}(t) - v_{i+1}(t)\} = 0 \end{cases} \dots (15)$$

in an N -masses-springs system. Here, m_i is the mass of i -th point and z_i , y_i , v_i have the same meanings as in section 2.1; k is a spring constant between the adjacent masses and all spring constants are assumed to equal to each other. And suffices 0 and $N+1$ mean the buildings 1 and 2 respectively. Equation (15) becomes

$$m_i \{\ddot{y}_i(t) + \alpha(t)\} + k\{2y_i(t) - y_{i-1}(t) - y_{i+1}(t)\} = 0 \dots (16)$$

using the relation $\ddot{z}_i(t) = \ddot{y}_i(t) + \alpha(t)$. Furthermore, Eq. (16) becomes

$$m_i \{\ddot{w}_i(t) + \ddot{v}_i(t)\} + k\{2w_i(t) - w_{i-1}(t) - w_{i+1}(t)\} + k\{2v_i(t) - v_{i-1}(t) - v_{i+1}(t)\} = -m_i \alpha(t) \dots (17)$$

Fig. 4 Coordinates of multi-masses-springs system

using the relation $y_i(t) = w_i(t) + v_i(t)$, and consequently the equations of motions of the whole system are obtained as

$$\ddot{w}_i(t) + \frac{k}{m_i} \{2w_i(t) - w_{i-1}(t) - w_{i+1}(t)\} \\ = -\alpha(t) - \frac{h_i}{H} \ddot{y}_{b1}(t) - \left(1 - \frac{h_i}{H}\right) \ddot{y}_{b2}(t) \dots \dots (18)$$

where $w_i(t)$, $v_i(t)$, $\alpha(t)$ have the same meanings as in section 2.1 and here the relation $\ddot{v}_i(t) = (h_i/H) \ddot{y}_{b1}(t) + (1 - h_i/H) \ddot{y}_{b2}(t)$ was already used. Thus the basic equation of modal analysis method for a lumped mass system was obtained same as Eq. (6).

3. Response spectrum estimation in statistical approach

In this chapter, responses of the system and computational scheme of response spectrum are studied by means of statistical methods in stationary random processes. In chapter 2, as the fundamental equation of a multi-masses-springs system was shown to be essentially similar to that of continuous system, here the authors discuss only the continuous system. Earthquake accelerations are assumed to be stationary random processes with zero mean. The analysis here is limited to the response acceleration within the range of $0 < \xi < L$ of the beam [for $\xi = 0$ or $\xi = L$, as equations of motion become those of a two-degrees-of-freedom system, for example, reference (6) can be cited]. Auto-correlation functions of the responses of pipings are equal to auto-covariance functions of them and can be written as

$$E\{\ddot{z}(\xi, t_1) \ddot{z}(\xi, t_2)\} = R_z(\xi, \tau) \\ = \int_{-\infty}^{\infty} |H_z(\xi, i\omega)|^2 A(\omega) e^{i\omega\tau} d\omega, \tau = t_1 - t_2 \dots \dots \dots (19)$$

where $A(\omega)$ is the power spectrum of the ground acceleration. Power spectrum of the response $\ddot{Z}(\xi, \omega)$ is written as

$$\ddot{Z}(\xi, \omega) = |H_z(\xi, i\omega)|^2 A(\omega) \dots \dots \dots (20)$$

then from Eq. (20), a Wiener-Khinchine relation⁽⁸⁾ for the beam response is obtained as follows:

$$\ddot{Z}(\xi, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_z(\xi, \tau) e^{-i\omega\tau} d\tau \dots \dots \dots (21)$$

Transfer function of the system $H_z(\xi, j\omega)$ in Eq. (19) or (20) is

$$H_z(\xi, i\omega) = \sum_{j=1}^{\infty} X_j(\xi) [H_{zj}(i\omega) \{\beta_{0j} H_{zb1}(i\omega) \\ + \gamma_{0j} H_{zb2}(i\omega)\}] \dots \dots \dots (22)$$

where $H_{zj}(i\omega)$ and $H_{zb1}(i\omega)$ ($i=1, 2$) are transfer functions of absolute acceleration through pipings in j -th mode and both buildings. Each of them has the same form, for example,

$$H_{zj}(i\omega) = \frac{2\zeta_j \nu_j(i\omega) + \nu_j^2}{(i\omega)^2 + 2\zeta_j \nu_j(i\omega) + \nu_j^2} \dots \dots \dots (23)$$

