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The Toms Effect of a Dilute Polymer Solution and Its Application*

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We analyzed the turbulent pressure loss in the pipe flow of a dilute polymer solution by a method similar to that of Kármán for Newtonian fluid or to that of Metzner et al. for non-Newtonian fluid, and found that the friction factor in the pipe flow was expressed by two non-dimensional numbers: R_* and S ($R_* = 2\rho RV/\eta_v$, $S = \eta_c/\rho R^2$, where η_v : viscosity, η_c : cross viscosity, ρ : density, R : pipe radius, V : mean velocity), and we showed that the analytical expression agreed with the experimental data well. In order to avoid the mechanical degradation, the concentrated polymer solution were injected into the water at the delivery side of the pump. Consequently, we obtained the same effect as the natural head flow.

By way of an example, we tried an experiment on a hydraulic conveyor. Then the concentrated polymer solution was injected into a flyash slurry. Apparently, the Toms effect may be also recognized in the case of hydraulic conveyor.

1. Introduction

The earliest observation of a remarkable reduction of the pipe friction in the turbulent flow of a dilute polymer solution was reported by B. A. Toms⁽¹⁾. Thus, Toms indicated that the pipe friction in the turbulent flow of polyethyl methacrylate in monochlorobenzene was much lower than that of the solvent only. With the development and the manufacturing of water soluble resin such as polyethylene oxide, Fabula⁽²⁾ and many other investigators^{(3)~(7)} verified the Toms effect of dilute polymer solutions by adding a small amount of polymer powder to water. Both for engineering and industrial field the phenomenon that the friction factor can be reduced by adding an extremely small quantity of polymer powder is quite interesting, and the practical application is widely expected.

The published reports concerned with the Toms effect in the pipe flow are almost experimental, and the outline of the experimental results is as follows.

(i) The smaller the diameter of the pipe is, the larger the drag reduction will be.

(ii) The drag reduction increases with an increasing concentration initially but eventually it takes a certain value which is independent of the con-

centration.

(iii) The drag reduction does not always occur in the turbulent flow. Namely, there is an onset of the drag reduction.

Moreover, the quantitative expression of the reduction of the turbulent pressure loss has hardly been obtained yet. W. A. Meyer⁽⁸⁾ has observed that the factor of the velocity profile for a Newtonian fluid in the turbulent pipe flow is a function of the frictional velocity and has expressed the pipe friction factor quantitatively. Tomita⁽⁹⁾ has analyzed and formularized the Toms effect from a viewpoint that the viscoelastic effect suppresses turbulence in the flow. Namely, Tomita indicated that the mixing length is decreased owing to the polymer additive showing the normal stress effect and therefore, the shear stress at the tube wall is decreased. After that, Tomita et al.⁽¹⁰⁾ verified the analytical result with the experiment. However, this analysis is not fully satisfactory yet as for the quantitative expression as the analytical result involves an unknown quantity which is the rheological constants involved in the constitutive equation, and is difficult to measure. As described above, it is a matter of importance for explaining the Toms effect analytically and quantitatively to choose the most suitable model showing the dynamical behaviour of a dilute polymer solution and to measure material constants involved in it.

In this paper, the turbulent pipe friction was measured by using three water soluble resins (polyethylen oxide) with various concentrations. Also,

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the coefficient of viscosity of these polymer solutions was measured by the capillary viscometer and the coaxial cylindrical viscometer and the normal stress was measured by the parallel plate rheogoniometer and the jet method apparatus⁽¹¹⁾. Consequently, a "simple Reiner-Rivlin model" was adopted as the constitutive equation for a dilute polymer solution. This model was adopted from the point of view that the Toms effect can be explained by it though it has weaknesses in the rheological field. The authors report that the turbulent pressure loss in the pipe flow of a dilute polymer solution could be expressed analytically by this model by means of a method similar to that of Kármán for the Newtonian fluid or to that of Metzner et al. for the power law fluid, and it was verified by the experimental results that the pipe friction reduction can be expressed quantitatively (very recently, Metzner et al.⁽¹²⁾ analyzed it for the viscoelastic fluid by the Maxwell model by means of

a similar method).

