Studies on Oil Hammer and Transient Response in Oil Pipeline*
(1st Report, In the Case of a Straight Uniform Pipeline)

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In this report, the pressure surge resulting from a step change of an outlet flow is called the oil hammer and that of an inlet flow is called the transient response in a pipeline. The response waves of the pressure at an arbitrary position of the pipeline have been investigated by theoretical analyses and experiments under various line-conditions. The theoretical solutions involving the influence of a reflecting wave in the oil hammer and the transient response have been derived by using the equivalent viscous resistance. The equivalent viscous resistance has been derived by comparing between the viscous resistance of an unsteady flow and that of a steady one. It is shown that the theoretical solutions are in good agreement with the experimental results under various line-conditions.

1. Introduction

Many studies on the dynamic characteristics of the oil pipelines have been performed recently. The oil hammers and the transient responses have not been studied so far because of the difficulty of a Laplace inverse transform in comparison with the frequency response. But, quite recently, numbers of studies have been executed by the methods of the approximate solutions obtained by limiting the applicable ranges and by the numerical analyses using an electronic computer(1)-(3,7).

F. T. Brown and S. E. Nelson(1,2) have investigated the step and impulse responses in the case of non-influence of a reflecting wave. D. N. Contractor(2) has numerically analyzed the partial differential equations by using the electronic computer and compared the theoretical values with the experimental results. E. L. Holmboe and W.T. Rouleau(4) have derived the analytical solutions which are applicable to the first cycle in the pressure pulse responses and the oil hammers, and they have compared the theoretical values with the experimental results. W. Zielke(5) has investigated the water hammer considering the frequency-dependent viscous resistance by the method of the numerical analysis using the electronic computer.

The objects of this report are to obtain the analytical solutions of the shape of the pressure response at an arbitrary position of the pipeline in the oil hammer produced by a quick-closing valve at the outlet of the pipeline and in the transient response due to the step change of inlet flow, and are to compare the theoretical calculations with the experimental results. The theoretical solutions have been analyzed by introducing the concept of the equivalent viscous resistance depending on the frequency.

Principal nomenclature

\[ P = \bar{P} + p: \text{ pressure (} \bar{P}: \text{mean pressure)} \text{ kg/cm}^2 \]
\[ P(s): \text{ Laplace transform of pulsating pressure} \]
\[ Q = \bar{Q} + q: \text{ flow rate (} \bar{Q}: \text{mean flow rate)} \text{ cm}^3/\text{sec} \]
\[ Q(s): \text{ Laplace transform of pulsating flow rate} q \]

Concerning the pressure and flow rate, suffixes 1 and 2 denote inlet and outlet of pipeline, respectively, and suffix x denotes distance from outlet of pipeline in the case of oil hammer or that from inlet of pipeline in the case of transient response.

\[ R_f: \text{ viscous resistance of steady laminar flow } 1/\text{sec} \]
\[ R_{fx}, R_{fx'}, R_{xx}: \text{ equivalent viscous resistance } 1/\text{sec} \]
\[ s: \text{ Laplace variable} \]
\[ t: \text{ time } \text{ sec} \]
\[ \zeta_x: \text{ damping factor} \]
\[ \omega_{xx}: \text{ natural angular frequency rad/sec} \]

2. Basic expressions for pipeline

In Fig. 1, the expressions of the transfer matrix for the pipeline are as follows(6):

\[
\begin{align*}
P(s) &= \begin{bmatrix} \cosh \gamma(s)l_1 & Z_0 \sinh \gamma(s)l_1 \end{bmatrix} P(s) \\
Q(s) &= \begin{bmatrix} \cosh \gamma(s)l_1 & Z_0 \sinh \gamma(s)l_1 \end{bmatrix} Q(s)
\end{align*}
\]

\[ \cdots \cdots \cdots \cdots \cdots (1) \]
\[ P(s) = \left[ \begin{array}{c} \cosh \gamma(s)l_2 \\ Z_0 \sinh \gamma(s)l_2 \end{array} \right] \frac{P_2(s)}{Q_2(s)} \]
\[ Q(s) = \left[ \begin{array}{c} (1/Z_0) \sinh \gamma(s)l_2 \\ \cosh \gamma(s)l_2 \end{array} \right] \frac{P_1(s)}{Q_1(s)} \]

where, \( Z_0 = \rho d^2 \sqrt{\gamma(s)} \), \( a \): velocity of pressure wave, \( \rho \): density of oil, \( d \): inner diameter of pipeline, \( f = \pi d^2/4 \), and \( \gamma(s) \) is defined as follows.