If the ground acceleration is assumed to be a Gaussian process and its transfer function is also the same, the amplification factor of the pipings λ is estimated as

$$\lambda = \sqrt{\frac{R_z(\xi, 0)}{R_a(\xi, 0)}} \dots \dots \dots (24)$$

This formula has been given by Tajimi⁽⁹⁾ and now is widely used in aseismic design. Here, $R_a(\xi, 0)$ is auto-correlation function of ground acceleration. From Eq. (24), λ^2 is obtained as

$$\lambda^2 = \frac{\int_{-\infty}^{\infty} |H_z(\xi, i\omega)|^2 |H_g(i\omega)|^2 \frac{(i\omega)^2}{\{\psi_1(i\omega) + 1\}^2} \frac{1}{\{\psi_2(i\omega) + 1\}^2} k d\omega}{\int_{-\infty}^{\infty} |H_g(i\omega)|^2 \frac{(i\omega)^2}{\{\psi_1(i\omega) + 1\}^2} \frac{1}{\{\psi_2(i\omega) + 1\}^2} k d\omega} \dots \dots \dots (25)$$

when the input acceleration is pseudo-earthquake, where $H_g(i\omega)$ is transfer function of the ground acceleration and $[(i\omega)^2 / \{\psi_1(i\omega) + 1\}^2] \cdot [1 / \{\psi_2(i\omega) + 1\}^2] \sqrt{k}$ is acceleration spectrum⁽³⁾⁽⁶⁾ of the basement. Here, the discussions on the $|H_z(\xi, i\omega)|^2$ are made on the basis of Eq. (22).

From Eq. (22), $|H_z(\xi, i\omega)|^2$ is

$$|H_z(\xi, i\omega)|^2 = \left| \sum_{j=1}^{\infty} X_j(\xi) [H_{zj}(i\omega) \{\beta_{0j} H_{zb1}(i\omega) + \gamma_{0j} H_{zb2}(i\omega)\}] \right|^2 \\ + \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} X_j(\xi) X_k(\xi) [H_{zj}(i\omega) H_{zk}(i\omega) \{\beta_{0j} H_{zb1}(i\omega) + \gamma_{0j} H_{zb2}(i\omega)\} \\ \times \{\beta_{0k} H_{zb1}(i\omega) + \gamma_{0k} H_{zb2}(i\omega)\}] \dots \dots \dots (26)$$

Tajimi already evaluated λ of a single-freedom system to a single input. In his paper, $|H_{zj}(i\omega) H_{zk}(i\omega)|$ in [] was evaluated as Q_{pq}^2 , and the values of λ_{pq}^2 introduced by integrating Q_{pq}^2 were tabulated in Table 3 in his paper. This value turns to zero, if the phase difference between the responses of j -th and k -th modes is $\pi/2$. If each mode is separable, the phase difference between a particular mode, which is in resonance, and the adjacent modes is close to $\pi/2$ under low damping condition. So, in such a condition, the contribution of this term to whole amplification factor is usually small. The authors followed this scheme and neglected the second term of Eq. (26), then

$$|H_z(\xi, i\omega)|^2 \doteq \sum_{j=1}^{\infty} X_j^2(\xi) |H_{zj}(i\omega)|^2 \{ \beta_{0j}^2 |H_{zb1}(i\omega)|^2 + \gamma_{0j}^2 |H_{zb2}(i\omega)|^2 + 2\beta_{0j}\gamma_{0j} |H_{zb1}(i\omega)| |H_{zb2}(i\omega)| \} \dots (27)$$

is introduced. In this relation again cross terms $[H_{zb1}(i\omega)H_{zb2}(i\omega)]$ are involved. If the vibration characteristics of both buildings are similar, the relation can be reduced to a single-input problem, and if the natural frequencies of both are separated enough, again the results of integrating these terms can be neglected with a similar discussion. The relation

$$|H_z(\xi, i\omega)|^2 \doteq \sum_{j=1}^{\infty} X_j^2(\xi) |H_{zj}(i\omega)|^2 \{ \beta_{0j}^2 |H_{zb1}(i\omega)|^2 + \gamma_{0j}^2 |H_{zb2}(i\omega)|^2 \} \dots (28)$$

also shows that the scheme of "root of sum of squares" would be effective for modal and response analysis to two inputs. But the authors should mention that this scheme is not always, on the safe side, because the relation of Eq. (28) is not $\geq$ but $\doteq$. And if buildings 1 and 2 are different floors of a same building, some restriction should be added to the above mentioned.