In general, the polymer solution is degraded in a high shear flow⁽¹³⁾⁽¹⁴⁾. Therefore, if the liquid is transported by the pump in an industrial application of the Toms effect, the effect of the polymer additives is lost due to the mechanical degradation when the solution flows through the pump. Then, concentrated polymer solutions were injected into the water at the delivery side of the pump in order to avoid the mechanical degradation in the pump. Consequently, we obtained the same effect as the natural head flow. By way of a practical example, the injecting method was used for a flyash slurry in order to apply to the hydraulic conveyor. Consequently, we obtained the above effect, in other words, a remarkable loss reduction.

2. The physical properties of dilute polymer solutions

Up to now, various constitutive equations have been proposed, but the experimental facts can not be described by these equations. Though it is considered that dilute polymer solutions possess the normal stress effect as the polymer melt and the concentrated polymer solution, it is very difficult to measure rheological constants involved in the constitutive equation because the concentration is very low and the shear rate in the pipe flow is high. However, if the constitutive equation which shows the flow property of a dilute polymer solution can not be selected or the rheological constants involved in the constitutive equation can not be measured, it is impossible to obtain the quantitative expression for the pressure loss, etc. Then, the authors measured the viscoelastic properties of dilute polymer solutions by various rheometers.

2.1 Materials

Three kinds of polyethylene oxides were used the solute. Their trade names are WSR 205, PEO 10 and PEO 16, of which the molecular weights are 6×10^5 , 2×10^6 and 3×10^6 respectively. These concentrations are shown in Table 1.

2.2 The coefficient of viscosity and the normal stress

The coefficient of viscosity measured by the viscometer is shown in Fig. 1(a), (b), (c). The viscosities for the shear rate below 10^2 sec^{-1} were measured by the coaxial cylindrical viscometer (inner cylinder diameter 18ϕ , outer diameter 26ϕ) and the capillary viscometer was used for the shear rate over 10^2 sec^{-1} . As is shown in Fig. 1, the viscosities are independent of the shear rate below 0.4, 0.08, 0.04% for WSR 205, PEO 10 and PEO 16 respectively. Namely, the viscosity of these solutions is considered to be the same as that of a Newtonian fluid. Therefore, the viscosities used for Reynolds numbers in order

Table 1 Solution

Material	Pipe Dia. mm	Concent. %	Temperature °C	η_c gr/cm	S
Peo 16	6.03	3×10^{-3}	12.5	3.3×10^{-6}	3.63×10^{-5}
		4.5×10^{-3}		5.4×10^{-6}	5.94×10^{-5}
	12.9	3.5×10^{-3}	12.5	3.98×10^{-6}	9.56×10^{-6}
		4.5×10^{-3}		5.40×10^{-6}	1.30×10^{-5}
		6.25×10^{-3}		8.1×10^{-6}	1.95×10^{-5}
		1.25×10^{-2}		2.1×10^{-5}	5.05×10^{-5}
	16.1	6.25×10^{-3}	12.5	8.1×10^{-6}	1.25×10^{-5}
		1.25×10^{-2}		2.1×10^{-5}	3.24×10^{-5}
		2.50×10^{-2}		6.1×10^{-5}	9.42×10^{-5}
		5.0×10^{-2}		2.2×10^{-4}	3.40×10^{-4}
	20.0	6.25×10^{-3}	12.5	8.1×10^{-6}	8.06×10^{-6}
		1.25×10^{-2}		2.1×10^{-5}	2.09×10^{-5}
		2.50×10^{-2}		6.1×10^{-5}	6.06×10^{-5}
		5.0×10^{-2}		2.2×10^{-4}	2.18×10^{-4}
Peo 10	6.03	1.25×10^{-2}	11.0	2.6×10^{-6}	2.86×10^{-5}
		2.50×10^{-2}		1.21×10^{-5}	1.33×10^{-4}
	12.9	1.25×10^{-2}	11.0	2.6×10^{-6}	6.25×10^{-6}
		2.50×10^{-2}		1.21×10^{-5}	2.91×10^{-5}
		5.00×10^{-2}		5.1×10^{-5}	1.23×10^{-4}
	16.1	2.50×10^{-2}	11.0	1.21×10^{-5}	1.87×10^{-5}
		5.00×10^{-2}		5.1×10^{-5}	7.87×10^{-5}
		1.0×10^{-1}		1.6×10^{-4}	2.47×10^{-4}
		1.5×10^{-1}		2.65×10^{-4}	4.09×10^{-4}
	20.0	2.50×10^{-2}	11.0	1.21×10^{-5}	1.20×10^{-5}
		5.0×10^{-2}		5.1×10^{-5}	5.07×10^{-5}
		1.0×10^{-1}		1.6×10^{-4}	1.59×10^{-4}
		1.5×10^{-1}		2.65×10^{-4}	2.63×10^{-4}
WSR 205	6.03	2.5×10^{-2}	21.5	5.7×10^{-7}	6.27×10^{-6}
		5.0×10^{-2}		1.88×10^{-6}	2.07×10^{-5}
	12.9	2.5×10^{-2}	21.5	5.7×10^{-7}	1.37×10^{-6}
		5.0×10^{-2}		1.88×10^{-6}	4.52×10^{-6}
		1.0×10^{-1}		7.40×10^{-6}	1.78×10^{-5}
		1.5×10^{-1}		1.87×10^{-5}	4.50×10^{-5}
	16.1	2.5×10^{-2}	21.5	5.70×10^{-7}	8.80×10^{-7}
		5.0×10^{-2}		1.88×10^{-6}	2.90×10^{-6}
		1.0×10^{-1}		7.40×10^{-6}	1.14×10^{-5}
		1.5×10^{-1}		1.87×10^{-5}	2.88×10^{-5}
	20.0	2.5×10^{-2}	21.5	5.70×10^{-7}	5.66×10^{-7}
		5.0×10^{-2}		1.88×10^{-6}	1.87×10^{-6}
		1.0×10^{-1}		7.40×10^{-6}	7.35×10^{-6}
		1.5×10^{-1}		1.87×10^{-5}	1.86×10^{-5}

