Swirl Flow in Long Pipes with Different Roughness*

By Yasutoshi SENOO** and Tetuzou NAGATA***

Swirl flow in a long pipe is experimentally investigated. In the major part of the pipe, that is, except at the entry zone with a very weak swirl, the circumferential component of the velocity is proportional to the radius near the wall. The axial component of the velocity is minimum at the center and the radial distribution is similar to the velocity profile of the wake behind a body in a uniform flow.

The intensity of swirl is indicated by a parameter. The variation of the parameter and the wall pressure along the length is examined for three pipes with different relative roughnesses and semi-empirical equations are derived. Using the equations, the velocity distribution and the wall pressure at any section may be predicted if the conditions at an upstream station and the relative roughness of the pipe are specified.

1. Introduction

When a swirling fluid flows through a long straight pipe, the velocity distribution in the pipe section depends upon the upstream condition. However, at a short distance downstream of the pipe entrance the radial distribution of the circumferential component of velocity in the pipe is approximately the same as solid vortex type at the core while it is approximately the same as free vortex type outside of the core, and such a pattern of swirl is maintained until the swirl becomes weak.

The authors measured the velocity distribution at many section of a pipe and examined the relationships between the decay of swirl, pressure distribution and wall friction coefficient and demonstrated that the behavior of swirl flow in a long pipe was nicely predicted using a parameter $m$, which represented the intensity of swirl.

2. Experimental apparatus

Figure 1 shows a general view of the experimental apparatus. Air was drawn by fan 2 through bellmouth 1 and sent into a plenum chamber 3 which has the air straightening vanes 4. A swirl flow was generated by the circular cascade 5 which was mounted at the outlet of the chamber, and then the swirl flow entered the test pipe 6. In order to examine the influence of the wall roughness on the swirl flow, three pipes with different relative roughnesses were used. They were:

- Smooth pipe: $k/D = 1.0 \times 10^{-3}$, $D = 150$ mm, $L/D = 60$, $R_s = 1.85 \times 10^3$
- Zinc coated steel pipe: $k/D = 1.8 \times 10^{-3}$, $D = 280$ mm, $L/D = 45$, $R_s = 1.95 \times 10^3$
- Rough pipe: $k/D = 2.5 \times 10^{-3}$, $D = 150$ mm, $L/D = 72$, $R_s = 1.50 \times 10^3$

The direction and the total pressure of flow in the pipe were measured with a cobra probe and the static pressure in pipe was measured with a sphere static probe. Although dynamic pressure could be measured by a calibration cobra probe, a more accurate measurement was made with the combination of a cobra probe and a sphere static probe. That is, the radial distribution of static pressure measured with the sphere static probe was more reasonable and consistent with the wall pressure than the pressure measured with the cobra probe alone.

3. Nomenclature

$B$: angular momentum flux per unit time

* Received 22nd June, 1970.
** Professor, The Research Institute of Industry Science, Kyushu University.
*** Assistant Professor, Kurume Technical College, Kurume.
\( C_f \): coefficient of friction  
\( D \): diameter of pipe  
\( F \): momentum flux per unit time  
\( L \): total length of the pipe  
\( l \): axial length  
\( p \): pressure  
\( R \): radius of pipe  
\( r \): radial distance from the center of pipe  
\( R_e = (\bar{v}D/\nu) \): Reynolds number  
\( \nu \): velocity  
\( \theta \): flow angle measured from the axial direction  
\( \psi \): kinematic viscosity  
\( \tau \): shear stress  

Suffixes  
1: upstream station  
2: downstream station  
\( R \): virtual value at the wall, which is calculated by extending the values at \( r/R = 0.85 \) and 0.90  
\( S \): static condition  
\( t \): stagnation condition  
\( u \): circumferential component  
\( z \): axial component  
\( \cdots \): mean value of a cross-section

4. Expression for the intensity of swirl

There are many ways to express the intensity of swirl. The flow angle at the wall \( \theta_R \) is commonly used in literature. However, it is not sufficient to express the intensity of swirl flow at various condition. The ratio of the angular momentum flux \( B \) to the axial momentum flux \( F \) through a cross-section of a pipe is defined as \( m \), or

\[
\frac{m}{FR} = \frac{B}{2\pi \rho \int_0^R \bar{v}_\psi r^2 dr} = \frac{2\pi \rho \int_0^R \bar{v}_\psi r^2 dr \times R}{2\pi \rho \int_0^R \bar{v}_\psi r^2 dr \times R} \tag{1}
\]

The variation of \( m \) along the pipe is indicated in Fig. 2.