4. Response calculus by an analog computer

Amplification factor of pipings to two inputs is analyzed by an electronic analog computer. Natural earthquakes and stationary pseudo-earthquakes are adopted in analog computer circuits of a piping-buildings system and their responses are calculated to these earthquakes. From these responses, amplification factors are obtained and compared with those to a single input.

4.1 Generation of pseudo-earthquake

The procedure for generation of a pseudo-earthquake is as follows (see Fig. 5): This is almost similar method to Sato's, that is, the output signal of a single-mass-spring system, which is the ground model, is used for a stationary Gaussian pseudo-earthquake when the acceleration of the basement to the ground model is a band limited Gaussian white noise. From stated above, the power spectrum of a stationary pseudo-earthquake $A(\omega)$ is described as

$$A(\omega) = |H_g(i\omega)|^2 |H_f(i\omega)|^2 k \dots (29)$$

where k is a constant; $H_g(s)$ and $H_f(s)$, ($s=i\omega$), are the transfer functions of the ground accelera-

Fig. 5 Flow chart of generation of pseudo-earthquake

tion and band limited filter of basement. These are

$$H_g(s) = \frac{2\zeta_g\omega_g s + \omega_g^2}{s^2 + 2\zeta_g\omega_g s + \omega_g^2} \dots (30)$$

and

$$H_f(s) = \frac{s^2}{(\psi_1 s + 1)^2} \frac{1}{(\psi_2 s + 1)^2} \dots (31)$$

where ζ_g and ω_g are the damping ratio and the undamped natural frequency of the ground model, $1/\psi_1$ and $1/\psi_2$ are low and high cut-off frequencies. Time duration of pseudo-earthquake is 20 seconds per single wave, one hundred waves are produced by using the parameters in Table 1 and recorded on magnetic tapes in a data recorder.

4.2 Multi-masses-springs system used in analog computation

An N -degrees-of-freedom linear chain system is adopted as multi-equi-masses-equi-springs system. Five masses are taken to analyze up to fifth mode in response waves. The following analog computer circuits are considered to make a comparison of schemes of superposing of each mode in amplification factor. The first method (direct method) is directly composed of the following equations of motion and the second one (modal method) is superposed on each mode obtained from the original equations of motion.

Equation of motion for building i is described as Eq. (1) and those for a multi-masses-springs system are

$$\left. \begin{aligned} \ddot{y}_1 + 2\zeta\omega\dot{y}_1 + 2\omega^2 y_1 - \omega^2 y_2 &= -\alpha + \omega^2 y_{b1} \\ \ddot{y}_2 + 2\zeta\omega\dot{y}_2 - \omega^2 y_1 + 2\omega^2 y_2 - \omega^2 y_3 &= -\alpha \\ \ddot{y}_3 + 2\zeta\omega\dot{y}_3 - \omega^2 y_2 + 2\omega^2 y_3 - \omega^2 y_4 &= -\alpha \\ \ddot{y}_4 + 2\zeta\omega\dot{y}_4 - \omega^2 y_3 + 2\omega^2 y_4 - \omega^2 y_5 &= -\alpha \\ \ddot{y}_5 + 3\zeta\omega\dot{y}_5 - \omega^2 y_4 + 2\omega^2 y_5 &= -\alpha + \omega^2 y_{b2} \end{aligned} \right\} \dots (32)$$

by direct method, where $\omega^2 = k/m$, $\zeta = c/2\sqrt{mk}$ and k , m , c are spring constant, mass, damping coefficient to the ground respectively. The solution of Eq. (32) is assumed to have the form of

$$y_i = \sum_{j=1}^5 B_i^{(j)} \varphi_j(t) \quad (i=1,2,\dots,5) \dots (33)$$

then the equations of motion for modal method are

$$\ddot{\varphi}_j + 2\zeta_j \nu_j \dot{\varphi}_j + \nu_j^2 \varphi_j = -I_j \alpha + \omega^2 J_j y_{b1} + \omega^2 K_j y_{b2} \quad (j=1,2,\dots,5) \dots (34)$$

for each mode, and I_j , J_j and K_j in Eq. (34) are

$$I_j = \frac{\sum_{i=1}^5 B_i^{(j)}}{\sum_{i=1}^5 B_i^{2(j)}}, \quad J_j = \frac{B_1^{(j)}}{\sum_{i=1}^5 B_i^{2(j)}}, \quad K_j = \frac{B_5^{(j)}}{\sum_{i=1}^5 B_i^{2(j)}} \dots (35)$$

