Synthesis of Spatial Multiple-Link Path-Generator*

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Among spatial path-generating mechanisms some four-link mechanisms have been analyzed and synthesized. However these mechanisms have only several parameters which determine the coupler motion, and hardly generate complex motions. In order to generate complex motions, the methods of using the spatial multiple-link mechanisms which have pairs of multiple degrees of freedom and the ones which are composed of several planar mechanisms are suggested.

In the present paper, the spatial motions with translations along three axes and rotations about two axes are classified. Taking such motions apart into several planar ones, a spatial multiple-link mechanism composed of the planar mechanisms which generate each planar motion is synthesized. Then making a mechanism as an example, theoretical and experimental analyses of the static and dynamic displacements are performed, and by use of these results, a dynamic synthesis is carried out.

1. Introduction

As planar path-generators, analyses and syntheses of the four- and six-link mechanisms were performed^{(1)~(7)}, but as spatial path-generators, only four-link mechanisms were synthesized^{(8)~(10)}. The shapes of the spatial four-link coupler curves are restricted, and the mechanisms have such few parameters that they cannot satisfy the complex motions, and further it is hard to take small dimensions and pressure angles of the mechanism compared to the given path. In order to satisfy the above conditions, a spatial multiple-link mechanism with pairs of multiple degree of freedom, and one obtained by composing many planar mechanisms three-dimensionally are suggested.

In this paper, the former mechanism is synthesized. The spatial motions with translations along three axes and rotations about two axes are classified about the paths and directions of a body. Such a motion is split into several planar ones, and composing the planar mechanisms which satisfy each planar motion, a spatial multiple-link mechanism is synthesized. Furthermore, the statical and dynamical displacements of the coupler of this

mechanism are analyzed theoretically and experimentally, and by use of these results, a dynamic synthesis is carried out.

2. Classification of motion of body

The spatial motion of a body is expressed as a combination of the translations of a point on the body and the rotations about this point. As a special case, the motions expressed by the translations along three fixed axes and the rotations about two axes fixed on the body can be classified as

Fig. 1 Classification of motion

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follows ;

"path surface" means the plane or surface described by a point on the body, and "direction surface" means the ones described by the body rotating about this point.

(i) Planar motion [the path and direction surfaces are the same plane, Fig. 1 (a)]

(ii) Spatial motion A [the path and direction surfaces are different planes, Fig. (b)]

(iii) Spatial motion B [the path surface is a plane, and the direction surface is a curved one, Fig. (c)]

(iv) Spatial motion C [the path surface is a curved one, and the direction surface is a plane, Fig. (d)]

(v) Spatial motion D [the path and direction surfaces are curved ones, Fig. (e)]

In Fig. 1, the thin solid lines represent the paths, and the bold arrows represent the directions of the body.

3. Synthesis of multiple-link path-generator

Taking a vector $\overline{HH'}$ on the body in the XYZ -space, we consider the motion of degree of freedom 5 except the rotation about $\overline{HH'}$, as shown in Fig. 2. Let $\overline{H_{XY}H_{XY'}}$ be the orthogonal projection of $\overline{HH'}$ to the XY -plane, and take the moving co-ordinate system $\xi\eta$ on the body, whose ξ axis is $\overline{H_{XY}H_{XY'}}$ and η axis is $\overline{H_{XY}H}$, then such motions can be expressed by five parameters which are the co-ordinate (X, Y, Z) of the point H , the angle θ between the axes ξ and Y , and the angle φ between the vector $\overline{HH'}$ and the axis ξ .