In the case of the unsteady laminar flow resistance,
\[
\gamma(s) = s/a \left[ 1 - \frac{2}{j \rho \sqrt{s}/v} \frac{J_0(j \rho \sqrt{s}/v)}{J_0(j \rho \sqrt{s}/v) \sqrt{s}/v} \right]^{1/2} 
\]
where, \( J_0, J_1 \): Bessel functions of first kind, \( \nu \): coefficient of kinematic viscosity of oil.

In the case of the steady laminar flow resistance,
\[
\gamma(s) = \sqrt{s^2 + R_f \delta s/a} 
\]
where, \( R_f = 32 \nu d^2 \).

When \( \gamma(j\omega) = \alpha^* + j\beta^* \) is introduced in the case of the unsteady laminar flow resistance and \( \gamma(j\omega) = \alpha^* + j\beta^* \) is introduced in the case of the steady laminar flow resistance, \( \alpha^*, \beta^*, \alpha' \) and \( \beta' \) are expressed as follows:
\[
\alpha^* = \frac{\omega}{\sqrt{2}} \frac{\chi}{\sqrt{2} \lambda + 1}, \quad \beta^* = \frac{\omega}{\sqrt{2}} \frac{\chi}{\sqrt{2} \lambda + 1} 
\]
\[
\alpha' = \frac{(\omega/a)(1 + (R_f/\omega)^2)}{\sqrt{2} \lambda + 1}, \quad \beta' = \frac{(\omega/a)(1 + (R_f/\omega)^2)}{\sqrt{2} \lambda + 1} \]

where, \( \chi = r_0 \sqrt{\omega/v}, \omega \): angular frequency.

As we have \( \beta^* = \beta' \equiv \beta^* = \beta^*(\omega) \) at the high angular frequency, putting \( R_f = R_{fr} \) at \( \alpha^* = \alpha' \), \( R_{fr} \) becomes as follows:
\[
R_{fr} = \frac{\omega}{\lambda} \sqrt{\lambda^2 + 2} 
\]
where, \( \lambda = \chi/\sqrt{2} \lambda + 1 \). \( R_{fr} \) is called the equivalent viscous resistance.

As we have \( \chi \geq 3 \) and \( \lambda \equiv 1/\chi < \sqrt{2} \) at the high angular frequency, Eq. (7) is rewritten as follows:
\[
R_{fr} = \frac{\sqrt{\lambda^2 + 2} \omega}{\lambda} \chi = \sqrt{2} \omega \sqrt{\omega} \sqrt{r_0} 
\]
where, we put \( R_{fr} \) at the high angular frequency as \( R_{fr}^* \). Therefore, according to Eq. (8), it is seen that the equivalent viscous resistance is larger than the angular frequency is higher. Figure 2 shows the comparisons of \( R_f, R_{fr} \) and \( R_{fr}^* \), and when \( \chi = 3.817, R_{fr} \) coincides with \( R_{fr}^* \).

Using \( R_{fr}^* \) in place of \( R_f \) in Eqs. (4) and (6), \( \gamma(s) \) and \( \gamma(j\omega) = \alpha + j\beta \) are redefined as follows:
\[
\gamma(s) = \sqrt{s^2 + R_f \delta s/a} 
\]
(9)
\[
\gamma(j\omega) = \alpha + j\beta \equiv \frac{\alpha}{(\omega/a)(1 + (R_f/\omega)^2)} \sin((1/2) \arctan(R_f/\omega)) 
\]
(10)
where, \( R_f \) is replaced with \( R_{fr} \) for \( \chi \leq 3.817 \).

In Eqs. (5) and (10), as \( \alpha \) coincides with \( \alpha^* \), and we have \( \beta = \beta^* \) at the high angular frequency, Eqs. (9) and (10) coincide with Eqs. (3) and (5), approximately. At the low angular frequency, namely \( \chi \geq 3.817 \), Eqs. (9) and (10) coincide with Eqs. (4) and (6). Table 1 shows the comparison between \( \beta \) and \( \beta^* \).