to arrange the data were measured by the Ubbelohde type viscometer.

Subsequently, the normal stress which characterizes the flow of polymer solution and polymer melt was measured. As an example, we obtained the relations between the normal stress difference and the shear rate as shown in Fig. 2. These values were measured by the parallel plate type rheogoniometer for the shear rate below 10^3 sec^{-1} and the jet method apparatus over 10^3 sec^{-1} . Then, the normal stress

difference ($P_{11} - P_{22}$) is converted to the rheological constant η_e involved in the constitutive equation (refer to section 3.2). The results are shown in Fig. 3(a), (b), (c). As is shown in Fig. 3, η_e is independent of the shear rate below 0.3, 0.06 and 0.03% for WSR 205, PEO 10 and PEO 16 respectively. Therefore, it can be considered that η_e is constant regardless of the shear rate within the range of the turbulent pipe flow experiment. Also, the normal stresses or η_e of the dilute concentration were obtained by extrapolation, because there was not any proper method for the normal stress measurement of the dilute solution at a high shear rate.

3. Experiments

3.1 Experimental apparatus

A reservoir with the dimensions $91 \times 145 \times 74$ (cm) was used as the head tank. Test pipes were set horizontally about 300 cm under the head tank. These are three polyvinylchloride tubes of 1.29, 1.61 and 2.0 cm in diameter and 400 cm in length and a brass tube of 0.6 cm in diameter and 300 cm in length. It was confirmed that these were hydraulically smooth by the preliminary experiment with pure water. Pressure tapes with 0.1 cm diameter were provided at the distance of 150 cm (50 cm for the brass tube) from the entrance and at the distance of 150 cm behind it (200 cm for the brass tube). Pressure was measured by the mercury or water manometer. The solutions were dissolved for a few days. Their physical properties are listed in Table 1. The solutions used once were poured out in order to avoid any degrada-

Fig. 1 Relation between the viscosity and the concentration

Fig. 2 Relation between $(P_{11} - P_{22})$ and shear rate

Fig. 3 Relation between the cross viscosity and the concentration

tion. Then, the liquid level reduction in the head tank is at most 2 cm and the error due to it can be ignored.

3.2 Experimental results

Since the dilute polymer solutions exhibit a Newtonian behaviour as was evident in Chapter 2, the experimental results were arranged for the relations between the pipe friction factor λ and Reynolds number Re . An example of them is shown in Fig. 4. The following are concluded from Fig. 4:

(i) For a polymer, the Toms effect increases with an increasing concentration of solution, and with a decreasing tube diameter. However, the Toms effect saturates when the concentration or the tube diameter reaches a certain value.