The relation of \( \theta_R \) to \( m \) changes considerably depending upon the radial distribution of the circumferential component of velocity. The relation is demonstrated in Fig. 3 for two types of model swirl flow, that is, a solid vortex pattern with uniform axial velocity and a free vortex pattern with uniform axial velocity. In many actual swirl flows, the radial distribution of the circumferential velocity component is a combination of a solid vortex pattern and a free vortex pattern, and it is expected that the value of \( m \) for an actual swirl flow is located between the two model swirl flows in Fig. 3. The marks in the illustration are the present experimental data. In this experiment the radial distribution of the circumferential component of velocity in the pipe is approximately solid vortex type at the core while it is approximately free vortex type outside of the core. As the axial velocity in the core is very small, the relationship between \( m \) and \( \theta_R \) is close to that of the free vortex type when \( m \) is larger than 0.3. At upstream stations \( m \) is large, and it decreases continuously along the pipe. The experimental points in the illustration fall on a curve which is parallel to the curve of the free vortex pattern until \( m \) decreases to 0.3. When \( m \) is equal to 0.3, the experimental point begins to deviate from the curve and to approach the curve of the solid vortex pattern. The deviation means that the velocity distribution radically changes for a weak swirl flow.

For a given value of \( \theta_R \) the intensity of swirl \( m \) varies depending upon the pattern of swirl, being the largest for a free vortex type and the smallest for a solid vortex type. For a given swirl flow with a wall flow angle \( \theta_R \) if we consider an imaginary free vortex pattern with uniform axial velocity which has the same \( \theta_R \) and calculate the intensity of swirl \( m_f \), the ratio \( m/m_f \) indicates the flow pattern to a certain extent. The ratio is 1.0 for the free vortex pattern.
and it is 0.5 for the solid vortex pattern with uniform axial velocity. In the present experiment it is 0.85 except for the weak swirl flow.

5. Circumferential velocity distribution

The radial distribution of non-dimensional angular momentum at various values of $m$ is shown in Fig. 4.

![Fig. 4 Non-dimensional angular momentum against the radius for each cross section](image)

The wall friction is the only cause of reducing the angular momentum of swirl flow, but the angular momentum decreases uniformly in the entire cross-section. If $v_r r$ near the wall becomes less than that at a smaller radius due to the wall friction, such a swirl flow is unstable and violent mixing occurs and the influence of the wall friction spreads over the entire cross-section.

The radial distribution of the circumferential component of velocity is presented in Fig. 5. Except at the central portion of the pipe the flow pattern is of the free vortex type and the maximum velocity is twice the velocity at the wall, providing that $m$ is larger than 0.3. For $m$ being less than 0.2, a solid vortex type flow pattern occupies the entire cross-section.

6. Radial distribution of the axial component of velocity

The radial distribution of the axial component of velocity is shown in Fig. 6. The distribution is little influenced by the wall roughness. The axial velocity at the core decreases as the intensity of swirl increases and a reverse flow is observed for $m$ being larger than 0.3. For $m$ being less than 0.25 the velocity defect is not observed at the core. The distribution of the defect of the axial velocity component looks like the wake behind an axisymmetric body, which may be expressed as $(U_0 - U_1)/U_0 = (1 - (y/b)^{1.3})^2$ where $b$ is the radius of the wake, $U_1$ is the velocity defect and $U_0$ is the maximum defect.

![Fig. 5 Radial distribution of non-dimensional circumferential velocity](image)

![Fig. 6 Axial velocity distribution at each cross section](image)
In Fig. 7 the present experimental data indicated as circles are compared with the above mentioned empirical equation and the experimental data of the wake behind a body of revolution, which are indicated by cross mark.

The maximum axial velocity and the maximum defect of the axial component of velocity as well as the radius of the defect zone are related to the intensity of swirl \( m \) in Fig. 8.

