621.43.03 : 621-50 : 681.142

Analysis of By-Pass Control Type Fuel Injection System for Small Diesel Engine by Digital Computer*

(Part 2, Experiments of the Physical Characteristics, Comparison
between Calculation Results and Experimental Results,
and Application of Calculation Method)

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It is necessary to decide the fuel compressibility, the specific weight, the sonic velocity, the flow coefficient and other physical characteristics when simulating a fuel injection system.

Until the present, it would seem that the relationship between these physical characteristics and the primary factors of these characteristics has been unclear. For this reason, proper application of these characteristics could not be made.

Therefore in this report, we measured the physical characteristics of the diesel gas oil and also of the fuel injection system used in this study. Moreover, after comparing the calculated values with the actual experimental values, a method of calculating the time interval for computation and the judgement values for permissible error, etc. were analyzed and were appropriately established.

By using the methods described above, the results of the calculation matched adequately with the specific experimental factors, even when a void condition was present.

1. Introduction

When simulating a fuel injection system, it is necessary to determine such physical values as the compressibility of fuel oil, the specific weight of fuel oil, the sonic velocity of fuel oil, and the flow coefficient, etc.

However, there are some questions retarding the values obtained experimentally hitherto in relation to the primary factors, making it difficult to determine their values at the time of adopting them. Therefore in this paper these values of the fuel (Table 1, The usual diesel gas oil used in Japan) and of the injection system were measured through experiments. The result of the comparison between the calculation, in which experimental data are used, and the experiment has proved that the actual characteristics of the injection system can be simulated qualitatively and quantitatively by the calculation, and application of the calculation is rendered possible. This paper deals with the experiments of the above mentioned physical

values, with comparisons between the calculated results and the experimental results and with the applications of the calculations.

2. Experiments on the physical values

2.1 Fuel oil used

The properties of the oil used in this trial are shown in Table 1. Those of the oil containing air are described below in section 2.2.

2.2 Compressibility of fuel oil β

Compressing rapidly the fuel contained in a pressure vessel made of steel and heated uniformly (inner volume V_0 : 435 cc, inner diameter: 6.5 cm, wall thickness: about 5 cm) by a plunger of 1.4 cm in diameter, using a universal material testing machine, and then recording the plunger displacement s cm and the oil pressure in the vessel p kg/cm² by $X-Y$ recorder as shown in Fig. 1, the value was obtained from the following Eq. (1).

Table 1 Properties of diesel gas oil

Specific weight (15.4°C) g/cm ³	0.835	Carbon residue	0.02
Flash point (PM) °C	85	Sulfur %	0.62
Cetane index	58	Carbon %	85.4
Viscosity Cs (30°C)	3.674	Hydrogen %	13.8
Pour point	-15	Nitrogen %	<0.2

* Received 10th August, 1970.

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$$\beta_e = \frac{1.4^2 \pi}{4 \{V_0 - (\pi/4) 1.4^2 s\}} \frac{\Delta s}{\Delta p} - 4.81$$

$$\times 10^{-6} \text{ cm}^2/\text{kg} \dots\dots\dots (1)$$

The second term on the right hand side means a differential deformation of the vessel due to pressure and is measured by the method⁽¹⁾ that compares the deformation between the use or disuse of steel balls (21 balls of 3/4 inch diameter) placed in the vessel. In this paper gauge pressure is used and is denoted by p . The compressibility is determined approximately from the gradient $\Delta s/\Delta p$ in Fig. 1. Therefore it is seen that due to compounded air the compressibility changes greatly below about 65 kg/cm² but above this pressure there is very little change. After comparing the compounded air of both the same kind of oil used on the market with that in this experiment, and finding them both to be saturated, the experiment was therefore carried out using this oil. Figure 2 shows the measured result of the compressibility β_e in the case that p and the oil temperature $t^\circ\text{C}$ are varied. Eq. (2) expresses the contents of Fig. 2 and reveals the effect of the primary factors.

$$\beta_e = 85.33 \times 10^{-6} \times 1.125^{0.05t} p^{-0.1127} \text{ cm}^2/\text{kg} \dots\dots\dots (2)$$

In order to simplify the propagation equation of the cumulative flow in the tube described in the first report⁽²⁾⁽³⁾, the following β obtained from substituting the sonic velocity a m/sec obtained from the Eq. (4) and the density ρ kg sec²/cm⁴ obtained from the Eq. (5) into the Eq. (3) is used as the compressibility.

$$\beta = \frac{1}{a^2 \rho} \text{ cm}^2/\text{kg} \dots\dots\dots (3)$$

Figure 3 shows the comparison of β obtained from

Fig. 1 Influence of mixed air on compressibility

Fig. 2 Measured compressibility

the Eq. (3) and β_e from the Eq. (2), and it is seen from the figure that β is approximately equal to β_e and the approximation and accuracy are sufficient for the simulation.