(ii) The drag reductions are classified into two types. One is that the friction factor agrees with Kármán-Nikuradse's curve until a certain Reynolds number and its reduction begins on the Reynolds number over its value, and the other is that the onset and the turbulent transition are not clear (it may occur on a higher Reynolds number) and the Toms effect begins.

4. Analytical consideration

It was reported as an analytical treatment that the Reynolds stress relaxed in comparison with that of Newtonian fluid as the normal stress effect may suppress the turbulence⁽⁹⁾⁽¹⁰⁾. Authors express the Toms effect quantitatively by means of expanding the analytical method of Kármán⁽¹⁵⁾ or Metzner et al.⁽¹⁶⁾. Then, the turbulent pipe flow is divided into the laminar sublayer in the neighbourhood of the tube wall, the turbulent core in the center of the tube and the transition zone in which two layers entangle. As was evident already in Chapter 2, the coefficient of viscosity of the dilute polymer solutions is the same as that of Newtonian fluid and at the same time, the flow of the dilute polymer solutions shows the normal stress effect and the physical properties expressing it, that is, {normal stress / (shear rate)²}, are independent

Fig. 4 Experimental results

of the shear rate. Therefore, authors adopt a "simple Reiner-Rivlin model" as the constitutive equation

$$\tau = -p\delta + \eta_v \mathbf{A} + (1/2)\eta_c(\mathbf{A} \cdot \mathbf{A}) \quad (1)$$

where τ : stress tensor, p : static pressure, δ_{ij} : Kronecker's delta, $\mathbf{A}$: rate of deformation tensor, η_v : coefficient of viscosity (constant), η_c : coefficient of cross viscosity (constant).

The following are reasons why authors used this model.

(i) The normal stress effect can be explained by it.

(ii) It is convenient for the analysis of the three dimensional flow.

(iii) It is very difficult to adopt many other models as the analysis is complex in other flows except the simple shear flow in this case.

However, this model has its weaknesses in the rheological field as follows.

(i) Weissenberg's assumption is not satisfied.

(ii) It is a pure viscous model which does not show any elastic effect.

4.1 Analysis

The authors analyze the Toms effect using Eq. (1) by a method similar to that of Kármán for Newtonian fluid or to that of Metzner et al. for non-Newtonian fluid.

Commonly, the turbulent pipe flow can be divided into the following three layers.

(1) Laminar sublayer: this layer is influenced by the viscosity only and is independent of the tube diameter.

(2) Transition zone: this zone is influenced by the viscosity as much as by the turbulence.

(3) Turbulent core: this layer is influenced by the turbulence. The velocity in the tube u is expressed as follows.

$$u/u^* = f_1(\xi, S, Z) \quad (2)$$

In the laminar sublayer,

$$u/u^* = f_2(SZ^2, \xi Z) \quad (3)$$

In the turbulent core,

$$(u_m - u)/u^* = f_3(\xi, S) \quad (4)$$

where

$$\xi = y/R, \quad S = \eta_c/\rho R^2, \quad Z = \rho u^* R/\eta_v, \quad u^* = \sqrt{\tau_w/\rho}$$

R : tube radius, ρ : fluid density, τ_w : shear stress at the tube wall, y : distance from the tube wall, u_m : maximum velocity at the tube axis, V : mean velocity. Though Eq. (4) is valid only in the turbulent core, if it is applied over the entire cross section of the tube, the volumetric flow rate Q is as follows, $Q = \pi R^2 V = 2\pi \int_0^R u r dr$ and from Eq. (4)

$$V = u_m - \frac{2u^*}{R^2} \int_0^R f_3(\xi, S) r dr \quad (5)$$

Now, $f_3(\xi, S)$ is dimensionless from Eq. (4) and both, the left hand side and the second term on the right

side in Eq. (5), have the dimension of the velocity. Therefore,

$$\int_0^R f_3(\xi, S) r dr = (\text{const.}) R^2$$

that is,

$$V = u_m - (\text{const.}) u^* \quad \text{or} \quad (u_m - V)/u^* = P_s \quad (6)$$