Table 1 Parameters of ground model for pseudo-earthquake

$T_g = 1.00$	sec	$\psi_1 = 3.00$	sec/rad
$\zeta_g = 0.30$		$\psi_2 = 0.008$	sec/rad

where I_j , J_j and K_j are exciting coefficients to the motions of ground, buildings 1 and 2 respectively. These values correspond to α_{0j} , β_{0j} , γ_{0j} in Eq. (10). Furthermore, ν_j and ζ_j are the undamped natural frequency and the damping ratio of the j -th mode vibration of a five-masses-springs system. Absolute acceleration of i -th mass $\ddot{z}_i(t)$ is

$$\begin{aligned}\ddot{z}_i(t) &= \sum_{j=1}^5 B_i^{(j)} \ddot{\phi}_j(t) + \alpha(t) \\ &= \sum_{j=1}^5 B_i^{(j)} [\ddot{\phi}_j(t) + I_j \alpha(t)] \quad \dots\dots\dots (36)\end{aligned}$$

where if $\ddot{z}_{\phi j}(t)$ is put as

$$\ddot{z}_{\phi j}(t) \equiv \ddot{\phi}_j(t) + I_j \alpha(t) \quad \dots\dots\dots (37)$$

, Eq. (37) becomes

$$\ddot{z}_i(t) = \sum_{j=1}^5 B_i^{(j)} \ddot{z}_{\phi j}(t) \quad \dots\dots\dots (38)$$

Table 2 Mode function of five-masses-springs system $B_i^{(j)}$

$j \backslash i$	1	2	3	4	5
1	1.000	1.732	2.001	1.732	1.000
2	1.000	1.001	0.000	-1.001	-1.900
3	1.000	0.003	-1.000	0.003	1.000
4	1.000	-0.996	0.000	0.996	-1.000
5	1.000	-1.726	1.980	-1.726	1.000

Table 3 Natural frequencies and damping ratios of five-masses-springs-system

j	ν_j rad/sec	ζ_j
1	6.274276	0.0386600
2	12.12767	0.0200101
3	17.15221	0.0141409
4	21.00922	0.0115479
5	23.43164	0.0103535

Table 4 Exciting coefficients of five-masses-springs system

$j \backslash i$	$I_j \equiv \frac{\sum_{i=1}^5 B_i^{(j)}}{\sum_{i=1}^5 B_i^{2(j)}}$	$J_j \equiv \frac{B_1^{(j)}}{\sum_{i=1}^5 B_i^{2(j)}}$	$K_j \equiv \frac{B_5^{(j)}}{\sum_{i=1}^5 B_i^{2(j)}}$
1	0.622	0.083	0.083
2	0.000	0.250	-0.250
3	0.335	0.333	0.333
4	0.000	0.251	-0.251
5	0.044	0.084	0.084

Table 5 Predominant periods of natural earthquakes

Earthquake	Predominant period of ground T_g sec
El Centro NS	0.20
Taft NS	0.20
Saitama EW	0.65
Kushiro NS	0.32

Here, the authors have named the method of superposing as in Eq. (38) the "method of sum of waves" and the methods which use the summation of absolute maximum values of each mode and the root of sum of square values respectively the "absolute summation method" and the "RMS method". Amplification factors corresponding to these above methods are named "sum of waves", "sum of absolute values" and "RMS value" respectively. $B_i^{(j)}$ in the above equations is a mode function in vibration. Several constants and mode shapes, which are necessary for analog computation, are as follows. Mode functions are given in Table 2, and the damped natural frequencies and the damping ratios of five-masses-springs system are in Table 3 when $T_{m1}=1.0$ sec (in this case ω in Eq. (32) becomes $\omega=12.1391$ rad/sec) and $\zeta=0.02$. The values of I_j , J_j , K_j are given in Table 4.

4.3 Selection of parameters and the results of analog computations

The parameters used in our calculation by an analog computer are intentionally set around $T_{b1}/T_g = T_{m1}/T_g = 1.0$ where T_g is the predominant period of ground motion and T_{bi} ($i=1,2$) is the undamped natural frequency of building i . And also, for a buildings-piping system, the parameters are set within the range $1/2 < T_{m1}/T_{b1} < 2.0$. The values of predominant period of natural earthquake records, that is, a period which gives the maximum amplification factor of the absolute acceleration, are shown in Table 5.