3.1 Construction of mechanism

Put the mechanism I of mobility 1 in the $\xi\eta$ -plane in Fig. 2, and let $\overline{J_1J_1'}$ on the mechanism

Fig. 2

coincide with $\overline{HH'}$. Moreover, let J_2 and J_3 which are the pivots of the mechanism in the XY -plane coincide with H_{XY} and a point on the axis ξ . The positive orientation of the axis ξ is the direction of $\overline{J_2J_3}$. Put two mechanisms II and III of mobilities 1 which are mutually independent in the XY -plane in Fig. 3 and let the coupler points of them coincide with J_2 and J_3 . Moreover put the mechanisms IV and V in the same plane, and combine the driven link of the mechanism V with the driving link of the mechanism IV, and the coupler link of the mechanism IV with the driving links of the mechanism II and III. By use of these mechanisms, above five parameters are controlled as follows :

(i) The height $h(=Z)$ and the angle φ of $\overline{HH'}$ are controlled by the mechanism I, the input of which is the length $\overline{J_2J_3}$.

(ii) The angle θ is controlled by the direction of $\overline{J_2J_3}$.

(iii) The locus of H_{XY} is described by the coupler point of the mechanism II.

(iv) The locus of J_3 is described by the mechanism III.

(v) The angular relation between the driving links of the mechanisms II and III, and the velocities and accelerations with stopping and dwelling of J_2 and J_3 are controlled by the mechanism IV and V.

In order to synthesize such spatial multiple-link mechanisms, the designer should synthesize each planar linkage according to the order of the number, and compose them.

3.2 Synthesis of the mechanism I

As the point H on a body always moves on

Fig. 3

the moving axis η , the positions of $\overline{HH'}$ in the $\xi\eta$ -plane are determined by h and φ , and the motion is classified as follows:

- (i) h is constant and φ varies.
- (ii) φ is constant and h varies.
- (iii) h and φ vary simultaneously.
- (iv) h and φ vary non-simultaneously.

In order to satisfy the above motions, mechanisms shown in Fig. 4 are applicable.

3.3 Syntheses of the mechanisms II and III

The locus of the pivot $J_2(H_{XY})$ is described by a four- or six-link mechanism, such that the generated curve should pass the edge points of the locus, and have the least error at the points. After the synthesis of the mechanism I, the length $\overline{J_2J_3}$, i. e., the position of J_3 , is determined at each position of the mechanism. This locus of J_3 is described by a planar four- or six-link mechanism similarly to the mechanism II.

3.4 Syntheses of the mechanisms IV and V

Joining the mechanism I to the mechanisms II and III at the pivots J_2 and J_3 respectively, as shown in Fig. 3, the inputs become the driving angles θ_2 and θ_3 of the mechanisms II and III, which have mutually a constant functional relationship. Joining two couplers to the driving links of the mechanisms II and III, and further joining them by a turning pair, the angular relation between θ_2 and θ_3 is transformed to the locus of the joint. The mechanism IV is determined as a four-link mechanism satisfying such locus.

When the given path is opened, the mechanism V is necessary for swinging the driving link b_4 of the mechanism IV, the number of links of which is 4 or 6 according to the magnitude of the swing angle of the link b_4 .

Fig. 4

4. Example

The spatial motion D may be generated here, which is the most complicated among five motions classified in Chapter 2. The fundamental path and directions of body of this motion are, for example, shown in Fig. 5. The maximum pressure angle⁽¹¹⁾ of the desired spatial multiple-link mechanism should be less than 60° .

4.1 Synthesis of the mechanism I

As the relation between h and φ in Fig. 5 satisfies the condition (iii) in section 3.2, the mechanism I is a six-link one shown in Fig. 4(c). This mechanism does not change its relative position in the $\xi\eta$ -plane between $H_2 \rightarrow H_4$, and has the maximum pressure angle at the edge point H_1 or H_4 . Then the mechanism, the pressure angles of which at edge points are 50° , is synthesized as follows. In Fig. 6, $J_{3,1}$ is the intersection of the line which passes through $H_1(0, 0)$ and whose inclination is $(50^\circ - 90^\circ)/2 = -20^\circ$, and the line which passes through $H_4(0, 1)$ and whose inclination is -45° . Let $J_{3,4}(J_{3,2}, J_{3,3})$ and J_2 be the intersections of the line which passes through $J_{3,1}$ and parallel to the axis ξ with the line which passes