From Weisbach-Darcy equation,

$$V \sqrt{\rho/\tau_w} = V/u^* = \sqrt{8/\lambda} \quad (7)$$

At the tube axis, Eq. (2) may be written as follows

$$u_m/u^* = f_1(1, S, Z) = F_1(Z, S) \quad (8)$$

Suppose that the velocity gradient in the laminar sublayer is linear,

$$u_m/u^* = k \xi Z f_2(SZ^2) \quad (9)$$

By considering Eq. (9)

$$\begin{aligned} u/u^* &= k Z \xi a_m S^m Z^{2m} = k a_m S^m Z^{1+2m} \xi \\ &= f_2(\alpha S^m Z^{1+2m} \xi) \end{aligned} \quad (10)$$

and from Eqs. (6), (7) and (8)

$$u_m/u^* = P_s + \sqrt{8/\lambda} = F_1(Z, S) \quad (11)$$

In the outer parts of the turbulent core, Eqs. (4) and (5) are assumed to be valid. Thus,

$$f_2(\alpha S^m Z^{1+2m} \xi) = F_1(Z, S) - f_3(\xi, S) \quad (12)$$

since

$$u/u^* = u_m/u^* - (u_m - u)/u^*$$

It seems that η_c , ρ and R^2 are constant when the polymer concentration and the tube diameter are decided, and by arranging the order of two sides

$$f_{2s}(Z\xi) = F_{1s}(Z) - f_{3s}(\xi) \quad (13)$$

The specific form of $F_{1s}(Z)$ can be determined by a procedure of involving the partial differentiation of Eq. (13) with respect to Z and to ξ^n as follows

$$\partial F_{1s}(Z)/\partial Z = A_s/Z$$

and integrating

$$u_m/u^* = F_{1s}(Z) = A_s \ln Z + B_s \quad (14)$$

Substituting Eq. (14) into Eq. (11) and arranging,

$$\begin{aligned} \frac{1}{\sqrt{\lambda}} &= 0.814 A_s \log R_s \sqrt{\lambda} + \frac{\sqrt{2}}{4} \\ &\times (B_s - P_s) - 0.614 A_s \end{aligned} \quad (15)$$

where A_s , B_s and P_s are functions of S only, and rewriting Eq. (15) as follows

$$\frac{1}{\sqrt{\lambda}} = A_{1s} \log R_s \sqrt{\lambda} + B_{1s} \quad (16)$$

where A_{1s} and B_{1s} are functions of S only. At this point, Meyer et al.⁽⁸⁾ reported that the value of A in the equation of velocity distribution $u/u^* = A \log(yu^*/\nu) + B$ is the same as with a Newtonian fluid, that is, the mixing length is constant. Also, Seyer et al.⁽¹²⁾ described f_3 in Eq. (12) as a function of ξ only. However, considering that Meyer et al. measured the velocity distribution by the Pito tube and that Eq. (5) was obtained by integrating Eq. (4) over the entire cross section of the tube, Eq. (16) is given as follows

$$\frac{1}{\sqrt{\lambda}} = \frac{2}{1 - aS^f} \log R_s \sqrt{\lambda} - \frac{0.8}{1 - bS^f} \quad (17)$$

where

$$A_{1s} = 2/(1 - aS^f), \quad B_{1s} = -0.8/(1 - bS^f) \quad (18)$$

in the case of $S=0$, Eq. (17) agrees with the Kármán-Nikuradse's equation for Newtonian fluid. It is evident from the analytical results that the pipe friction factor in the dilute polymer solutions is dependent on the dimensionless numbers R_e and S .

4.2 The determination of A_{1s} and B_{1s}

A_{1s} and B_{1s} were determined by the experimental results as follows.

Exchanging Eq. (18)

$$1 - 2/A_{1s} = aS^e, \quad 1 - 0.8/B_{1s} = bS^f$$

The relations between $\log(1 - 2/A_{1s})$, $\log(1 - 0.8/B_{1s})$ and $\log S$ are described by linear line, respectively.