One hundred pseudo-earthquakes were produced from a low-frequency noise generator and recorded in a data recorder to use them respectively. Computing time for each input is 20 sec, and one hundred waves are input successively for a set of parameters.

In order to obtain the knowledge on the effect of small difference of both input waves, the ratio of the natural period of buildings is taken as $T_{b1}/T_{b2}=1.00, 1.01, 1.10, 1.50, 2.00$ or 2.73 . The last value 2.73 is the ratio of the first natural period to the third one of piping consisting of the five-masses-springs lumped system. The choice of this value means simultaneous occurrences of quasi-resonances of both modes. Furthermore, a vital point in analog computation is using many earthquakes and selecting parameters in a wide range to obtain the general characteristics of a two inputs problem.

Figure 6 shows the response factors of absolute acceleration of the middle point of the masses $\ddot{z}_{3\max}/\alpha_{\max}$ to natural earthquakes (El Centro NS, 18 May 1940; Taft NS, 21 July 1952; Kushiro

NS, 24 Dec. 1961; Saitama EW, 14 Feb. 1956)*. T_{m1} , T_{b1} and T_{b1}/T_{b2} are selected as the parameters and the condition $T_{m1}=T_{b1}$, which is a severe condition for pipings, is used. When T_{b1} coincides with T_g and T_{b1}/T_{b2} is equal to unity, the response factor becomes maximum similarly to the case of a single input problem. For the most severe condition, the response factor to Kushiro earthquake shows the largest one. The authors interpret this as follows: Stationarity and power concentration near the predominant period of Kushiro earthquake⁽¹¹⁾ are stronger than those of the other earthquake. A change in the pattern of response spectrum curve with an increase in T_{b1}/T_{b2} is not so remarkable and the amplification factor tends to a constant value when T_{b1}/T_{b2} exceeds 1.5. The difference between natural earthquakes and pseudo-earthqua-

(a) Response spectra of absolute acceleration to natural earthquakes

(b) Response spectra of absolute acceleration to El Centro earthquake

Fig. 6

Fig. 7 Response spectra of absolute acceleration

* The digitized data in SERAC report⁽¹⁰⁾ were used by converting to analog signals with an optical curve tracer.

kes, which will be described later, is a peak number in power spectral density. Natural earthquake has more peaks than pseudo-earthquake produced above and consequently the pattern of response spectrum curve of natural one is more complicated than that of pseudo-one. When $T_{m1}=T_{b1}=T_g$, amplification factors of pseudo-earthquakes almost equal to those of natural ones, but when $T_{m1}=T_{b1}>T_g$, amplification factors of natural ones are larger than those of pseudo-ones. These results would be caused as explained above. Figure 7 shows the response spectrum computed by means of the method of sum of waves for $T_{m1}=T_{b1}$ to natural earthquakes. The resonance can be seen to turn rapidly off near $T_{b1}/T_{b2}=1.10$. As the system is symmetrically composed, there should be a peak for $T_{m1}=T_{b2}$ same as for $T_{m1}=T_{b1}$, but no peak appears in this figure because of the wide interval for T_{b1} in computation. To study this point concretely, responses were computed and response spectra were made as in Fig. 8. This figure shows a two-peak-characteristic, and this is the most typical characteristic of two input problem. When the first natural period of pipings coincides with that of building 1 (i.e. $T_{m1}=T_{b1}$), the amplification factor decreases monotonously with an increase in T_{b1}/T_{b2} from 1.00, but when $T_{m1}\neq T_{b1}$, it is not monotonous with an increase in T_{b1}/T_{b2}

Fig. 8 Response spectra of absolute acceleration

Fig. 9 The difference of mean values of response spectra due to the difference of superposing method

This means that the influence of the building 2 on the whole system in amplification factor, is not always monotonous with an increase in T_{b1}/T_{b2} when $T_{m1} \neq T_{b1}$. This state can be also seen in Fig. 8. Figure 9 shows the amplification factors of several schemes of superposing each mode in modal analysis method and in direct method. These curves are average values from one hundred amplification factors. Figure 10 shows the relation between the convergence of average value and sample size in amplification factor. About twenty samples guarantee an almost constant average value. In view of this, it would be said that the reliability of amplification factors, which have been calculated using one or two samples, is considerably low. Figure 11 shows a histogram produced from the

data for the parameter set of the worst condition to pipings. And for a set of other parameters, almost a similar tendency is obtained. Relative dispersion of amplification factor is about 0.25~0.30. Many reasons can be thought of for the fluctuations of amplification factor. Dominant reason is power spectral fluctuations of short sample waves. The difference in amplification factors caused by the difference in the schemes of superposing all modes is smaller than the width in their fluctuations, because of a large fluctuation due to a large filtering effect in a buildings-piping system near the resonance of a single mode of vibration.