Fig. 5

Fig. 6

through H_4 and whose inclination is -70° , and the axis η , then $(\overline{J_2 J_3})_{\max} = \overline{J_2 J_{3,1}} = 1.57225$, $(\overline{J_2 J_3})_{\min} = \overline{J_2 J_{3,4}} = 0.57225$. $J_{de1,1}$ and $J_{de1,4}$ are the points on the lines which pass through H_1 and H_4 and whose inclinations are 45° and -45° , respectively, which satisfy $\overline{H_1 J_{de1,1}} = \overline{H_4 J_{de1,4}} = d_1$. Moreover, $J_{ef1,1}$ and $J_{ef1,4}$ are the mid points of $\overline{H_1 J_{3,1}}$ and $\overline{H_4 J_{3,4}}$, respectively, and $J_{de1,4}'$ is obtained by rotating $J_{de1,4}$ through $\angle J_{ef1,4} J_2 J_{ef1,1}$ about the pivot J_2 , in which $c_1' = J_{3,1} J_{ef1,1}$, $f_1'' = \overline{J_2 J_{ef1,1}}$ and $c_1' = f_1'' = c_1/2$. Let $J_{ef1,1}$ be the intersection of the line passing through J_2 and inclining to the link f_1'' by ν_1 with the perpendicular bisector of $\overline{J_{de1,1} J_{de1,4}'}$, then the six-link mechanism $a_1 b_1 c_1 d_1 e_1 f_1$ is synthesized as shown in the figure.

4.2 Synthesis of the mechanism II

The locus of H_{XY} which is the orthogonal projection of H to the XY -plane is $H_{XY1} H_{XY3} H_{XY4}$, as shown in Fig. 7. It is described by a six-link stephenson mechanism instead of a four-link one, because the dimensions of links, pressure angles, and the swing angle of the driving link are desired to be smaller. The mechanism synthesized⁽⁵⁾ is shown in the figure by solid lines, in which the swing angle of the driving link is 100.87° , and the maximum pressure angle is 47.50° . The generated curve is shown by a bold solid line, which passes through H_{XY1} and H_{XY4}

precisely, and has the maximum error 0.14 at H_{XY3} compared to $\overline{H_1 H_2} = 1$.

4.3 Synthesis of the mechanism III

From the results in section 4.1, the coupler point J_3 should pass through four points $J_{3,1}(1.00000, 1.57225)$, $J_{3,2}(1.00000, 0.57225)$, $J_{3,3}(0.57225, 0.00000)$, and $J_{3,4}(0.00000, 0.42775)$. The four-link mechanism which satisfies the above condition, is obtained in reference (3), and modified as shown in Fig. 7. The swing angle is 158.96° , and the maximum pressure angle is 39.00° . The generated curve is shown by a bold broken line, which passes $J_{3,k}(k=1, 3, 4)$ precisely, and has the error 0.086 at $J_{3,2}$.

4.4 Synthesis of the mechanism IV

In Fig. 7, the pivot J_2 stands still when J_3 moves between $J_{3,1} \leftrightarrow J_{3,2}$, and when it moves between $J_{3,2} \leftrightarrow J_{3,4}$, $\overline{J_2 J_3} = 0.57225$ (=constant). The driving link b_2 is transformed to g_4' by use of a uniform motion mechanism $g_4' h_4 b_2''$ ($g_4' = b_2''$). Ad-

Fig. 7 Mech II, III, IV

Fig. 8 Mech. V

Fig. 9

ding the links e_4 and f_4 to the points on g_4' and b_3 , respectively, by turning pairs, and joining them by a turning pair J_{ef4} , the locus of J_{ef4} satisfying the above relationships is shown by a thin broken line. The four-link mechanism satisfying this locus is determined⁽³⁾ as in Fig. 7, similarly to the mechanism III. The swing angle is 155.99° , and the maximum pressure angle is 41.00° .