### Table 1: Comparison between \( \beta \) and \( \beta^* \)

<table>
<thead>
<tr>
<th>( \chi )</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta*(\omega/a) )</td>
<td>1.1948</td>
<td>1.1340</td>
<td>1.0699</td>
<td>1.0353</td>
<td>1.0235</td>
</tr>
<tr>
<td>( \beta/\omega )</td>
<td>1.0657</td>
<td>1.0173</td>
<td>1.0033</td>
<td>1.0007</td>
<td>1.0003</td>
</tr>
<tr>
<td>Error %</td>
<td>10.811</td>
<td>10.291</td>
<td>6.221</td>
<td>3.342</td>
<td>2.272</td>
</tr>
</tbody>
</table>

### 3. Oil hammer

In this section, we deal with the shape of pressure response produced at an arbitrary position by closing quickly the valve at the outlet of the pipeline in Fig. 3. The boundary condition at the inlet of the pipeline is \( P_1 = \text{constant}, \)
\[
P(z) = 0 
\]
(11)
From Eqs. (1), (2) and (11), the transfer function \( G_0(z) = P_2(z)/Q_2(z) \) becomes as follows:

![Fig. 1 Pipeline](image1.png)

![Fig. 2 Viscous resistances](image2.png)
where, \( l \): length of pipeline, \( K = f / \rho a^2 \).

Expressing Eq. (12) as an infinite product with the aid of Eq. (9), it is rewritten as follows:

\[
G_0(s) = \frac{\gamma(s)l \sinh \gamma(s)(1-x)}{Ks \cosh \gamma(s)l} \quad \text{(12)}
\]

where, \( \omega_{nk} = \frac{\gamma_{nk}}{T_n}, \quad \gamma_{nk} = \frac{R_n k^2}{2 \omega_{nk}}, \quad \xi_{nk} = (2k-1)\pi / 2, \quad T_n = l / \alpha, \quad \omega_{nk} = \xi_{nk} / T_n, \quad \gamma_{nk} = R_n k^2 / 2 \omega_{nk}, \quad \xi_{nk} = k \pi, \quad T_{ss} = (l-x) / \alpha, \quad \alpha, \quad R_n \text{ and } R_k \text{ are the values of } R_\alpha \text{ or } R_\beta^* \text{, when } \omega_{nk} \text{ and } \omega_{nk} \text{ are substituted into the angular frequencies } \omega \text{ in Eqs. (7) and (8), respectively.}

The shape of the pressure response generated by the quick-closing valve at the outlet of the pipeline corresponds to one due to the step change \( Q_\alpha s = -Q_\beta s = -Q_0 s \). Therefore, \( P_\alpha(s) / \rho u_0 a \) becomes as follows:

\[
P_\alpha(s) = \frac{\gamma(s)l \sinh \gamma(s)(1-x)}{T_n s^2 \cosh \gamma(s)l} \quad \text{(14)}
\]

where, \( u_0 = Q_\alpha / f = Q_\beta / f \).

Finding the expressions of the inverse transform, \( (P_\alpha(t) - (P_\beta - P_\alpha)) / \rho u_0 a \), from Eq. (14), they are written as follows finally.

For \( \xi_{nk} < 1 \)

\[
P_\alpha(t) - (P_\beta - P_\alpha) = -2 \sum_{k=1}^{\infty} \frac{e^{-\xi_{nk}t} / T_n}{\omega_{nk} \sqrt{1 - \xi_{nk}^2}} \cos \left( \frac{\pi x}{2} \right) \times \sin \left( \xi_{nk} t / T_n + 2 \phi_{nk} \right) \quad \text{(15)}
\]

For \( \xi_{nk} = 1 \)

\[
P_\alpha(t) - (P_\beta - P_\alpha) = \frac{4}{\pi} \frac{e^{-t / T_n}}{T_n} \cos \left( \frac{\pi x}{2} \right) \times 2 \sum_{k=1}^{\infty} \frac{e^{-\xi_{nk}t} / T_n}{\omega_{nk} \sqrt{1 - \xi_{nk}^2}} \cos \left( \xi_{nk} t / T_n + 2 \phi_{nk} \right) \quad \text{(16)}
\]