5. Conclusions

The authors obtained the following conclusions for the two input problem of a bridged piping. Piping systems in aseismic design were examined as a two inputs problem and then the analytical solution was obtained. Fundamental idea of response calculus was derived from the solution. The effect of the vibration characteristics of a buildings-piping system on the amplification factors was surveyed by analog computer analysis of four well known earthquake records and one hundred pseudo-earthquakes. And the results were discussed from a statistical point of view.

(1) The amplification factor of piping becomes the largest one when the period of piping coincides with that of building, according to the result of a single input problem which was studied by Sato.

(2) And the amplification factor is the largest when the periods of both supporting buildings coincide with each other. Then the worst condition for piping occurs when these above three periods coincide with the predominant period of ground motion.

(3) As for the comparison between the schemes of superposing the responses of each mode, the scheme of the square root of the sum of squares is said to be theoretically reasonable in a statistical sense, but it is not always on the safe side.

(4) The fluctuations of amplification factors to one hundred pseudo-earthquakes obtained by an analog computer which are bigger than the difference between them come from the difference in the schemes of superpositions. The reason may be considered that the responses of higher mode are rather low under the effect of filtering high frequency components of ground motion through building structures. But in some cases, the scheme of superposition might be more significant than the effect of fluctuations, *i.e.* unbalanced motions of two

Fig. 10 Moving average and standard deviation of response spectra

Fig. 11 Frequency distribution of amplification factor

supporting buildings may excite a second mode of vibration when a resonance of first and second modes occurs or $T_{b1}=T_{m1}$, $T_{b2}=T_{m2}$.

(5) The sensitivity of the amplification factor to a difference of natural frequencies of supporting buildings is considered as follows: the difference within 1 % has almost no effect, but if it reaches up to 50%, two inputs can be treated as independent inputs. The effect is the strongest under about 10% difference.

The authors have felt strongly the necessity for further analytical study on the non-stationary response analysis, especially the effect of time duration of inputs on the fluctuation of response curves. The result of their study in this field will be reported in a next paper.

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The authors should note in addition that the computers used are NEC-A300 for analog computa-

tions and HITAC-5020 in the Computing Center of University of Tokyo for digital computations.

References

- (1) Shibata, H. and Shigeta, T., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol. 29, No. 200 (1963-4), p. 783.
- (2) JSME, *Report on Aseismic Design Method of Reactor Piping Systems* (in Japanese), (Vol. 1-1962), (1963-12), p. 405.
- (3) JSME, *Report on Aseismic Design of Reactor Vessels and Piping Systems* (in Japanese), (1967), Chapt. 7, (1969-3), p. 126.
- (4) Watari, A., et al., *Proceeding of JSME 1967, Semi-int. Symp.*, Vol. 1 (1967-9), p. 215.
- (5) Shibata, H., *Science of Machine* (in Japanese), Vol. 19, No. 1 (1967-1), p. 173; Vol. 19, No. 2 (1967-2), p. 311.
- (6) Sato, H., *Rept. of the Inst. of Ind. Sci.*, Univ. of Tokyo, (in Japanese), Vol. 15, No. 1 (1965-11), p.109.
- (7) Shimogo, T., *Science of Machine* (in Japanese), Vol. 22, No. 6 (1970-6), p. 907.
- (8) Miller, K. S., *Engineering Mathematics I* (translated by Sato and Fujii), (in Japanese), (1962-10), p. 168, Kyoritsu Co.
- (9) Tajimi, H., *Rept. of the Inst. of Ind. Sci.*, Univ. of Tokyo, (in Japanese), Vol. 8, No. 4 (1959-3), p. 46.
- (10) *SERAC Report* (Faculty of Engng., Univ. of Tokyo), (in Japanese), No. 6 (1966-10), p. 251 and others.
- (11) Kanai, K., et al., *Report on the Relation between Damages of Buildings and the Record of SMAC in Kushiro at Hiroo-oki Earthquake* (in Japanese), (1969-3), p. 130.