4.5 Synthesis of the mechanism V

As the given path is opened, the mechanism V is necessary for swinging the driving link of the mechanism IV. The desired mechanism is a six-link one, which has the same input displacement angles between the edge points.

The driving link b_4 is transformed to b_4' by the rotation about J_{ab4} through $\lambda_{45} = -97.00^\circ$, and, in Fig. 8, $J_{eb5,1}$ and $J_{eb5,4}$ are the moving pivots on b_4' at the positions 1 and 4, then $\angle J_{eb5,4}J_{ab4}J_{eb5,1} = 155.99^\circ$. The link e_5 is added to $J_{eb5,k}$ ($k=1, 4$), and further d_5 is to e_5 . The fixed pivot J_{da5} on d_5' is determined such that the angle between b_4' and e_5 may be 55° , and the swing angle of d_5' between the edge points may be 60° . Moreover M is a mid point of $\overline{J_{cd5,1}J_{cd5,4}}$, where $J_{cd5,k}$ lies on d_5' ($J_{da5}J_{cd5,k} = d_5$), and N is the point satisfying $\angle J_{da5}MN = 40^\circ$ and $\overline{MN} = d_5$. Let J_{ab5} be the intersection of $\overline{J_{cd5,1}J_{cd5,4}}$ with the hyperbola which satisfies $|\overline{J_{ab5}J_{da5}} - \overline{J_{ab5}N}| = \overline{J_{cd5,1}J_{cd5,4}}/2$, then the mechanism V is synthesized as shown by the bold solid lines.

Composing the above mechanisms, the spatial multiple-link mechanism is synthesized as shown in Fig. 9. The maximum pressure angle is 50.44° , and the total number of links is 26.

5. Theoretical analysis of displacement

About the 26-link mechanism shown in Fig. 9, the analyses of the static and dynamic displace-

ments are performed, and a dynamic synthesis is carried out. The analyses are applied only to the neighbourhoods of the edge points H_1 and H_4 , because of the actual requirement in general. The XY -plane is assumed horizontal, so the gravities acting on the mechanisms II, III, IV, and V are perpendicular to the moving plane, and the inertia forces are parallel, while the gravities and inertia forces acting on the mechanism I are both parallel to the moving plane near the edge points. Now we assume the mechanisms II and III to be separated from I at J_2 and J_3 , and the pivots J_i' ($i=2, 3$) to be supported on the mechanism I elastically by two coil springs, as shown in Fig. 10. Then the influences of the gravities and inertia forces acting on the mechanisms in the XY -plane are representable by the flexions ΔJ_i of the pivots J_i ($i=2, 3$), which are negligibly small, and the ones acting on the mechanism I are by the flexion $\Delta J_i'$ ($i=2, 3$) of the springs, which are obtained as follows.

In the figure, m_c and $W_c(0, -m_cg)$ are the mass and gravity of the link a_c , a_c is the acceleration of the center of gravity G_c , I_c' is the inertia moment about G_c , and $\overline{J_3J_{cd}} = C(C_z, C_y)$, $\overline{J_3G_c} = G_c$, and ψ_c is the displacement angle of C from the axis ξ . About the links d , e , and f , the parameters are determined similarly to the above. Let F_{pq}^n be the force acting from the link q to p at the pivot J_{pq} which joins p and q ($p, q=c, d, e, f$), and F_i be the force acting from the mechanism I to the spring at J_i' , then the equations of motion for the translations of centers of gravity are

$$\left. \begin{aligned} A_c &= F_{cf}^c + F_{cd}^c - F_3 \\ A_d &= F_{cd}^d + F_{de}^d \\ A_e &= F_{de}^e + F_{ef}^e \\ A_f &= F_{cf}^f + F_{ef}^f - F_2 \end{aligned} \right\} \dots\dots\dots (1)$$