7. Estimation of the wall shear stress

Concerning a control volume between two sections, the balance of forces in the axial direction is

\[
2\pi p \int_0^R v_1 r^2 dr + 2\pi \int_0^R \rho v_1 r dr = 2\pi \rho \int_0^R v_1 v_1 r^2 dr
\]

\[
\times \int_0^R v_0 v_2 r^2 dr + 2\pi \int_0^R \rho v_0 r dr + \tau_s 2\pi R l
\]

\[
\cdots \cdots \cdots \cdots \cdots (2)
\]

and the balance of moment is

\[
2\pi p \int_0^R v_1 v_0 r^2 dr = 2\pi \rho \int_0^R v_0 v_2 r^2 dr
\]

\[
+ \tau_s 2\pi R l
\]

\[
\cdots \cdots \cdots \cdots \cdots (3)
\]

Using the measured values of \( v_0, v_2 \) and \( p \), the wall shear stress components \( \tau_s \) and \( \tau_r \) are calculated with the above two equations. The direction of shear stress \( \theta_s \) is the ratio of \( \tau_s \) and \( \tau_r \), and it is compared with the direction of the flow at the wall in Fig. 9. Agreement is good. The wall friction coefficient is defined as \( \tau_s = \frac{c_f \rho u_v^2}{2} \) where \( u_v \) is the virtual velocity at the wall as defined as section 3. The friction coefficient is plotted against the intensity of swirl \( m \) in Fig. 10 where the parameter is the relative roughness of the pipe. The value of \( c_f \) at \( m = 0 \) was calculated from the pipe friction coefficient data considering that the representative velocity was the virtual wall velocity instead of the mean velocity. As the ratio is approximately 0.9, \( c_f \) in this definition is 1.25 times the friction coefficient of conventional definition. According to Fig. 10, the friction coefficient increases with \( m \) as expected and it is expressed as
\[ c_f = c_f0 (1 + 0.196 m^2 - 0.41 m^3 + 0.344 m) \]  \hspace{1cm} (4)

where \( c_f0 \) is the friction coefficient without a swirl. Using this equation we can estimate the wall friction coefficient of swirl flow in pipes with any roughness.

8. Decay of swirl along the pipe

Differentiating Eq. (1) with respect to the length of pipe

\[ \frac{dm}{dl} = \frac{1}{FR} \left( B \frac{dF}{dl} - F^2 \frac{dR}{dl} \right) \]  \hspace{1cm} (5)

As the experimental results in Fig. 11 show that \( |dF/|dl| \) is very small compared to \( |dR/|dl| \), the second term is negligible and Eq. (5) becomes as follows.

\[ \frac{dm}{dl} = \frac{2\pi \rho}{FR} \int_0^R v_s r^2 dr \]  \hspace{1cm} (6)

According to Eq. (3) the derivative of the parenthesis is \(-2\pi R^2 \tau_s\) and \( \tau_s = 1/2 \rho c_f \nu_0 R^2 \).

Thus Eq. (6) becomes

\[ (1/2) \rho c_f v_s R^2 \int_0^R v_s r^2 dr dl = -dm FR \]  \hspace{1cm} (7)

Figure 12 shows that \( v_s R^2 / \nu_0^2 \) is proportional to \( m \), or \( v_s R^2 / \nu_0^2 = \xi_1 m \) where \( \xi_1 \) is a constant.

Substituting this relation into Eq. (7) and integrating it, one arrives at

\[ \log \frac{m_2}{m_1} = c_f \xi_1 \left( \frac{F_A}{F} \right) \frac{l_2 - l_1}{R} \]  \hspace{1cm} (8)

In this integration the change of \( c_f \) with respect to \( m \) was neglected and \( F_A \) means \( \rho \pi \nu_0 R^2 \). Figure 11 and 12 show that \( F_A / F = 0.98 - 0.90 = 0.94 \) and \( \xi_1 = 1.35 \). As the result Eq. (8) becomes

\[ \log_{10} \frac{m_2}{m_1} = -1.10 \xi_1 \frac{l}{D} \]  \hspace{1cm} (9)

The lines in Fig. 13 were calculated with Eq. (9) and the circles show experimental values.

