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Generalized Oldham Coupling of Turning Block Double Circular-Arc-Slider Mechanism*

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This paper deals with a new mechanism, the generalized Oldham coupling. First the existence of the mechanism is deduced through the inversion of chain. This is a turning block double-slider mechanism with circular-arc-sliders and is a cyclical varying speed coupling. Next the relative angular motion between two shafts connected by the coupling is analysed and is verified experimentally. This coupling has one to one correspondence to the double crank mechanism on the condition that two mechanisms yield the same angular motion. In particular, the coupling corresponding to that having one crank of infinite length generates a new motion. The Oldham coupling corresponds to one of that having both cranks of infinite length and is considered to be a special case of the generalized Oldham coupling.

1. Introduction

The well-known Oldham coupling is one of the practical applications of the fixed block double-slider crank mechanism, as shown in Figs. 1(a) and 1(b). In Fig. 1(a) the connecting rod L can make one revolution relative to each of the sliders A and B and the disk C . The kinematic inversion produced by holding link L is the basis of the Oldham coupling⁽¹⁾⁽²⁾.

Now let us consider a fixed block double-slider crank mechanism which has circular-arc-sliders with R_1 and R_2 as radii [Fig. 1(c)]. This mechanism also must be applied to the coupling when the connecting rod L can make one revolution to each of the sliders A and B and the disk C , as the mechanism in Fig. 1(a) can be done. What mechanism can be applied to the coupling then? To answer the question, first all the mechanisms of this kind are classified and their motions are investigated. As a result, it becomes clear that this mechanism can be applied to the coupling when the motion of the connecting rod L is identical with the motion of the coupler of the double lever mechanism satisfying the Grashof condition⁽³⁾⁽⁴⁾. Next the relative angular motion between two shafts connected by this coupling is deduced to be identical with the motion between two cranks of a double crank mechanism with L as fixed link and R_1 and R_2 as cranks. All the motions are analysed and are shown in the figures. Experimental values

agree well with theoretical ones.

Finally if Fig. 1(c) shows the fixed block double-slider crank mechanism with arbitrary curved-sliders, this mechanism cannot be applied to the coupling, because in this case sliders must be roller or ball such as A_1 or B_1 and the rotation of the slider A_1 cannot be transmitted to the disk C even when the connecting rod L is fixed.

2. Classification of the fixed block double circular-arc-slider crank mechanism

This mechanism, as shown in Fig. 2, is composed of the sliders A and B , the disk C with circular-arc-slots, and the connecting rod L . The basic dimensions are the radii R_1 , R_2 , the center distance S_0 , and the length of connecting rod L . The classification of the mechanism based upon the slot form is as follows.

1. Two circles intersected; Fig. 2(b)
 - (1) $r = R_2/R_1 = 0$, $R_1 = \infty$, $0 < R_2 < \infty$
 - (2) $0 < r < \infty$, $0 < R_1 < \infty$, $0 < R_2 < \infty$
 - (3) $r = \infty$, $0 < R_1 < \infty$, $R_2 = \infty$
 - (4) $R_1 = \infty$, $R_2 = \infty$
2. Two circles separated; Fig. 2(c)
 - (5) $r = 0$, $R_1 = \infty$, $0 < R_2 < \infty$
 - (6) $0 < r < \infty$, $0 < R_1 < \infty$, $0 < R_2 < \infty$
 - (7) $r = \infty$, $0 < R_1 < \infty$, $R_2 = \infty$
 - (8) $R_1 = \infty$, $R_2 = \infty$
3. One circle within the other; Fig. 2(a)
 - (9) $0 < r < 1$, $0 < R_1 < \infty$, $0 < R_2 < \infty$

By taking L different in value for each type above mentioned, all the mechanisms of this kind are obtained. The examination of the motion of the

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connecting rod L showed that for the rod L to make one revolution relative to each of the sliders A and B and the disk C , it is necessary to satisfy the following conditions: (1) two circular-arc-slots intersect each other and (2) the length of rod is within a certain limit.