2.3 Sonic velocity a of the fuel oil

This value a is obtained from the time elapsing in the measured distance 4 m by the use of a synchroscope. At that time, three various measuring tubes (inner/outer diameter: 1.5/6, 2/6 and 4/10 unit in mm) are used having the same diameter as the pressured tube used in practice, and the pressured feeding section for the oil is immersed in the oil tank with the temperature regulator. Among the measured results there hardly exists any difference in the sonic velocity due to the different inner diameters of the measuring tubes as shown in Fig. 4. This figure shows the value in the case that the back pressure in the tube p kg/cm² and the oil temperature $t^\circ\text{C}$ are varied. The relation shown in this figure is expressed as the following Eq. (4).

$$a = 1390 + 0.414p - 2.9t + 0.0005tp \text{ m/sec}$$

$$\dots\dots\dots (4)$$

2.4 Specific weight γ of the fuel oil

This is measured also by the compressibility measuring device mentioned in section 2.2. The measured result is shown in Fig. 5 and the relation in the figure can be expressed as follows.

$$\gamma = (0.8430 + 0.000058p - 0.000665t + 3.5tp \times 10^{-7}) \times 10^{-3} \text{ kg/cm}^3 \dots\dots\dots (5)$$

In addition, when these Eqs. (3), (4) and (5) are used for the simulation, the residual pressure in the injection system and the average oil temperature in the high pressure tube during fuel injection are used as

Fig. 3 Comparison between β and β_e

Fig. 4 Measured sonic velocity

p and t respectively, according to the estimation given in section 4-1.

The above description makes clear the physical values dependent on the fuel oil properties. The physical values influenced by structural factors are stated below.

2.5 Flow coefficient of throttle

The flow coefficient was measured in a steady flow. At that time the change of the flow rate, dependent on the oil temperature, was hardly perceivable although the oil temperature varied from 40°C to 60°C. Therefore the flow coefficient μ is related to the oil pressure p kg/cm², valve lift h cm and the minimum passage area F cm². The measured results are shown in Fig. 6 and Eqs. (6), (8), (9), (11), (12) and (13). Figure 6 shows the conical by-pass throttle valve and the flow coefficient μ_s which corresponds to the change of valve lift h_s and pressure p . The theoretical calculations⁽⁴⁾ indicated by the chain line in Fig. 4 are somewhat larger than the experimental values. Based on the iteration procedure, this calculation is complicated and lengthy, and therefore the relation of the experimental results has been tried as is suggested in the Eq. (6). The actual flow coefficient when the valve lift is smaller than 0.1 mm, is estimated to be larger than the value in steady flow. As the result of the factor analysis in

Fig. 5 Measured specific weight

Fig. 6 Flow coefficient of by-pass control valve

section 4-1 and the comparison between the calculation and experiment, the modified Eq. (7) is used for the simulation. The other flow coefficients are discussed in the same manner, and the values of the Eqs. (8), (10), (11), (12) and (13) are used for the simulation.

1) By-pass control valve seat

$$\mu_s = \frac{1}{3} \tan^{-1} \left\{ (4p+300) \left(h_s - \frac{0.0235}{\sqrt[3]{p}} \right) \right\} - \frac{1}{3} \tan^{-1} \left\{ -(4p+300) \frac{0.0235}{\sqrt[3]{p}} \right\}$$

.....(Empirical equation).....(6)

$$\mu_s = 0.94(1 - e^{-300h_s}) \dots \text{(Modified equation)} \dots (7)$$

2) Nozzle outlet

$$\mu_d = 0.82 \dots \text{(Empirical value)} \dots (8)$$

3) Feed valve seat

$$\mu_v = 0.26 \tan^{-1} \left\{ (4p+500) \left(h_v - \frac{0.014}{\sqrt[3]{p}} \right) \right\} + 0.315$$