For \( \xi_{nk} > 1 \) (When \( k = m, \xi_{nk} = 1 \))

\[
P_\alpha(t) - (P_\beta - P_\alpha) = \frac{m-1}{2} \sum_{k=1}^{\infty} \frac{e^{-\xi_{nk}t} / T_n}{\omega_{nk} \sqrt{1 - \xi_{nk}^2}} \cos \left( \xi_{nk} t / T_n \right) \times \left( 2 \xi_{nk} \sqrt{\xi_{nk}^2 - 1} \cos \xi_{nk} - \xi_{nk}^2 - 1 \right) / \xi_{nk} \quad \text{(17)}
\]

\[
-4 \frac{T_n}{T_n} \sum_{k=1}^{\infty} \frac{e^{-\xi_{nk}t} / T_n}{\omega_{nk} \sqrt{1 - \xi_{nk}^2}} \cos \left( \xi_{nk} t / T_n \right) \times \left( 1 - \phi_{nk} \right) \quad \text{(18)}
\]

\[
\text{where, } \phi_{nk} = \arctan \left( \sqrt{1 - \xi_{nk}^2} / \xi_{nk} \right). \text{ When } \xi \leq 3.817, \text{ namely } \xi_{nk} \leq 0.2745, \text{ } R_n = R_\alpha \text{ or } R_\beta^*.
\]

Neglecting the viscous resistance, Eq. (15) is rewritten as

\[
P_\alpha(t) = \sum_{k=1}^{\infty} \frac{4}{(2k-1) \pi} \cos \left[ \frac{(2k-1)\pi x}{2} \right] \times \sin \left[ \frac{(2k-1)\pi t}{2T_n} \right] \quad \text{(18)}
\]

Equation (18) is identical with the well-known expression of the water hammer neglecting the viscous resistance. In Eqs. (15)~(18), the shape of the pressure response at \( x = 0 \) becomes one at the outlet of the pipeline.

4. Transient response

In this section, we treat the shape of the pressure response generated by the step change of inlet flow rate, as shown in Fig. 4. The boundary condition at the outlet of the pipeline is \( P_\alpha = 0 \) constant, namely

\[
P_\alpha(s) = 0 \quad \text{(19)}
\]

From Eqs. (1), (2) and (19), the transfer function \( G_\alpha(s) = P_\alpha(s)/Q_\alpha(s) \) becomes as follows:

\[
G_\alpha(s) = \frac{\gamma(s)l \sinh \gamma(s)(1-x)}{Ks \cosh \gamma(s)l} \quad \text{(20)}
\]

Expressing Eq. (20) in a nondimensional form, it is rewritten as

\[
G_\alpha^*(z) = \frac{P_\alpha(s)}{(P_\alpha - P_\beta) / Q_\alpha(s)} \quad \text{(21)}
\]

where, \( Q_\alpha / (P_\alpha - P_\beta) = f / (32(l-x) - K R_T T_{ss} \mu) \text{ coefficient of viscosity of oil.} \)

As the expression obtained by multiplying Eq. (12) by \(-K R_T T_{ss}\) is identical with Eq. (21), the nondimensional expression of Eq. (21) is identical with that obtained by multiplying Eq. (13) by \(-K R_T T_{ss}\).

The boundary condition at the inlet of the pipeline is the step input of flow rate,
\[ q_1(t) = \begin{cases} \frac{|q_0|}{\mathcal{Q}_1} & (t \geq 0) \\ 0 & (t < 0) \end{cases} \]  \hspace{1cm} (22)

The following expression is obtained from Eqs. (21) and (22).

\[ \frac{P_2(s)}{(P_2 - P_3)} = \frac{\gamma(s)l \sinh [\gamma(s)(l-x)]}{R_l T_e s^2 \cosh \gamma(s)l} \]  \hspace{1cm} (23)

Equation (23) is identical with the expression obtained by multiplying Eq. (14) by \(1/R_l T_e\).

\[ (P_2(t)/(P_2 - P_3))/(q_0/\mathcal{Q}_1) \]  \hspace{1cm} (23)

is identical with the expression adding unity to those obtained by multiplying Eqs. (15) \sim (17) by \(1/R_l T_e\), respectively.