2.1 Basic dimensions β , r , l and slot form dimensions S_0 , S_1 , S_2 , and S_3

Figure 3 is a detailed view of Fig. 2(b), where R_1 , R_2 , r , L and β are limited in expression (1) and S_0 , S_1 , S_2 and S_3 are expressed by Eq. (2).

$$\left. \begin{aligned} 0 < R_1, R_2 \leq \infty, 0 \leq r \leq \infty \\ 0 < L < S_0 + R_1 + R_2, 0 < \beta = \angle O_1 P O_2 < 2\pi \end{aligned} \right\} \dots (1)$$

$$\left. \begin{aligned} S_0 &= \sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos \beta} \\ &= R_1 \sqrt{1 + r^2 - 2r \cos \beta} \\ S_1 &= R_1 - R_2 + S_0 \\ &= R_1(1 - r + \sqrt{1 + r^2 - 2r \cos \beta}) \\ S_2 &= R_2 - R_1 + S_0 \\ &= R_1(r - 1 + \sqrt{1 + r^2 - 2r \cos \beta}) \\ S_3 &= R_1 + R_2 - S_0 \\ &= R_1(1 + r - \sqrt{1 + r^2 - 2r \cos \beta}) \end{aligned} \right\} \dots (2)$$

Let $L = R_1 l$ (or $L = R_2 l$), then β , r , l may be considered the basic dimensions instead of R_1 , R_2 , S_0 and L . Here β and r determine S_0 , S_1 , S_2 and S_3 , in other words, the slot form. From Eq. (2), we have

$$\left. \begin{aligned} \cos \beta < 1/(2r), \quad S_3 < S_1 \\ \cos \beta = 1/(2r), \quad S_3 = S_1 \\ \cos \beta > 1/(2r), \quad S_3 > S_1 \\ \cos \beta < r/2, \quad S_3 < S_2 \\ \cos \beta = r/2, \quad S_3 = S_2 \\ \cos \beta > r/2, \quad S_3 > S_2 \\ r < 1, \quad S_2 < S_1 \\ r = 1, \quad S_2 = S_1 \\ r > 1, \quad S_2 > S_1 \end{aligned} \right\} \dots (3)$$

Fig. 1 Fixed block double-slider crank mechanism and turning block double-slider mechanism

Fig. 2 Classification of the fixed block double circular-arc-slider crank mechanism

Expression (3) is illustrated in Fig. 4, which shows the relations among β , r , S_1 , S_2 and S_3 ; that is, the classification of the slot form.

2.2 Fixed block double circular-arc-slider crank mechanism applicable to the coupling

After the investigation on the motion of the connecting rod, it became clear that the rod L can make one revolution relative to each of the sliders A and B and the disk C only when $L \leq S_1$, S_2 , S_3 ; then we have

$$\left. \begin{aligned} L \leq S_1 = R_1 - R_2 + S_0 \\ L \leq S_2 = R_2 - R_1 + S_0 \\ L \leq S_3 = R_1 + R_2 - S_0 \end{aligned} \right\} \begin{aligned} L + R_2 \leq R_1 + S_0 \\ L + R_1 \leq R_2 + S_0 \\ L + S_0 \leq R_1 + R_2 \end{aligned} \therefore \dots (4)$$

Expression (4) is the Grashof condition of the four-bar linkage consisting of the links R_1 , R_2 , S_0 and L , of which L is the smallest. The fact mentioned above shows that the mechanism applicable to the coupling is one in which the motion of the connecting rod is identical with the motion of the coupler of the double lever mechanism with fulfillment of the Grashof condition.

3. The generalized Oldham coupling

The deduction process of the generalized Oldham coupling is shown in Fig. 5. Figure 5(a) shows a double crank mechanism. Figure 5(b) does a double lever mechanism which is the inversion produced by holding the link S_0 of Fig. 5(a), so this mechanism satisfies the Grashof condition. Figure 5(c) does a fixed block double-slider crank mechanism, in which the motion of the connecting rod L is identical with the motion of the coupler L of Fig. 5(b). The respective motions of the sliders A and B relative to the rod L in Fig. 5(c) are identical with those of the

links R_1 and R_2 relative to the fixed link L in Fig. 5(a). Besides, if the rod L of Fig. 5(c) is fixed and the slider A is turned about its pin, the slider B can be rotated though the angular velocity ratio varies cyclically. The same relative angular motion as that between the sliders A and B in Fig. 5(c) is also obtained between the disks A and B of the turning block double-slider mechanism in Fig. 5(d); the relative angular motion between the links R_1 and R_2 in Fig. 5(a) is equal to one obtained between the turning disks A and B in Fig. 5(d). In other words, this turning block double-slider