9. Pressure distribution

Pressure varies in the radial direction due to the centrifugal force of swirl. Therefore the pressure at the wall is different from the mean pressure at the section.

9-1 Mean pressure at a section

Dividing Eq. (2) with respect to the cross sectional area of the pipe,

\[ \tilde{\rho}_s - \tilde{\rho}_w = \frac{F_2 - F_1}{\pi R^2} + 4 \pi \tau_s \frac{D}{2} \]  \hspace{1cm} (10)

where \( \tilde{\rho} \) is the mean pressure at a section and defined as

\[ \tilde{\rho} = 2 \int_0^R \rho rdr / R \]  \hspace{1cm} (11)

According to Fig. 11 the variation of \( F \) along the pipe is expressed as

\[ \frac{A(F/F_A)}{A(I/D)} = 1.43 \times 10^{-3} \]  \hspace{1cm} (12)

Although this term is not accurate, the first term in Eq. (10) is small compared to the second term and the inaccuracy in Eq. (12) becomes insignificant for the evaluation of the mean pressure. Using these relation Eq. (10) becomes

\[ \tilde{\rho}_s - \tilde{\rho}_w = -1.43 \times 10^{-3} \frac{l}{D} \rho \nu_0^2 + c_f \frac{l}{D} \rho v_s R^2 \]  \hspace{1cm} (13)

According to Fig. 14, \( v_s R^2 = 0.8(1 + m^3) \nu_0^2 \). Therefore Eq. (13) may be written as
As the variation of $m$ along the pipe axis is evaluated with Eq. (9) the variation of the mean pressure in Fig. 15 were calculated using Eqs. (9) and (14), while the circles in the illustration denote the experimental points. The mean pressure at the exit of the pipe is below the atmospheric pressure. Such a condition is common for a swirl flow.

### 9-2 Wall pressure

In a swirl flow there is the following relation between the wall static pressure and the static pressure at radius $r$:

$$\dot{p}_s = \rho_R - \rho \int_0^R \frac{p_r}{r} dr$$ ......................................(15)

$$\dot{p}_s = \frac{2\pi}{\pi R^2} \int_0^R \int_0^R p_r r dr$$ ......................................(16)

For the sake of simplicity of analysis it is assumed that the circumferential component of velocity is approximated as a Rankine vortex or

$$v_x = \frac{v x R}{r} (1-e^{-10(r/R)^2})$$

which is shown as a broken line in Fig. 5. Then, Eq. (16) becomes

$$\dot{p}_s = \frac{2\pi}{\pi R^2} \int_0^R \int_0^R p_r r dr$$ ......................................(17)

$$\frac{2\pi}{\pi R^2} \int_0^R \rho R^2 dr$$ ......................................(18)

Figure 16 shows that $v_x R/\dot{p}_s$ is proportional to $m$ and $v_x R = 1.12 \dot{p}_s$. Consequently Eq. (18) becomes

$$\frac{p_R - \dot{p}_s}{(1/2) \rho \dot{R}^2} = 2.61 m^2$$ ......................................(19)

The variation of intensity of swirl $m$ along the axis was calculated with Eq. (9), the variation of the mean static pressure along the axis was calculated with Eq. (14) and then the axial distribution of the wall static pressure was calculated with Eq. (19). The results are the curves in Fig. 17 and the circles show the experimental values.
10. A method of computation of intensity of swirl and pressure distribution along the pipe axis

As for the initial condition, the radial distribution of velocity and the wall static pressure must measured at an upstream station. From the velocity distribution, \( m \) is calculated. From the direction of the virtual wall velocity \( \theta_R \) and Fig. 3, \( m_f \) is determined. If the ratio \( m/m_f \) is close to 0.85, the flow pattern is stable and the present method is applicable. At first the wall friction coefficient \( \epsilon_f \) is assumed and the distribution of \( m \) along the axis is calculated with Eq. (9). The wall friction coefficient without a swirl is adjusted for a swirl flow using \( \epsilon_f = 1.25 \lambda / 4 \), Eq. (4), and the calculated distribution of \( m \). Using the wall friction coefficient for the swirl flow, the distribution of \( m \) and the mean pressure along the axis are calculated with Eqs. (9) and (14), and the wall static pressure is calculated with Eq. (19). The radial distribution of circumferential component of velocity is determined from \( m \), Fig. 16 and Fig. 5. The radial distribution of axial component of velocity is decided from \( m \), Fig. 8 and Fig. 7. The present method is not applicable to the range where \( m \) is less than 0.30.