5. Damping properties

For \(\zeta_{st} < 1\), the shape of the pressure response becomes a damped vibration. Figure 5 shows an example of the numerical calculations. The shape of the pressure response is indicated as the sum of the series. Since the equivalent viscous resistance is larger as the angular frequency becomes higher, the ratio of damping becomes larger as \(k\) is larger. In the case of the theoretical calculation, it is necessary to use many terms of the series in the range of small values of \(t/T_e\), but it is not necessary to use many terms of the series in the range of larger values of \(t/T_e\), because the shape of the pressure response is chiefly governed by only the term of \(k=1\) in the range of large values of \(t/T_e\). Therefore, it is possible to indicate the expressions of the period, maximum and minimum of the shape of the pressure response by only the term of \(k=1\), approximately.

From Eq. (15), the expression indicated by only the term of \(k=1\) becomes as follows:

\[ F_1(t/T_e) = -\frac{4e^{-2\zeta_{st} t/T_e}}{\pi \sqrt{1-\zeta_{st}^2}} \times \sin \left( \frac{\pi}{2} - \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (24)

From Eq. (24), the period \(T\) of the shape of the pressure response is written as

\[ T = \frac{4}{\pi \sqrt{1-\zeta_{st}^2}} \]  \hspace{1cm} (25)

Putting \(t/T_e\) which gives the maximum and minimum of it as \(t_1/T_e\), we have

\[ \frac{t_1}{T_e} = \frac{(2i+1)\pi - 4\zeta_{st}}{\pi \sqrt{1-\zeta_{st}^2}} \]  \hspace{1cm} (26)

where, when \(i\) is an odd number, \(t_1/T_e\) gives the maximum to the shape of the pressure response, and when \(i\) is an even number, \(t_1/T_e\) gives the minimum to it.

The envelope joining the maximums of \(F_1(t/T_e)\), \(f_1(t/T_e)\), and one joining the minimums of it, \(f_1^*(t/T_e)\), are written as follows, respectively:

\[ f_1(t/T_e) = \frac{-4}{\pi \sqrt{1-\zeta_{st}^2}} \cos \left( \frac{\pi}{2} - \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (27)

\[ f_1^*(t/T_e) = \frac{-4}{\pi \sqrt{1-\zeta_{st}^2}} \cos \left( \frac{\pi}{2} + \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (28)

From Eq. (26), \(t/T_e\), giving the first maximum value of \(F_1(t/T_e)\) becomes as follows:

\[ \frac{t_1}{T_e} = \frac{3\pi - 4\zeta_{st}}{\pi \sqrt{1-\zeta_{st}^2}} \]  \hspace{1cm} (29)

and the first maximum value is written as

\[ f_{max} = f_1(t_1/T_e) = \frac{-4}{\pi \sqrt{1-\zeta_{st}^2}} \times \cos \left( \frac{\pi}{2} + \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (30)

Substituting Eq. (29) into Eq. (30), we have

\[ f_{max} = \frac{4}{\pi \sqrt{1-\zeta_{st}^2}} \times \cos \left( \frac{\pi}{2} + \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (31)

Figures 6 and 7 show the damping properties, \(t_1/T_e\) and the first maximum versus an arbitrary damping factor \(\zeta_{st}\), respectively.

The consecutive amplitude ratio \(f_1(t_1/T_e)\) \(f_1(t_1, t_1/T_e)\) and logarithmic decrement \(\delta\) become as follows:

\[ f_1(t_1/T_e) \approx \cos \left( \frac{\pi}{2} + \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (32)

\[ f_1(t_1/T_e) \approx \cos \left( \frac{\pi}{2} + \frac{2\pi t}{T_e} \right) \]  \hspace{1cm} (33)

Fig. 6 Damping properties

Fig. 7 The first maximum values and \(t_1/T_e\) versus an arbitrary damping factor \(\zeta_{st}\)
6. Experiments and considerations on oil hammer

6.1 Experimental apparatus and method

The apparatus used in the experiment of the oil hammer is schematically shown in Fig. 8. The accumulator and the quick-acting solenoid directional control valve manufactured for this experiment are attached at the inlet and outlet of the test pipeline, respectively. The pressure transducers used in the experiments are of the strain gage type and are mounted in the valve and in the middle of the pipeline, respectively. The shapes of the pressure response

in the valve and in the middle of the pipeline resulting from an instantaneous valve-closure have been recorded by the oscilloscope. Figure 9 shows the movement of a spool, namely, the time required for the valve-closure and the rising state of the pressure surge at the valve. The time required has been between 0.003 and 0.005 seconds. According to this figure, it is seen that the rising of the pressure surge begins as soon as the valve closes perfectly. Figure 10 shows an example of the record of the pressure surge, namely, the shape of the pressure response. Table 2 shows the specifications of the pipeline systems used in the experiments. Satisfying (mean pressure)/$\rho_{100}$ > 1 in the experimental conditions, the vacuum state does not come out in the shape of the pressure response. Therefore, the experiments have been performed in the range where the mean pressure is higher than $\rho_{100}$.