.....(Empirical equation).....(9)

$$\mu_v = 0.72 \dots \text{(Modified value)} \dots (10)$$

4) Fuel intake of plunger chamber

$$\mu_p = 0.71 \dots \text{(Experimental value at the time of fuel intake)} \dots (11)$$

$$\mu_p = 0.65 \dots \text{(Experimental value at the time of fuel discharge)} \dots (12)$$

5) Joining of oil tube

$$\mu_f = 0.80 \dots \text{(Experimental value)} \dots (13)$$

2.6 Damping coefficient and spring constant of the fuel injection nozzle

These values are obtained from the repulsion between the nozzle valve and valve seat by comparing the calculation and experiment as shown in Fig. 7. The solid line in this figure is the measured value by the nozzle valve lift transducer described in section 3-1 when the situation is such that the nozzle valve of the injection nozzle is pushed up periodically by the notched cam and the nozzle valve falls from the notch and strikes the valve seat. The dotted and chain lines in the figure show the calculated value by solving the equation of motion of the nozzle valve

Fig. 7 Rebounding of nozzle valve at nozzle valve seat

movable parts by the use of Runge-Kutta-Gill method⁽²⁾⁽³⁾. As the result of the comparison, it is seen that the damping coefficient of the nozzle valve movable parts λ_n is about 0.03 kg sec/cm, the nozzle spring constant δ_n is equal to that of a single spring measured by the spring tester, the damping coefficient of the nozzle valve seat λ_{ns} is infinitely small, and the spring constant of the seat δ_{ns} is about 13 700 kg/cm. The elasticity of the valve seat has very little effect on the final properties of the injection system, and is therefore not taken into consideration in this paper.

After investigation of the condition of the damping oil pressure in the nozzle oil chamber by comparing the calculation with the experiment, the fluid resistance coefficient in the high pressure tube k_r takes the following corrected value.

$$k_r = 150 \text{ 1/sec} \dots\dots\dots (14)$$

The main physical values used in the simulation of the injection system can be determined as stated above.

3. Comparison between the calculated results and experimental results

3.1 Measuring apparatus of nozzle valve lift, nozzle oil chamber pressure and injection rate⁽⁵⁾

Applicability of the simulation to the actual phenomenon is investigated through the nozzle valve lift, oil chamber pressure and injection rate which are the final characteristics of the injection system. The measuring apparatuses for this purpose are described in the following. The transducer of the valve lift works on the principle of cutting the light, and uses a photo transistor PD-3L. For simplicity and to make the output of the transducer proportionate with the valve lift, a straight filament is used as the source for the light. In order to measure the pressure of the nozzle chamber directly and eliminate the vibration from the nozzle body, the pressure transducer uses a germanium strain gauge with a compensating temperature, and the diameter of the pressure membrane

Fig. 8 Transducer of nozzle oil chamber pressure

is 3 mm as shown in Fig. 8. The prepared measuring tubes are 4 mm in inner diameter but their lengths are 4.5 m, 9 m and 13.5 m, and used appropriately in order to prevent the overlapping of the reflected wave. The pressure in the tube is measured by a semi-conductor pressure transducer having a pressure membrane of 10 mm in diameter. The pressure regulating tube is 4 mm in inner diameter and 4.5 m long. The pressure is regulated by a rubber damper placed near the pressure regulating valve at the tube end. In this paper, the back pressure in the tube was kept under about 0.01 kg/cm² (gauge pressure), and the injection rate was measured under the condition of atmospheric pressure. Figure 9 shows the measured result of this apparatus, which is three cycles being photographed repeatedly. The line in Fig. 9 corresponds to the oil chamber pressure P_d , nozzle valve lift h_d , injection rate $\Delta Q'$ and rotary angle mark of the fuel cam shaft (lift starting point: 0 degree) respectively from the above.

As mentioned above, the final characteristics are calculated and can be compared with the experiment. The result of this comparison is stated in the following.

3.2 Comparison between the calculation and experiment

In order to obtain good conformity between the calculated results and experimental results, it is exceptionally necessary to determine adequately (1) calculation equations, (2) physical values and (3) calculation pitch (or number of divisions of oil tube and high pressure one) and the constant values of judging convergence for the calculation. However, it is difficult to decide the priority of these factors.