By this figure, A_{1s} and B_{1s} are determined as follows

$$\left. \begin{aligned} A_{1s} &= 2/(1 - 1.278S^{0.0358}), \\ B_{1s} &= -0.8/(1 - 1.02S^{0.0037}) \end{aligned} \right\} \dots\dots\dots(19)$$

(a) WSR 205

(b) PEO 10

(c) PEO 16

Fig. 5 Comparison Eq. (20) with experimental results

Therefore,

$$\frac{1}{\sqrt{\lambda}} = \frac{2}{1 - 1.278S^{0.0358}} \log R_e \sqrt{\lambda} - \frac{0.8}{1 - 1.02S^{0.0037}} \dots\dots\dots(20)$$

In Fig. 5(a), (b), (c), the experimental values agree well with Eq. (20).

Further, the analytical results by Tomita et al.,⁽¹⁰⁾

$$\frac{1}{\sqrt{(1 + \alpha N_e^*)\lambda}} = \frac{2.0}{n^{0.75}} \log(1 + \alpha N_e^*)^{-n/2} \times R_e^* \lambda^{1-n/2} - \left\{ \frac{0.2}{n^{1.2}} + \frac{1.2}{n^{0.75}} \left(1 - \frac{2}{n} \right) \right\} \dots\dots\dots(21)$$

at $n=1.0$, $m=0$

$$\left. \begin{aligned} A_{1s} &= 2.0\sqrt{1 + \alpha N_e^*} \\ B_{1s} &= \sqrt{1 + \alpha N_e^*} \{ 2 \log \sqrt{1 + \alpha N_e^*} - 0.8 \} \end{aligned} \right\} \dots\dots\dots(22)$$

The estimated value of α by the authors' experimental results is about 2.

4.3 The limits of application for Eq. (20)

As described in Chapter 3, there are two types of the turbulent drag reduction behaviours. Figure 6 shows the relation between λ and S at $R_e=10^4$ and 3×10^4 . As is evident from this figure, Eq. (20) agrees well with the experimental values for S below 1.9×10^{-5} , but it does not agree with them as it draws closer to 1.9×10^{-5} and the drag reduction is independent of S and becomes for S much larger than 1.9×10^{-5} . From these results, it follows that Eq. (20) is applicable only when the relation between the friction factor λ and Reynolds number R_e once agrees with Kármán-Nikuradse's curve and a drag reduction occurs, that is, when S is below 1.9×10^{-5} . It is not applicable in the case of S over 1.9×10^{-5} where the onset and the turbulent transition are not clear and a drag reduction occurs. In other words, it may be broadly said that the saturation of the Toms effect occurs for $S > 1.9 \times 10^{-5}$. The collection of many detailed experimental data is desired for the clarification of saturation value of the Toms effect, the transition from laminar to turbulent and the onset of drag reduction.

Fig. 6 Relation between the pipe friction factor and non-dimensional number

Fig. 7 Experimental apparatus by way of injection

(a) PEO 10

(b) PEO 16

Fig. 8 Experimental results with the injecting method

Fig. 9 Toms effect influence upon the slurry flow

5. Application

5.1 A scheme to avoid the mechanical degradation

In general, the polymer solution are degraded by the shear. In the industrial use, the mechanical degradation takes place while polymer solutions are transported through the pump. Accordingly, it is not desirable to flow the polymer solutions through the pump directly. Therefore, a concentrated polymer solution was injected into the water at the delivery side of the pump.

Materials are PEO 10 (2.0×10^{-2} , 1.0×10^{-2} and $5.0 \times 10^{-3} \%$) and PEO 16 (2.5×10^{-2} , 1.0×10^{-2} , 5.0×10^{-3} and $1.0 \times 10^{-3} \%$). The testing pipe is a polyvinylchloride tube of 1.29 cm in diameter. The concentrations of concentrated solutions are 0.1~1.0% and 0.02~1.0% for PEO 10 and PEO 16 respectively and their injected volume was determined such that the desirable concentration might be obtained in the tube. The concentrated solution were injected through the nozzle inspected previously by handling the piston pump. Arranged experimental data are shown in Fig. 8. They indicated a larger drag reduction with the natural head flow and also, the drag reduction was saturated at lower concentration, that is, lower S . The causes are considered to be that the turbulent control effect of polymer solution varies depending on the injected position in the radial direction of the pipe. The problem of this position is a subject for future study. Anyway, it was verified that the same effect as that by the natural head flow was obtained by the injecting method.