and the ones for the rotations about them are

$$\left. \begin{aligned} M_c &= \overline{G_cJ_{cd}} \times F_{cd}^c + \overline{G_cJ_{cf}} \times F_{cf}^c - \overline{G_cJ_3} \times F_3 \\ M_d &= \overline{G_dJ_{cd}} \times F_{cd}^d + \overline{G_dJ_{de}} \times F_{de}^d \\ M_e &= \overline{G_eJ_{de}} \times F_{de}^e + \overline{G_eJ_{ef}} \times F_{ef}^e \\ M_f &= \overline{G_fJ_{cf}} \times F_{cf}^f + \overline{G_fJ_{ef}} \times F_{ef}^f - \overline{G_fJ_2} \times F_2 \end{aligned} \right\} \dots\dots\dots (2)$$

where

$$A_p = m_p a_p - W_p, \quad M_p = I_p \ddot{\psi}_p \quad (p=c, d, e, f) \dots\dots\dots (3)$$

Eliminating E_{pq}^n in these equations, F_2 and F_3 are obtained as follows:

$$\left. \begin{aligned} F_2 &= -\frac{1}{D} \{ C_{11}M + (LC_{12}^2 + GC_{13})A, \quad C_{21}M + (LC_{22}^2 + GC_{23})A \} \\ F_3 &= -\frac{1}{D} \{ -C_{11}M + (LC_{12}^3 - GC_{13})A, \quad -C_{21}M + (LC_{22}^3 - GC_{23})A \} \end{aligned} \right\} \dots\dots\dots (4)$$

where

$$D = \frac{f''}{f'} \sin(\psi_d - \psi_e) \sin(\psi_e - \psi_f)$$

$$\begin{aligned}
 M &= \begin{bmatrix} M_c \\ M_d \\ M_e \\ M_f \end{bmatrix} & A &= \begin{bmatrix} A_c \\ A_d \\ A_e \\ A_f \end{bmatrix} & L &= [c \ d \ e \ f] \\
 & & & & G &= [G_c \ G_d \ G_e \ G_f] \\
 C_{j1} &= [C_{jc} \ C_{jd} \ C_{je} \ C_{jf}] & C_{j3} &= \begin{bmatrix} C_{jc} & 0 & 0 & 0 \\ & C_{jd} & & \\ & & C_{je} & \\ 0 & & & C_{jf} \end{bmatrix} \\
 C_{j2} &= \begin{bmatrix} 0 & C_{jc} & C_{jd} & C_{je} \\ 0 & 0 & C_{jd} & C_{je} \\ 0 & 0 & 0 & C_{je} \\ 0 & 0 & 0 & 0 \end{bmatrix} & C_{j2} &= \begin{bmatrix} C_{jc} & 0 & 0 & 0 \\ C_{jd} & C_{jd} & 0 & 0 \\ C_{je} & C_{je} & C_{je} & 0 \\ C_{jf} & C_{jf} & C_{jf} & C_{jf} \end{bmatrix} \\
 & & & & & j=1, 2 \\
 \begin{bmatrix} C_{1c} & C_{2c} \\ C_{1d} & C_{2d} \\ C_{1e} & C_{2e} \\ C_{1f} & C_{2f} \end{bmatrix} &= \begin{bmatrix} 0 & \Psi_{c'd} & -\Psi_{c'e} & \Psi_{c'f'} \\ -\Psi_{c'd} & 0 & \Psi_{de} & -\Psi_{df'} \\ \Psi_{c'e} & -\Psi_{de} & 0 & \Psi_{ef'} \\ -\Psi_{c'f'} & \Psi_{df'} & -\Psi_{ef'} & 0 \end{bmatrix} & \begin{bmatrix} c_z & c_\eta \\ d_z & d_\eta \\ e_z & e_\eta \\ f_z & f_\eta \end{bmatrix} \\
 \Psi_{c'd} &= \frac{\sin(\Psi_e - \Psi_{f'})}{c'd}, & \Psi_{c'e} &= \frac{\sin(\Psi_d - \Psi_{f'})}{c'e}, & \Psi_{c'f'} &= \frac{\sin(\Psi_d - \Psi_e)}{c'f'} \\
 \Psi_{de} &= \frac{\sin(\Psi_c - \Psi_{f'})}{de}, & \Psi_{df'} &= \frac{\sin(\Psi_c - \Psi_e)}{df'}, & \Psi_{ef'} &= \frac{\sin(\Psi_c - \Psi_d)}{ef'}
 \end{aligned}
 \tag{5}$$