In the following three points, these will be explained in detail.

(1) Of course the calculation equations must be accurate in a compressive situation and it must also be accurate when a void is generated. If a void is not dealt with properly when it is generated, the calculation results will be somewhat different from the experiment in the case of the continuous

Fig. 9 Actual example (1 500 rpm, 25 mm³/stroke)

multi-cycle calculation and in a condition where a void is too easily generated, for example a) high speed and profuse injection, b) irregular injection, c) injection system with unloading delivery valve. Figure 10 shows an example of comparing the conformity of the calculation with the experiment under the condition where a void is generated. In this figure, the upper curve corresponds to the amount of the void V_{cd} in the oil chamber, the middle curve corresponds to the oil chamber pressure P_d and the lower curve corresponds to the nozzle valve lift h_d . The dotted line obtained from the calculation agrees well in general with the solid line of the measurement. The right half of the P_d curve pulsates due to the reflection of oil pressure in the high pressure tube after the main injection and shows that a void is generated. Even in this part, the dotted line agrees quite well with the solid line. The chain line near P_d curve shows when a void is not considered in the calculation equation, and it differs largely from the solid line where a void is generated. Therefore, this calculation equation can also be considered suitable in the treatment of a void.

(2) Because it was expected that the physical values for sonic velocity of fuel, specific weight of fuel, compressibility of fuel and flow coefficient largely affected the results of the calculation and the values that had been used up to this time were found unsatisfactory, the experiment described in section 2 was carried out. When these values are used in calculation, the calculated results sometimes differ from the experiment results. Thus, the effect that these values have on the final characteristics described in section 4-1 is evaluated, and the values that are largely affected are corrected by comparing calculation with experiment. The method of modification is described below. The flow coefficients (by-pass control valve seat, nozzle port) are the most seriously affected, and in sequence the compressibility, sonic velocity, specific weight, damping coefficient of the moving part (nozzle valve moving part) and flow coefficients (delivery valve seat, plunger chamber fuel

Fig. 10 Comparison between experiment and calculation when a void is generated

intake) are also affected. The flow coefficient μ_s of the by-pass control valve that is seriously affected is modified as shown in Fig. 11. This figure shows the relation between the opening of the adjusting angle of the by-pass control and the injection quantity when only μ_s is changed, and the dotted line corresponds to the direct application of the experimental values and differs from the actual performance indicated by the solid line when only a small opening is used. Therefore through the comparison of the calculation and experiment the modified Eq. (7) is obtained. Experimental values are used for the physical values that have little influence on the final characteristics. As the result of these modifications, the chain line of the calculated result agrees well with the actual performance indicated by the solid line.

(3) As for the calculation, the calculation pitch and the constant values of judging convergence are considered below. Though these should be determined according to computational accuracy, the calculation alone is so complicated and lengthy that this is not practical. Therefore they are determined from the comparison between the calculation and experiment. Figure 12 shows an example of a case where the computational pitch ΔT is determined when the high pressure tube is divided first into 3 sections, then 9 sections and 26 sections, and the effect is investigated by comparing the nozzle oil chamber pressure, nozzle valve lift and injection rate. As a result, when the high pressure tube of a small engine is divided into 9 sections, the calculation agrees well with the experiment under the condition of rated performance. It is seen as well that when the revolution of the fuel cam shaft is changed greatly, satisfactory results are obtained by varying the division approximately in reciprocal proportion to the revolutions. Figure 13 shows the effects when the convergence evaluation constants for the nozzle valve lift h_d and the variation of the volume in the plunger chamber Y_k and that of delivery valve chamber Y_k' are varied. As the result, it can be seen

Fig. 11 Correction of flow coefficient at by-pass control

that the constants below 10^{-6} cm³ and 10^{-4} cm are suitable for Y and h_d respectively.

When the investigated result of these three conformities is applied to the calculation under the experimental condition shown in Fig. 14, the calculation agrees well with the experiment even under a condition of low revolution and scarce injection, even though the conformity is not thought to be good. Fig. 14 is obtained by taking three superimposed photographs repeatedly, and Fig. 15 shows the calculated value after 5 cycles. In addition the experimental conditions of 500 rpm of the fuel cam shaft speed

and 10 mm³/stroke of injection quantity correspond to the operational condition at almost the lowest speed and no load in case of small engine. Furthermore, that of 2000 rpm and 25 mm³/stroke is the operational condition for super high speed and maximum loading.