mechanism has one to one correspondence to the double crank mechanism on the condition that two mechanisms yield the same motion. The length of each link of the double crank mechanism is obtained by giving the basic dimensions β , r , and l . Figure 5(e) shows the slots on the disk C in Fig. 5(d), which is the same as that in Fig. 5(c). The slot with radius R_1 is on one side to fit the disk A and the slot with radius R_2 on the other side to fit the disk B. The details of the slot form are shown in Figs. 3, 7~10. In this paper, the mechanism of Fig. 5(d) is called the generalized Oldham coupling. The following chapter describes this coupling.

4. Analysis

4.1 Input and output angle of rotation when $0 < \beta < \pi$

This section deals with a mechanism with $0 < \beta < \pi$. A general type of the fixed block double-slider crank mechanism is shown in Fig. 3. The relative angular motion between the input and the output shaft in Fig. 5(d) is identical with that between

Fig. 3 Fixed block double circular-arc-slider crank mechanism ($0 < \beta < \pi$)

○: mechanism, the motion of which is analysed theoretically
●: mechanism, the motion of which is analysed theoretically and is verified experimentally

Fig. 4 Classification of slot form based on β and r

two cranks R_1 and R_2 in Fig. 5(a), so it is easy and simple to analyse the motion of the latter. Figure 6 shows a double crank mechanism, where the input Φ is related to the output angle of rotation Ψ by the following equations as generally known⁽⁵⁾. The angles Φ and Ψ are positive when respective cranks rotate counter-clockwise.

$$\left. \begin{aligned} \Psi &= F(\Phi) = \delta_1 + \delta_2 \\ &= \tan^{-1} \left(\frac{R_1 \sin \Phi}{L + R_1 \cos \Phi} \right) + \cos^{-1} \left(\frac{N + 2LR_1 \cos \Phi}{2R_2 M_1} \right) \\ M_1 &= \sqrt{L^2 + R_1^2 + 2LR_1 \cos \Phi} \\ \omega_2 / \omega_1 &= d\Psi / d\Phi = F'(\Phi) \end{aligned} \right\} \dots\dots\dots (5)$$

$$\left. \begin{aligned} \Phi &= G(\Psi) = \pi - (\gamma_1 + \gamma_2) \\ &= \pi - \tan^{-1} \left(\frac{R_2 \sin \Psi}{L - R_2 \cos \Psi} \right) \\ &\quad - \cos^{-1} \left(\frac{N - 2LR_2 \cos \Psi}{2R_1 M_2} \right) \\ M_2 &= \sqrt{L^2 + R_2^2 - 2LR_2 \cos \Psi} \\ \omega_2 / \omega_1 &= d\Psi / d\Phi = 1/G'(\Psi) \end{aligned} \right\} \dots\dots\dots (6)$$

where ω_1 : input angular velocity

ω_2 : output angular velocity

$N = L^2 + R_1^2 + R_2^2 - S_0^2 = L^2 + 2R_1 R_2 \cos \beta$: constant of the mechanism

4.1.1 The mechanism with $r=0$

The fixed block double-slider crank mechanism

Fig. 5 Deduction process of the generalized Oldham coupling

Fig. 6 Double crank mechanism

of this type is shown in Fig. 7. Here $R_1 = \infty$, $0 < R_2 < \infty$, and by taking $L = lR_2$, the basic dimensions are β , $r = 0$, and l . From Eqs. (5) and (6), we get

$$\left. \begin{aligned} \Psi = F(\Phi) &= \Phi + \cos^{-1}(\cos \beta + l \cos \Phi) \\ F'(\Phi) &= \frac{d\Psi}{d\Phi} = 1 + \frac{l \sin \Phi}{\sqrt{1 - (\cos \beta + l \cos \Phi)^2}} \\ \omega_2/\omega_1 &= F'(\Phi) \end{aligned} \right\} \dots\dots\dots (7)$$

$$\left. \begin{aligned} \Phi = G(\Psi) &= \pi - \tan^{-1} \left(\frac{\sin \Psi}{l - \cos \Psi} \right) - \cos^{-1} \left(\frac{\cos \beta}{m_2} \right) \\ G'(\Psi) &= \frac{d\Phi}{d\Psi} = \frac{1 - l \cos \Psi}{m_2^2} - \frac{l \cos \beta \sin \Psi}{m_2^2 \sqrt{m_2^2 - \cos^2 \beta}} \\ \omega_2/\omega_1 &= 1/G'(\Psi) \\ m_2 &= M_2/R_2 = \sqrt{1 + l^2 - 2l \cos \beta} \end{aligned} \right\} \dots\dots\dots (8)$$