Substituting each parameter into Eqs. (4) and (5), the desired forces $F_i = (F_{i\xi}, F_{i\eta})$ ($i=2, 3$) are determined.

Let $k_i (k_{i\xi}, k_{i\eta})$ be the spring constants at J_i' , then the flexions $\Delta J_i' (\Delta \xi_i', \Delta \eta_i')$ are obtained as follows;

$$\left. \begin{aligned} \Delta \xi_i' &= F_{i\xi} / k_{i\xi} \\ \Delta \eta_i' &= F_{i\eta} / k_{i\eta} \end{aligned} \right\} (i=2, 3) \tag{6}$$

and the static and dynamic displacements of $\overline{J_1 J_1'}$ can be determined.

Among ΔX , ΔY , ΔZ , $\Delta \theta$, and $\Delta \varphi$ which are the differences between the static and dynamic errors, ΔZ and $\Delta \varphi$ are larger than the other errors

at the neighbourhood of H_1 . In order to reduce such errors, we suggest to supply a leaf spring between the links c and f, as shown in the figure. Let $\Delta F_{i\xi}$ be the difference between the static and dynamic forces which the mechanism I applies on the springs at J_i' in the direction of ξ -axis, then the moment around J_{cf} due to the leaf spring is

$$M_{cf} = \frac{1}{2} (-\overline{J_{cf} J_2'} \times \Delta F_{2\xi} + \overline{J_{cf} J_3'} \times \Delta F_{3\xi}) \tag{7}$$

Owing to the supplement of such a spring, the dynamic error of displacement can be reduced.

6. Experimental results

The mechanism synthesized in Chapter 4 is constructed, the displacements of the coupler is obtained, and the theory is confirmed experimentally.

6.1 Trial making

The unit length of the co-ordinate in Fig. 5 is 100 mm. The radial ball bearings are used for the turning pairs, and the mechanisms are constructed as shown in Fig. 11. The arrow in Fig. (a), (b), (c), or (d) represents each position of $\overline{HH'}$ in Fig. 5. The driving link is turned uniformly through the angle Θ by a V-belt.

6.2 Experimental results

The static displacements of $\overline{J_1 J_1'}$ measured at the positions $\Theta = 10^\circ \times n (n=0, 1, 2, \dots, 35)$ are shown in Fig. 12 by broken lines. The solid lines express the theoretical displacements obtained by use of the ideal values in Table 1, in which the weight of each link is neglected. The displacement of J_1 has errors ± 1.5 mm, ± 2.3 mm, ± 2.5 mm, respectively along the axes X , Y , and Z , in Fig. (a),

Fig. 10

at the edge points H_1 and H_4 . The angular displacement θ almost coincides with the ideal values, and φ has errors $\pm 2.0^\circ$, in Fig. (b). These errors are due to the elastic deformations of links and shafts caused by the weights of them as well as the structural errors.