4. Application of the calculation

4.1 Factor analysis of the physical value

The result of analysing the oil temperature t and pressure p , namely, the primary factors that affect the sonic velocity, weight and compressibility of the fuel oil, is shown in Fig. 16. As seen from this figure, when the oil temperature is 60°C and the oil pressure is 40 kg/cm² the compressibility β becomes equal to 79.9×10^{-6} cm²/kg according to the equation (3), and the sonic velocity a and the specific weight γ take the value given in the figure. When the final characteristics are calculated through the use of these values, the injection quantity attains 26.7 mm³/stroke, and the beginning and end of injection are 22.6 degrees and 33.6 degrees respectively. These values are taken as a tentative standard for the evaluation. If only t is lowered to 40°C in the Eqs. (3), (4) and (5), β becomes smaller than the standard value by 10.1%, a and γ larger by 4.7% and 1.6% respectively. If the factors are varied in the equations, it is seen that the injection quantity increases by 1.9%, and the beginning and end of injection are advanced by 2% and 1.2% respectively. When only the oil pressure is lowered from 40 kg/cm² to 14 kg/cm², a similar effect takes place as indicated by the half tone in the figure. The upper and lower limits of the oil temperature and pressure remain within the range of average oil temperature and residual pressure in the high pressure tube used in this experi-

Fig. 12 Calculation accuracy by a number of divisions of high pressure pipe

Fig. 13 Calculation accuracy by difference of convergence constant

Fig. 14 Experimental results

Fig. 15 Comparison between calculation and experiment

Primary factors	Temperature t		40(Lower Limit)	60°C	88 (Upper Limit)	
	Pressure P		14 (Lower Limit)	40kg/cm²	134 (Upper Limit)	
Scale of evaluation			4% 3 2 1	Bases of evaluation	1 2 3 4 5 6 %	
Variation of characters	β	Vary: only	t		79.9 × 10 ⁻⁶	
		P		cm²/kg		
	α	t		1234		
		P		m/sec		
	γ	t		0.8063		
		P		g/cm³		
	Injection quantity	t		26.7		
		P		mm³/stroke		
	Beginning of injection	t		22.6°		
		P				
	End of main injection	t		33.6°		
		p				

Fuel cam speed 1500rpm

Fig. 16 Evaluation of primary factors in physical values

Flow coefficient	0.6			0.8			1.0		
	Scale of evaluation			Bases of evaluation			0 %		
	Average flow			0 %			10 20 30 %		
Injection quantity	Only μs	10585	m/sec	+26 %					+25.5 %
	" μd	15416	"	-3 %					+1 %
	" μv	7173	"	+0.3 %			29.0		-0.3 %
	" μp	3046	"	+1.7 %			mm³/stroke		-0.7 %
	" μf	21	"	-0.3 %					+0.3 %
	" μs	10585	"	-1.3 %					+2.5 %
Beginning of injection	" μd	15416	"	0 %			Cam angle		+1.3 %
	" μv	7173	"	+1.3 %			22.3°		0 %
	" μp	3046	"	0 %					+1.3 %
	" μf	21	"	0 %					0 %
	" μs	10585	"	0 %					0 %
	" μd	15416	"	+0.86 %			Cam angle		0 %
End of main injection	" μv	7173	"	0 %			33.6°		0 %
	" μp	3046	"	0 %					0 %
	" μf	21	"	0 %					0 %

Fuel cam speed 1500rpm

Fig. 17 Evaluation of flow coefficient

ment of the injection system. As seen from this figure, the effect of the oil temperature is considerably large as indicated by the solid bar graph, and it is therefore necessary to choose appropriately the oil temperature in the empirical equation so that the injection system can be kept at average oil temperature in the

high pressure tube under continuous performance. However, as the pressure is hardly affected as shown by the half tone bar in the above figure, the pressure in the empirical equation may be considered to be the same as the residual pressure used in the simulation.