4.1.2 The mechanism with $0 < r < \infty$

The fixed block double-slider crank mechanism of this type is shown in Fig. 3. Here $0 < R_1 < \infty$, $0 < R_2 < \infty$, and by taking $R_2 = rR_1$ and $L = lR_1$, we get

Fig. 7 Fixed block double circular-arc-slider crank mechanism ($r=0$)

Fig. 8 Fixed block double circular-arc-slider crank mechanism ($r=\infty$)

$$\left. \begin{aligned} \Psi = F(\Phi) &= \tan^{-1} \left(\frac{\sin \Phi}{l + \cos \Phi} \right) + \cos^{-1} \left(\frac{n + 2l \cos \Phi}{2rm_1} \right) \\ F'(\Phi) &= \frac{1 + l \cos \Phi}{m_1^2} + \frac{(2m_1^2 - 2l \cos \Phi - n)l \sin \Phi}{m_1^2 \sqrt{(2rm_1)^2 - (n + 2l \cos \Phi)^2}} \\ \omega_2/\omega_1 &= F'(\Phi) \\ \Phi = G(\Psi) &= \pi - \tan^{-1} \left(\frac{r \sin \Psi}{1 - r \cos \Psi} \right) - \cos^{-1} \left(\frac{n - 2lr \cos \Psi}{2m_2} \right) \\ G'(\Psi) &= \frac{r(r - l \cos \Psi)}{m_2^2} + \frac{(2m_2^2 + 2lr \cos \Psi - n)lr \sin \Psi}{m_2^2 \sqrt{4m_2^2 - (n - 2lr \cos \Psi)^2}} \\ \omega_2/\omega_1 &= 1/G'(\Psi) \end{aligned} \right\} \dots\dots\dots (9)$$

$$\left. \begin{aligned} \Psi = F(\Phi) &= \tan^{-1} \left(\frac{\sin \Phi}{l + \cos \Phi} \right) + \cos^{-1} \left(\frac{\cos \beta}{m_1} \right) \\ F'(\Phi) &= \frac{1 + l \cos \Phi}{m_1^2} - \frac{l \cos \beta \sin \Phi}{m_1^2 \sqrt{m_1^2 - \cos^2 \beta}} \\ \omega_2/\omega_1 &= F'(\Phi) \\ m_1 &= M_1/R_1 = \sqrt{1 + l^2 + 2l \cos \beta} \end{aligned} \right\} \dots\dots\dots (10)$$

where

$$\begin{aligned} m_1 &= M_1/R_1 = \sqrt{1 + l^2 + 2l \cos \beta} \\ m_2 &= M_2/R_1 = \sqrt{l^2 + r^2 - 2lr \cos \beta} \\ n &= N/R_1^2 = l^2 + 2r \cos \beta \end{aligned}$$

4.1.3 The mechanism with $r = \infty$

The fixed block double-slider crank mechanism of this type is shown in Fig. 8. Here $0 < R_1 < \infty$, $R_2 = \infty$, and by taking $L = lR_1$, the basic dimensions are β , $r = \infty$, and l . From Eqs. (5) and (6), we get

$$\left. \begin{aligned} \Psi = F(\Phi) &= \tan^{-1} \left(\frac{\sin \Phi}{l + \cos \Phi} \right) + \cos^{-1} \left(\frac{\cos \beta}{m_1} \right) \\ F'(\Phi) &= \frac{1 + l \cos \Phi}{m_1^2} - \frac{l \cos \beta \sin \Phi}{m_1^2 \sqrt{m_1^2 - \cos^2 \beta}} \\ \omega_2/\omega_1 &= F'(\Phi) \\ m_1 &= M_1/R_1 = \sqrt{1 + l^2 + 2l \cos \beta} \end{aligned} \right\} \dots\dots\dots (11)$$

$$\left. \begin{aligned} \Phi = G(\Psi) &= \Psi - \cos^{-1}(\cos \beta - l \cos \Psi) \\ G'(\Psi) &= 1 + \frac{l \sin \Psi}{\sqrt{1 - (\cos \beta - \cos \Psi)^2}} \\ \omega_2/\omega_1 &= 1/G'(\Psi) \end{aligned} \right\} \dots\dots\dots (12)$$