6.2 Experimental results and considerations

Figures 11–14 show the shapes of the pressure response to the various pipeline conditions. In these figures, the theoretical calculations have been performed using Eqs. (15), (18) and putting $R_{ok}$ as $R_f$ in Eq. (15). They are called the values being based on the equivalent viscous resistance, the non-viscous resistance and the resistance of steady laminar flow, respectively. The theoretical calculations have been performed by considering the terms until $k=4$ for $\zeta_{\alpha}=0.058, 0.178$, and until $k=3$ for $\zeta_{\alpha}=0.379, 0.601$. The pressure pulsations of high frequency and small amplitude appear in the theore-

![Fig. 8 Experimental apparatus in oil hammer](image)

![Fig. 9 Movement of spool and trace of pressure surge](image)

![Fig. 10 Shapes of pressure response in oil hammer](image)

<table>
<thead>
<tr>
<th>Table 2 Specifications of pipeline systems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length of pipeline $l$ m</strong></td>
</tr>
<tr>
<td>------------------------------------------</td>
</tr>
<tr>
<td>10.8</td>
</tr>
<tr>
<td>11.8</td>
</tr>
<tr>
<td>11.8</td>
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<td>18.7</td>
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</table>
tical curves. These pressure pulsations are due to the number of the terms of the series used in the calculation. It is possible that they are taken away by calculating with the aid of more terms. Agreement between the theoretical calculations and the experimental results is comparatively good, even if the pressure pulsations appear a little in the theoretical curve. It is a good method of the theoretical calculation to take consideration of many numbers of the terms of the series in the range of small values of \( t/T_e \) and to decrease them as \( t/T_e \) becomes larger, because the damping of the high frequency is fast. The damping properties are shown by Eqs. (25)\textasciitilde(33), Figs. 6 and 7. According to Figs. 11\textasciitilde14, the states of the damping of the theoretical calculations being based on the equivalent viscous resistance are in good agreement with those of the experimental results, even if \( \zeta_{n1} \) is small. In the range of \( \zeta_{n1} < 0.2745 \), the agreement between the theoretical calculations being based on the viscous resistance of steady laminar flow and the experimental results are good in the case of \( t/T_e \leq 2 \), but they are poor in the case of \( t/T_e \geq 2 \). In the range of \( \zeta_{n1} \geq 0.2745 \), the theoretical calculations being based on the viscous resistance of steady laminar flow well agree with the experimental results. In the case of \( \zeta_{n1} \geq 0.2745 \) and \( \zeta_{n2} < 0.2745 \), comparing the theoretical calculations using \( R_{n1} = R_f \) in the first term of the series and \( R_{n2} \) in the second and succeeding terms with ones using \( R_{n1} = R_f \) in all terms, a remarkable difference didn't appear in both procedures. Hence, the theoretical curves in Figs. 13 and 14 are calculated only by the latter procedure. Consequently, it is possible to estimate the shape of the pressure response by the theoretical calculation being based on the viscous resistance of steady laminar flow in all terms of the series for \( \zeta_{n1} \geq 0.2745 \). In the case of the nonviscous resistance, a damping does not happen, and the theoretical calculations comparatively well agree with the experimental results only in the range of the limited conditions, namely \( \zeta_{n1} < 0.1 \), \( \nu/L=0 \), \( t/T_e \leq 2 \).

7. Experiments and considerations of transient response

Figure 15 shows schematically the experimental apparatus of the transient response. The hydraulic oils delivered from an oil hydraulic pump flow through the test pipeline and the quick-acting solenoid directional control valve. Closing this valve quickly, the hydraulic oil flow to the test pipeline. It means that the step changing flow rate is added at the inlet of

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**Fig. 13** Shapes of pressure response in oil hammer

**Fig. 14** Shapes of pressure response in oil hammer
8. Conclusions

The conclusions drawn from this study are summarized as follows:

(1) The expression of the equivalent viscous resistance depending on the frequency has been derived by comparing the viscous resistance of unsteady laminar flow with that of steady one.