The dynamic displacements are measured in the ranges of $-40^\circ < \theta < 40^\circ$ and $140^\circ < \theta < 220^\circ$, when the input angular velocity is $\dot{\theta} = \pi \text{ rad/sec}$. As the velocities and accelerations of $J_i (i=2, 3)$

(a) Position 1 in Fig. 5

(b) Position 2 in Fig. 5

(c) Position 3 in Fig. 5

(d) Position 4 in Fig. 5
Fig. 11

are negligibly small in the range of $-40^\circ < \theta < 40^\circ$, only ΔY , ΔZ , and $\Delta \varphi$ are obtained as shown in Fig. 13 by solid lines. The frequency of the fluctuation of each displacement is 9 c/sec, which coincides

(a)

(b)

Fig. 12 Static displacement

(a)

(b)

Fig. 13 Dynamic displacement

Fig. 14 Dynamic displacement

es with the natural frequency of the system in Fig. 10. The errors at H_1 are $\Delta Y = -0.5$ mm, $\Delta Z = -0.2$ mm, $\Delta \varphi = 2.0^\circ$. On the other hand, the spring constants of $J_{i'}$ are $k_2 = (1500, 800)$ g/mm, $k_3 = (1500, 500)$ g/mm at H_1 , then the error ΔZ calculated from Eqs. (4), (5), and (6) is shown in Fig. (a) by a dotted line, which approximates to the mean displacement obtained by excluding the natural vibration from the experimental value.

The errors in the range of $140^\circ < \theta < 220^\circ$ are shown in Fig. 14, in which the large fluctuations of displacements are the natural vibrations similar to the above case, and the errors at H_1 are $\Delta X = -0.3$ mm, $\Delta Y = 1.0$ mm, $\Delta Z = -0.3$ mm, $\Delta \theta = 0.0^\circ$, and $\Delta \varphi = 0.6^\circ$. The theoretical value of ΔY is shown in Fig. (a) by a dotted line, which is smaller than the mean experimental value by the amount of 18%, because the inertia forces of the mechanisms in the XY -plane are neglected. As is evident from Figs. 13 and 14, ΔZ and $\Delta \varphi$ are greater than the other errors at H_1 .

The moment around J_{cf} by the leaf spring at H_1 is 6825 g·mm which is obtained by Eq. (7). Adding such a spring to the mechanism I, the errors ΔZ and $\Delta \varphi$ in the range of $-40^\circ < \theta < 40^\circ$ are as shown in Fig. 13 by the broken lines, which have been reduced to 1/3~1/5 at H_1 .

7. Conclusions

The results obtained in this paper are summarized as follows:

(1) The spatial motions with translations along three axes and rotations about two axes have been classified into five kinds about the paths and directions of body.

(2) Such a motion has been expressed by five parameters which are three co-ordinates of a point on the body and two displacement angles, and composing the planar four- or six-link mechanisms satisfying the fluctuations of these parameters, a spatial multiple-link mechanism has been synthesized.

(3) Making the mechanism as an example, the influences of the gravities and inertia forces acting on the mechanisms in the vertical plane on the displacements and directions of the coupler have been theoretically and experimentally clarified.

(4) In order to reduce the dynamic errors due to the inertia forces, addition of a leaf spring to the mechanism has been suggested, and as a result, the dynamic errors have been reduced to 1/3~1/5 of the initial ones at the edge point, and a precise dynamic synthesis of spatial multiple-link path-generator has become possible.

Moreover, even in the case of the motion with the rotations about three axes, such mechanism synthesis as shown in this paper is possible.

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Errata (Vol. 15, No. 87, September, 1972)