4.1.4 The mechanism with $R_1 = \infty$, $R_2 = \infty$

The fixed block double-slider crank mechanism of this type is shown in Fig. 9. From Eqs. (5) and

Fig. 9 Fixed block double circular-arc-slider crank mechanism ($R_1 = \infty$, $R_2 = \infty$)

(6), we get

$$\left. \begin{aligned} \Psi &= \Phi + \beta \\ \omega_2/\omega_1 &= d\Psi/d\Phi = 1 \end{aligned} \right\} \dots\dots\dots (13)$$

The turning block double-slider mechanism with $\beta=90^\circ$ is the Oldham coupling. From the facts mentioned above, the turning block double circular-arc-slider mechanism is considered to be a generalized form of the Oldham coupling, so this mechanism will be called the generalized Oldham coupling.

4.2 Input and output angle of rotation when $\pi < \beta < 2\pi$

A general type of the fixed block double-slider crank mechanism is shown in Fig. 10, where the term β' is adopted newly instead of β , and $2\pi - \beta' = \bar{\beta}$. The mechanism as viewed towards Fig. 10 from the back side of the paper is the same as one with $0 < \beta (= \bar{\beta}) < \pi$ in Fig. 3, except that both sliders A and B are reversed. Consequently, the relations among the input, the output angle of rotation and the angular velocity ratio are obtained by substituting Φ , Ψ and β in Eqs. (5)~(13) by $2\pi - \Phi$, $2\pi - \Psi$, and $\bar{\beta}$ respectively.

When $r=0$, the fixed block double-slider crank mechanism is the same as one in Fig. 7 and when $r=\infty$, the mechanism is the same as one in Fig. 8.

Taking $\beta' = \beta + \pi$, and $\bar{\beta} = 2\pi - \beta'$, we have $0 < \beta$, $\bar{\beta} < \pi$, and $\beta + \bar{\beta} = \pi$. The mechanism with β' and one with β are the same and yield the same motion. The motion yielded by the mechanism with β' is equal to the reverse motion by one with β of the value $\bar{\beta}$. So it is generally concluded that when $0 < \beta_1, \beta_2 < \pi$, and $\beta_1 + \beta_2 = \pi$, the reverse angular motion generated by the mechanism with β_1 is equal to that by the mechanism with β_2 . When $\beta_1 = \beta_2$,

Fig. 10 Fixed block double circular-arc-slider crank mechanism ($\pi < \beta < 2\pi$)

Fig. 11 Output angle of rotation Ψ versus input Φ

Fig. 13 Angular velocity ratio ω_2/ω_1 versus output ψ

Fig. 12 Angular velocity ratio ω_2/ω_1 versus input ϕ

two motions agree precisely. These facts are clearly shown in Figs. 12 and 13.

When $R_1 = \infty$, $R_2 = \infty$, the fixed block double-slider crank mechanism is identical with one of Fig. 9; this coupling transmits rotation with a constant angular velocity ratio of unity.

4.3 Numerical calculated results

In Fig. 4, the mark $\circ$ means the mechanism for which only its theoretical values of Φ , Ψ and ω_2/ω_1 are calculated, while the mark $\bullet$ does one for which both theoretical and experimental values are obtained. In order to get a whole picture of the variable speed angular motions yielded by this mechanism or those by the double crank mechanism, theoretical values of Φ , Ψ and ω_2/ω_1 are calculated for many mechanisms. The theoretical values of $\Psi = F(\Phi)$, $\omega_2/\omega_1 = F'(\Phi)$, and $\omega_2/\omega_1 = 1/G'(\Psi)$ are shown in Figs. 11~13, where the mechanism is called M1-mechanism or M2-mechanism according as $0 < \beta < \pi$ or $\pi < \beta < 2\pi$. Let $S_{\min}$ be the minimum length of S_1 , S_2 , and S_3 , then the distance L between two shafts is in the range of $0 \leq L \leq S_{\min}$. The coefficient k is obtained by taking $L = kS_{\min}$.