The evaluation of the flow coefficient for each throttle is described in the following. Since the flow coefficient is affected greatly by the structural factors, the flow coefficient should be assumed at the time of designing a new injection system. Thus, when it is assumed as 1, 0.8 and 0.6 on trial, the evaluation result can be listed as shown in Fig. 17. When the flow coefficient is taken as 0.8 for the five cases, the injection rate of the final characteristics becomes 29.0 mm³/stroke, and the beginning and end of the injection are 22.3 degrees and 33.6 degrees respectively. These values are then taken as the standard. When only the flow coefficient of the by-pass throttle valve seat μ_s is lowered from 0.8 to 0.6, the injection quantity is increased by 26% from the standard. Similarly, the effect of the flow coefficient on the injection rate is made clear as indicated by the black bar in the figure. Also it is seen that the effect is increased with the average flow velocity when the valve seat is of a

Fig. 18 Evaluation of structural factors of worn-out nozzle

500 rpm, 25 mm³/stroke

1500 rpm, 25 mm³/stroke

Fig. 19 Experimental results of developed pump

Fig. 20 Comparison between calculation and experiment of developed pump

simple shape and every throttle is opened. Therefore, the flow coefficient of each throttle is assumed in advance as 0.8, and the essential throttle is decided depending on the magnitude of the average velocity at each throttle according to trial calculation when the result based on the above assumption differs largely from the conventional value, and it seems preferable to choose the value of flow coefficient of the essential part by comparing simply the conformity of the injection quantity between the calculation and experiment.

4.2 Evaluation of the structural factors

Figure 18 shows the result of evaluating structural factors by use of the dimensions of the nozzle and the pump that have been on the market. As seen in this figure, the injection quantity of $26 \text{ mm}^3/\text{stroke}$ for the standard nozzle is reduced by 21.5% when the wear is considered in the calculation. Furthermore, the black bar in this figure shows the effect of the wear factor. For example, when the fit shaft is assumed to be worn by 12μ from the standard condition, the injection quantity is decreased by 31%. In addition, if the wear of the valve seat corresponds to a clearance of 1μ , the injection quantity during the main injection interval is decreased by 7.1% because of a leakage from the valve seat. This investigation makes it quantitatively clear that the leakage from the valve seat and the wear of the nozzle fit shaft affect the degradation of engine performance, and as a result various effective improvements become possible.

4.3 Example of new development

In this section, a pump is presented where the oil leakage is lowered by reducing the residual pressure in the high pressure tube. When developing this pump, it was necessary to meet the following conditions, in order not to deteriorate engine performance.

1) The injection rate and the pressure during injection should hardly differ from the usual ones.

2) The decrease of the residual pressure should be small during the increase of the fuel cam shaft revolutions and during the decrease of the injection.

3) The change of the residual pressure due to the wear in the injection system should be small.

4) In order to prevent corrosion by cavitation, a void should not remain in the high pressure tube. Thus such a pump is devised that meets the above conditions. This pump is equipped with a by-pass control throttle in the delivery valve chamber, and it is expected that the residual pressure during an

operation is kept at the shut pressure of the by-pass control valve spring. The simulation result of this injection system is indicated by the dotted line in Fig. 20, and the oil leakage is reduced as expected, resulting in the similar final characteristics during fuel injection as with former pumps. Figure 19 shows the final characteristics of this pump made on trial, and the oil leakage is seen to be very little. Figure 20 shows the comparison between the calculated result and the experimental one. Since the calculation indicated by the dotted line hardly differs from the measurement indicated by the solid line, it can be concluded that the actual performance is estimated through the simulation.

5. Conclusions

The results of this research can be summarized as follows:

1) Experiments were carried out on the main physical values used in this simulation, and the method of applying the values was decided.

2) It was found that the calculation equations based upon the variation of the volume hold well in the case of a void being initiated in the injection system.

3) A method of carrying out the simulation is investigated, and the related factors are determined appropriately.

4) It is seen that the calculated results agree well with the actual performance, evidencing the sufficient practicability of the simulation. Some improvements were carried out and an example is presented.

In conclusion, the authors wish to express their sincere thanks to Emeritus Professor Tokio Uematsu and Professor Mitsunobu Ogasawara of the University of Osaka and Professor Hiroyuki Hiroyasu of the University of Hiroshima for their valuable suggestions given in the course of this research.

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