5.2 Application to the hydraulic conveyor of slurry

If the same effect as the polymer solution is obtained in the flow of a polymer resin-added slurry, it may be applied to the hydraulic conveyor of small coal-dust, dredging work and others. Then, as an example of the industrial applications, the authors made experiment with the apparatus show in Fig. 7 for a flyash slurry. The testing pipe was a polyvinylchloride tube of 12.9 cm. The concentration of slurry and the concentration of polymer for the slurry are shown in the inserted table of Fig. 9. The concentrations of added polymer solution are 0.02~1.0%. The relation between λ/λ_w or λ_s/λ_w and Re (calculated by the kinematic viscosities of polymer solutions) are shown in Fig. 9. λ is the pipe friction factor in the case of injecting a concentrated solution into the slurry. λ is the friction factor of the slurry only. λ_w is the friction factor of the water. As is evident in this figure, the higher the concentration of the polymer solution is, the more the drag reduction in-

creases. The friction factor λ is reduced over 50% by adding a polymer resin at the high shear rate and it is lower than that of the water only. From these results, it may be considered possible to apply the Toms effect to the hydraulic conveyor of the solid particles.

6. Conclusions

(1) The turbulent pipe friction factor for dilute polymer solutions was expressed quantitatively with the tube diameter, the flow velocity and the physical properties of polymer solution (η_v , η_e).

(2) In order to avoid the mechanical degradation caused when the polymer solutions flow the pump, concentrated polymer solutions were injected into the water at the delivery side of the pump and the same effect as the natural head flow was obtained.

(3) As an example of application of the Toms effect to the hydraulic conveyor of solid particles, the injection of concentrated polymer solutions into a flyash slurry was tried. Consequently, a drag reduction appeared, that is, the usefulness of the Toms effect in the hydraulic conveyor of the solid particles was proved.

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Discussion

H. ITO (The Institute of High Speed Mechanics, Tohoku University):

(1) According to the authors' theory, by what formula is the onset of drag reduction expressed?

(2) It has been reported in reference (6) that there are two asymptotes for the limit of drag reduction when the concentration of solution is increased. How does the authors' theory interpret these experimental results? The discussor hopes that the experimental data of the friction factors for higher Reynolds numbers be shown.

(3) How does the authors' theory predict the velocity profiles for the turbulent pipe flow of dilute polymer solutions when they are expressed by the usual u/u^* vs. u^*y/ν relations? The discussor wishes that the authors present the relation graphically by varying the parameter within the range of their experiments.

T. TAGORI (University of Tokyo):

(4) The past experimental data state that the Toms effect saturates at higher Reynolds number also when the relation between the pipe friction factor and Reynolds number agrees once with Kármán-

Nikuradse's curve and a drag reduction occurs, that is, that the gradients of $\lambda-R_e$ curve decrease by degrees and they fall into one line. In Fig. 4, this tendency is observed. It thus appears that the limits of applicability of Eq. (20) are from the onset on Kármán-Nikuradse's curve to the saturating points.

(5) Were the injecting nozzles set on the circumference of the tube wall? As shown by Wells and Spangler's experiment of adding aqueous polymer solution, the pressure gradient in downstream after the tube wall injection was lower than the expected value obtained in the case of full diffusion, though for the tube center injection they agreed well. Therefore, do the authors think that a larger drag reduction than that in the natural head flow is caused by that the concentration at tube wall becomes higher because of very slow diffusion of the concentrated polymer solution and non-uniform concentration at the measuring positions?

T. HASEGAWA (Tokyo Institute of Technology):

(6) The discussor wants the authors to show concretely the jet method apparatus used to measure the cross viscosity, and in this case, what are the

effects of surface tension?

Authors' Closure

(1) Though it is considered possible to express the onset by the shear stress at the tube wall like Virk et al. did, the experimental accuracy at the neighbourhood of the transition point and the onset is not satisfactory enough to discuss the transition point and the onset quantitatively. Therefore, the onset points are expressed roughly by the junction of Eq. (20) and Kármán-Nikuradse's equation, since Eq. (20) is applicable only in the case when $\lambda-R_e$ relation agrees once with Kármán-Nikuradse's equation and a drag reduction occurs.