In the present experiment the Reynolds number was not systematically changed. The wall friction coefficient without a swirl varies depending upon the Reynolds number as well as the relative roughness, and the friction coefficient is taken into account in the present method. The authors believe that the present method is applicable for a wide range of Reynolds numbers providing that the flow is turbulent.

11. Conclusions

1. The ratio of the angular momentum flux to the axial momentum flux is defined as \( mR \), and \( m \) is used as the indication of intensity of swirl.

2. Except at the entrance region, the pattern of swirl is somewhat similar and the degree of free vortex \( m/m_f \) is 0.85. Such condition is maintained until \( m \) decreases down to 0.3.

3. The axial velocity is low at the center of pipe and the defect of velocity is similar to the wake behind a body of revolution.

4. A simple theory predicts that \( m \) decreases exponentially and the relation has been experimentally proved.

5. If the flow rate, intensity of the swirl and the relative roughness of the pipe are given at a section, the distribution of the wall static pressure and the variation of the swirl along the pipe are predicted.

Discussion

S. Kamiyama (Tohoku University):

(1) For the purpose of applying the present experimental results to general swirl flows, I would like to know how far away from the entrance of a pipe does the swirl flow proceed before the pattern of swirl flow becomes stable \( (m/m_f = 0.85) \).

(2) Judging from Fig. 3, \( m \) in this report is well related to \( \theta_R \) which has been widely used in literature. Is it acceptable to assume that we can use \( \theta_R \) as a parameter to indicate the intensity of swirl for pipes with different roughnesses?

H. Itô (Tohoku University):

The authors are to be congratulated on presenting useful results after difficult experiments. I would like to comment on the following points of interest.

Let us divide the flow field into two parts, one being the central core where the axial component of velocity is defective like a wake and the other being the boundary layer near the pipe wall.

(3) Concerning the central core, if \( y/b \) is used as the abscissa and suitable non-dimensional variables are chosen as the ordinate, then both the distribution of angular momentum at various values of \( m \) in Fig. 4 and the circumferential component of velocity at various values of \( m \) in Fig. 5 may be better organized, and even though they may not converge to a single curve the results may help to understand the problems.

(4) Concerning the skewed boundary layer along the pipe wall, what did you add by your experiments to the knowledge in literature?(*)?(?)

Author's closure

(1) We name that region the entrance range of swirl flows. We have examined the entrance range of swirl flows experimentally and published the results in Japan Soc. Mech. Engrs. Book of preprints No. 700-7, 1971.

In that experiment the swirl flow at the entrance of the pipe was of a solid vortex \( (m/m_f = 0.5) \) type and the value of \( m \) ranged 1.0 to 1.7. In all of them the


entrance length was less than 12 times the pipe diameter.

(2) In the major part of the pipe, except the entrance length and the region of weak swirl, $\theta_B$ may be used as a parameter to indicate the intensity of swirl. However, Fig. 3 shows that in the region of weak swirl a single value of $\theta_B$ may correspond to many values of $m$, and $\theta_B$ is not suitable to indicate the intensity of the swirl.

(3) According to your advice we have reported the radial distributions of the angular momentum and the circumferential component of velocity, where the ordinate was normalized by $v_*$ at $v=b$. The spread of the curves at different values of $m$ is only twenty percent of the value. It may be possible to deduce some important information from the trend of these curves.

(4) Although you name the outer part a skewed boundary layer, we consider that the outer part is a kind of main flow with a free vortex pattern. At the outer edge of this main flow there is a boundary layer close to the pipe wall. Figure 9 shows that the direction of wall shear is different from the direction of virtual velocity at the wall by only a few degrees. It means that the skewness of the boundary layer is little. We were not much interested in this boundary layer in the present paper and our results are little related to the literature of skewed boundary layer.

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<th>Author</th>
<th>Page</th>
<th>Column</th>
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