(2) The theoretical solutions for the shape of the pressure response have been derived by indicating the transfer function in an infinite product and making a Laplace inverse transform. The theoretical calculations comparatively well agree with the experimental results.

(3) In the case of $\zeta_{st} \geq 0.274.5$, the theoretical calculations using the viscous resistance of steady laminar flow in all terms of the series sufficiently well agree with the experimental results.

(4) Considering the terms of the series until $k=4$ for $\zeta_{st}<0.274.5$ and until $k=3$ for $0.274.5 \leq \zeta_{st} < 1$ in the theoretical calculations, they are in good agreement with the experimental results.

(5) Considering many terms of the series in the range of small values of $t/T_e$ and decreasing the terms of the series as $t/T_e$ becomes larger, it is possible to avoid a waste of the numerical calculation.

(6) The damping properties have been obtained from the expression of the first term of the series, approximately.

(7) When the pressure is maintained constant at the outlet of the pipeline in the transient response, the expressions of the shape of the pressure response have coincided with ones in the oil hammer. The experimental results in the transient response also have been nearly similar to ones in the oil hammer.

Acknowledgement

The authors sincerely wish to thank Mr. Akira Hibi and Mr. Seitchi Nihashi of Shizuoka University and Mr. Kazuyoshi Imai of Yuken Kogyo Ltd., for their cooperations.

References


Discussion

M. Fukuda (Yamaguchi University):

(1) In the practical problem, what do you think is the extent of the values of (amplitude of pulsating pressure)/(mean pressure) under which the values calculated in accordance with this theory can hold? Is it possible to apply the theory* of the cascade connection of the case of the oil hydraulic wave?

Y. Ishigaki (Shinshu University):

(2) Indicate the values of pressure, flow rate, velocity and Reynolds’ number in the experiments.

Authors’ closure

(1) \( \bar{P} \) and \( p \) denote mean pressure and pulsating pressure, respectively. As the vacuum state appears in the case of \( \bar{P} < |p| \), the authors deal with the case of \( \bar{P} > |p| \) in this report. If the pipeline systems consist of linear element, it is possible to use \( \bar{P} = P + p \) under the condition of \( \bar{P} > |p| \) in the theoretical analysis. In the systems including nonlinear elements, it is possible to treat them with the linear theories by linearizing them approximately. The linearization of the nonlinear function is estimated by the values of \( |p|/\bar{P} \). According to the binomial theorem, \( (1 + |p|/\bar{P})^n = 1 + n|p|/\bar{P} + (n(n-1)/2)! \) \( (|p|/\bar{P})^2 \) + \( \ldots \); and according to the Taylor series, \( f(\bar{P} + p) = f(\bar{P}) + f'(\bar{P})p + (f''(\bar{P})/2!)p^2 + \ldots \). For \( |p|/\bar{P} < 1 \), the value of the sum until the second term is larger than one of each value over the third term, even if the value of \( |p|/\bar{P} \) is large comparatively. For example, when the nonlinear element is an orifice, \((1 + 0.5)^{1/2} = 1 + 0.25 - 0.03125 + \ldots \) for \( |p|/\bar{P} = 0.5 \); and \((1 + 0.8)^{1/2} = 1 + 0.4 - 0.08 + \ldots \) for \( |p|/\bar{P} = 0.8 \). Therefore, it is possible to linearize the nonlinear element by considering until the second term of the series. T. Ichikawa and A. Hibi** have indicated that the calculated values of the linearized theoretical expression well agree with the experimental results for \( |p|/\bar{P} < 1 \). Also it is possible to apply the theory of the cascade connection to the case of the oil hydraulic wave.

(2) The experiments have been made under various conditions. Some example are shown as follows:

Example 1. mean pressure = 30 kg/cm², flow rate = 109 cm³/sec, velocity = 54.6 cm/sec, Reynolds’ number = 132.3.

Example 2. mean pressure = 35 kg/cm², flow rate = 61.2 cm³/sec, velocity = 184 cm/sec, Reynolds’ number = 181.3.

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Errata (Vol. 15, No. 86, August, 1972)

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