5. Experimental verification

5.1 Experimental apparatus and method

The experimental apparatus is diagrammatically shown in Fig. 14. This apparatus measures Φ or Ψ , ω_1 , and ω_2 at the same time. Rotation is transmitted from the motor to the input shaft by the worm gearset. The input angular velocity ω_1 and the output one ω_2 are measured by the generators G_1 and G_2 , respectively. These generators are low speed tachometers which generate the direct current proportional to the angular velocity. In experiments, the input angular velocity ω_1 is constant (45 rpm) and the velocity ω_2 may be regarded as the velocity ratio ω_2/ω_1 when both velocities are recorded on the scale taking $\omega_1 = 1$. The input Φ and the output angle of rotation Ψ are detected by the potentiometers. The distance L between two shafts is measured by the dialgauge. In this case, the standard position of the output shaft with $L=0$ is determined by $\omega_2 = \omega_1$. Experiments were performed on the mechanism with k of 0, 0.1, 0.2, ..., in this order. Figure 15 shows one of the coupling specimens.

5.2 Results

The theoretical and the experimental angular velocity ratios ω_2/ω_1 of M1-mechanism are shown in Figs. 16 and 17. Both values agree well. The experimental results of other mechanisms shown by the mark $\bullet$ in Fig. 4 also agree well with the theoretical ones, though these results are not illustrated here. From the results of this experiment, it is inferred that the relation among Φ , Ψ and ω_2/ω_1 obtained by

the analysis holds true for the generalized Oldham coupling with any β , r , and l . When $k=1.0$, the distance L tends to maximum. The practical maximum value of k was about 0.8 in this experiment. The maximum and the minimum value of ω_2/ω_1 were approximately 3.0 and 0.2 respectively. These limits are considered to depend on the dynamical conditions such as friction, pressure angle, etc.

Fig. 14 Experimental apparatus

Fig. 15 Generalized Oldham coupling specimen ($\beta = 90^\circ$, $r = 0.5$)

Fig. 16 Theoretical and experimental angular velocity ratio ω_2/ω_1 ($\beta = 60^\circ$)

Fig. 17 Theoretical and experimental angular velocity ratio ω_2/ω_1 ($\beta = 90^\circ$)

6. Conclusions

The existence of the generalized Oldham coupling is deduced. The relative angular motion between two shafts connected by this coupling is analysed and is verified by experiments. This motion will be called briefly "the motion by the coupling" in the following.

(1) The generalized Oldham coupling is a turning block double-slider mechanism having circular-arc-sliders. This coupling is a new mechanism which generates a cyclical varying speed angular motion. This mechanism is an inversion obtained from the double crank mechanism through the double lever mechanism and the fixed block double circular-arc-slider crank mechanism (see Fig. 7).

(2) The motion by the generalized Oldham coupling is identical with the relative angular motion between two cranks of the double crank mechanism; that is, this coupling has one to one correspondence to the double crank mechanism on the condition that two mechanisms yield the same motion. The length of each link of the double crank mechanism is obtained by giving the basic dimensions of the coupling β , r , and l .

(3) Both when $r=0$ and when $r=\infty$, the generalized Oldham coupling corresponds to the double crank mechanism having one crank of infinite length. In this case, though the latter mechanism cannot practically exist, the coupling does. Consequently,

the motion by these couplings is considered to be a new one. Besides, if $L \leq S_{\min}$, the fixed block double-slider crank mechanism also has been unknown so far (Figs. 7 and 8).

(4) The Oldham coupling corresponds to one of the double crank mechanism having both cranks of infinite length ($R_1 = \infty$, $R_2 = \infty$, $\beta = 90^\circ$, $0 \leq L \leq \infty$) and is considered to be a special case of the generalized Oldham coupling.

(5) All the motions by the generalized Oldham coupling are analysed (Figs. 11~13). The experimental values agree well with the theoretical ones (Figs. 16 and 17).

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