Author	Page	Column	Line	For	Read
M. HAMADA et al.	1030	right	Eq. (16)	$C = d\sigma_f/d\epsilon^p =$	$C = d\sigma_f/d\epsilon^p =$
	1031	"	Eq. (26)	$[D^p] = [D_p] - \dots$	$[D^p] = [D^e] - \dots$
	1032	"	Eq. (47)	$\dots - (\bar{\psi}_V \sin \beta \dots$	$\dots - (\bar{\psi}_V \sin \beta \dots$
	1033	"	Eq. (62)	$[\dot{\beta} \bar{\psi}_H], \dots$	$[\dot{\beta} \bar{\psi}_H], \dots$
	1037	"	Raf. (2)	<i>Int. Jour.</i>	<i>Int. Jour.</i>
T. OMORI et al.	1052	left	9	condition	conditions
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	1060	"	4~2 from the bottom	$\frac{1}{EJ}$	EJ
	1060	right	2	$\frac{1}{EJ}$	EJ
	1075	left	14	$m_1 \ddot{X}_1$	$m_1 \ddot{X}_1$
K. SUZUKI et al.	"	"	16	$m_2 \ddot{X}_2 + C_2(X_2 - \dot{X}_1)$	$m_2 \ddot{X}_2 + c_2(\dot{X}_2 - \dot{X}_1)$
	"	"	30	$(\ddot{X} - Y_2) + \dots$	$(\ddot{X} - \ddot{Y}_2) + \dots$
	"	"	31	$-Y_2$	$-\ddot{Y}_2$
	"	"	33	$(X_1 - Y_1) + \dots$	$(\ddot{X}_1 - \ddot{Y}_1) + \dots$
	"	"	38	$(\ddot{X}_1 - \ddot{Y}_2) + \dots$	$(\ddot{X}_1 - \ddot{Y}_2) + \dots$
	"	"	41	$(\ddot{X}_2 - Y_2) + \dots$	$(\ddot{X}_2 - \ddot{Y}_2) + \dots$
	"	"	42	$-2h_m \omega_m (X_1 - \dot{Y}_2)$	$-2h_m \omega_m (\dot{X}_1 - \dot{Y}_2)$
	1076	"	6	$(Y_1 - \ddot{Y})$	$(\ddot{Y}_1 - \ddot{Y})$
	"	"	11	$-Y + \gamma \frac{c}{m} X (\ddot{X}_2 - Y_2)$	$-\ddot{Y} + \gamma \frac{c}{m} (\ddot{X}_2 - \ddot{Y}_2)$
	"	"	12	$(\dot{X}_1 - Y_1) + \dots$	$(\ddot{X}_1 - \ddot{Y}_1) + \dots$
	"	"	13	$(\ddot{X}_2 - Y_1) - \dots = -Y_1$	$(\ddot{X}_2 - \ddot{Y}_1) - \dots = -\ddot{Y}_1$
	"	"	14	$(\ddot{X}_2 - Y_1)$	$(\ddot{X}_2 - \ddot{Y}_1)$
	"	"	16	$= -Y_1 + \dots$	$= -\ddot{Y}_1 + \dots$
	"	"	17	$(\dot{X}_1 - Y_2) + \dots$	$(\ddot{X}_1 - \ddot{Y}_2) + \dots$
	"	"	19	$= -Y_2 + \frac{c}{m} (Y_1 - \dot{Y}_2)$	$= -\ddot{Y}_2 + \frac{c}{m} (\ddot{Y}_1 - \ddot{Y}_2)$
	"	"	20	$(\ddot{X}_2 - Y_2) + \dots$	$(\ddot{X}_2 - \ddot{Y}_2) + \dots$
	"	"	21	$-\frac{c}{m} (X_1 - \dot{Y}_2)$	$-\frac{c}{m} (\dot{X}_1 - \dot{Y}_2)$
	"	"	3 from the bottom	$\omega_{bI}^2 = \frac{1}{2M} \{(K_1 + \dots$	$\omega_{bI}^2 = \frac{1}{2M} \{(K_1 + \dots$
	1081	right	2 from the bottom	Y_1 and $\ddot{Y}_2$	$\ddot{Y}_1$ and $\ddot{Y}_2$
	"	"	last line	Y	$\ddot{Y}$
	1083	"	last line	(in japanese)	(in Japanese)
H. HONDA et al.	1109	left	11	borling	boiling
	1114	"	28	expressed	expressed
	"	"	43	$\pi R^2 \Delta P$	$\pi R^2 \Delta P$
	1115	right	3~4	Fig. 2	Fig. 12