(2) It is considered that the saturation of drag reduction over a certain concentration does depend not only on the concentration but also on the tube diameter. As described in this paper (Fig. 6), it seems that the drag reduction saturates in the neighbourhood of $S=1.9 \times 10^{-5}$ within the limits of authors experiment. The interpretation for this problem has not been established yet. The authors must leave it until the experimental result at higher Reynolds number are obtained.

(3) The analysis was made by the dimensional analysis and the factors of Eq. (20) were determined by experimental values. Therefore, it becomes a problem to show the velocity distribution quantitatively from Eq. (20). But, the velocity distribution is shown in Append.-Fig. 1 by putting $P_s=3/2k$ and using the values of A_{1s} and B_{1s} . Then, in the equation of velocity distribution $u/u^*=A_s \log(u^*y/\nu) + B_s$, A_s and B_s are as follows.

$$\left. \begin{aligned} A_s &= \frac{2.303}{k} = 2\sqrt{2} A_{1s} \\ B_s &= 2\sqrt{2} \left\{ B_{1s} - A_{1s} \log \frac{\sqrt{2}}{8} \right\} + \frac{3}{2k} \end{aligned} \right\}$$

where k is the factor of mixing length.

It has been reported recently that the velocity distribution measured by the Pito-tube is parallel to that of Newtonian fluid ($k=0.4$, $A_s=5.75$)*. But, authors' results predict that its gradients have larger

Append.-Fig. 1 Velocity distribution

value with an increasing non-dimensional number S . It seems to be due to the values of S and u^*y/ν .

(4) The asymptote by $\lambda-R_e$ curve at the high Reynolds number occurred at $S > 1.9 \times 10^{-5}$ as described in this report. Also, there is an asymptote at a high Reynolds number. However, this point could not be discussed because it was impossible to obtain the data at higher Reynolds numbers by the natural head flow. Therefore, the limits of applicability of Eq. (20) are from the onset to this saturation point and within the limits of S described above.

(5) As shown in Fig. 7, there is only injecting nozzle of 1.5 mm in diameter at the tube wall. Also, it is anticipated that the concentrated solutions could not be diffused fully, since the concentrations were 0.02~1.0% for PEO 16 and 0.1~1.0% for PEO 10. For this reason, it seems that the gradients of $\lambda-R_e$ curve are higher in the neighbourhood of the onset and that the drag reduction is larger and is saturated at lower concentration.

(6) The jet diameter was measured by photograph and the tool marker's microscope and the normal stress difference was calculated from it. This normal stress difference was converted to the cross viscosity in a "simple Reiner-Rivlin model". As shown in Append.-Fig. 2, the normal stress could not be measured by the jet method in the whole range of concentrations used in this experiment. Thus, the normal stresses used in this arrangement were obtained by the extrapolation of the values measured by the parallel plate rheogoniometer and the jet method apparatus.

The effect of the surface tension was ignored. Though this point has been discussed, it is considered

Append.-Fig. 2 Relation between $(P_{11}-P_{22})$ and the concentration

Append.-Fig. 3

simply as follows,

$$\int_0^{r_{in}} 2\pi r \rho v_{in}^2 dr - \pi r_j \rho v_j^2 - \int_0^{r_{in}} 2\pi r \times \left\{ (P_{11} - P_{22}) - \frac{2\sigma}{d_{in}} \right\} dr = 0$$

(see Append.-Fig. 3). The surface tension of polymer solution is about 60 dyn/cm though it varies slightly with the concentration and the tube diameter is 1.5×10^{-1} cm in this case.

Thus,

$$2\sigma/d_{in} \doteq 8.0 \times 10^2 \text{ dyn/cm}^2$$

As shown in Append.-Fig. 3, within the limits of concentrations for the jet experiment, the values of normal stress difference ($P_{11} - P_{22}$) are as follows.

$$3 \times 10^3 \text{ dyn/cm}^2 \leq P_{11} - P_{22} \leq 1.3 \times 10^5 \text{ dyn/cm}^2$$

Therefore, if the surface tension is ignored, for the lowest concentration the error is about 25%, but it seems that the error for a higher concentration is very little.

* Tomita, *Bull. JSME*, Vol.13, No. 61 (1970-7), p. 935; Wells, C.S., et al., *AIAA Jour.*, Vol. 6 No. 2 (1968), p. 250.