Upward Liquid Flow in a Small Tube into which Air Streams

(4th Report, Pipe Friction II)

By Takio ŌYA**

A study of pressure gradient of an air-liquid two-phase flow which streams upward after confluence in a small tube is made. In this paper, the pressure gradients of flows in the region of high ratio of supplied air and liquid flow, that is, piston, froth, annular and long piston flows are reported. In these flow patterns, the tendency of an unestablished flow is strong, and especially the difference of pressure gradient between upper and down streams in forth flow is large. Depending on conditions of mixing air and liquid, there is a region in the part of froth flow in which a steady rate of air flow can not exist. The pressure gradient of piston flow is small, and we must be cautious about the definition of its pipe friction. The nondimensional empirical equations for the pressure gradient of each flow pattern are obtained, and one equation which covers the whole region, is also obtained for supplementary purpose, though it is an estimation somewhat short of preciseness.

1. Introduction

This is a report of experimental investigation on the pipe friction of a two-phase flow in the case that air streams through holes of the wall into the liquid flowing upwards in small vertical tubes of 6.3,2 mm of inner diameter. Because it is immediately after the confluence, the phenomena have a tendency of an unestablished flow. The effects of air and liquid flow rate, diameter of tube, air holes, the kind of liquid and temperature in each flow pattern (cf. the first report\(^\text{(1)}\)) are examined. The pipe friction of bubble flow and slug flow whose ratio of air flow rate to liquid flow rate \(N\) is small has been described in the 3rd report\(^\text{(2)}\). In this report, we shall examine the pipe friction of flow whose \(N\) is large, that is, piston flow, froth flow, annular flow or long piston flow, and present the nondimensional equations of pipe friction in each flow pattern and one covering the whole flow region.

Nomenclature

- \(A\): inner cross area of tube
- \(d\): inner diameter of tube
- \(f_s\): void fraction
- \(f_l\): holdup of liquid \((=1-f_s)\)
- \(N\): \(Q_A/Q_l\)

\( (\Delta P/\Delta Z)_{f1} \): pressure drop gradient by pipe friction
\( (\Delta P/\Delta Z)_f \): gradient of total static pressure drop
\( Q_A/Q_l \): flow rate of air and liquid
\( R_{a0} = v_{a0}/\nu_0 \)
\( R_{d0} = v_{d0}/\nu_1 \)
\( v_{a0} = Q_A/A \)
\( v_{d0} = Q_l/A \)
\( Z\): distance from the air hole
\( \gamma_{d1} \): specific weight of air and liquid
\( \nu_{a1} \): kinematic coefficient of viscosity of air and liquid
\( \sigma\): surface tension
\( \beta\): degree of unestablishedness of flow

2. The way of expressing pipe friction and arranging the experimental values

The way of expressing of pipe friction has been described in the third report. In general, using the head term of liquid \( \gamma_{d1} \), many investigators have defined the pipe friction \((\Delta P/\Delta Z)_{f1}\) by the following equation.

\[
(\Delta P/\Delta Z)_{f1}\equiv(\Delta P/\Delta Z)_f-\gamma_{d1}f_l \tag{1}
\]

But, this definition may not always be applicable to all flow patterns. As described later, a special consideration is necessary for expression of the pipe friction of piston flow.

Now, since the measured part is in a so-called entrance length, it is considered that there are some
differences of pipe friction between the upper and
down stream part, so we shall examine again these
differences in this report as examined in previous
report. The part of measuring the pressure drop is
the region from 3.5d to 11.5d apart from air holes.
We presume that the pipe friction is \((\Delta P/\Delta Z)_T -
\tau_i((N+1))\), and that of the upper stream, half of
the whole, as \((\Delta P/\Delta Z)_{1/2}\), and that of the down stream,
half of the whole, as \((\Delta P/\Delta Z)_D\). We will provisionally
use \(\tau_i/(N+1)\) as the head term of liquid. We
define the degree of unestablishedness as \(\beta = (\Delta P/
\Delta Z)_T/((\Delta P/\Delta Z)_D)\), which, when \(\beta\) is apart from unity,
means that this flow has a strong tendency of un-
established flow. Table I shows \(\beta\) of each flow pattern
obtained from experiments. As described in the
author’s previous paper, \(\beta\) is nearly equal to unity
in case of small \(N\), but as shown in Table 1, in case of
large \(N\), the degree of unestablishedness is large.
In Table 1, the values of \(\beta\) of froth flow and long
piston flow especially depart from unity. The \(\beta\) of
piston flow of 6 mm dia’s tube is not always nearly
equal to unity, and this is because the relative error
of \(\beta\) becomes larger with a decreasing pipe friction
\((\Delta P/\Delta Z)_T\), as will be described later. In the long
piston flow, there is same cause as in the piston flow,
and still more the variation of \((\Delta P/\Delta Z)_T\) is large,
and therefore \(\beta\) of this flow departs from unity.
Accordingly, the flow which has an apparent tendency of
unestablished flow is a froth flow. Nevertheless, \(\beta\)
of froth flow varies very much with \(v_{10}, v_{90}\) and tube’s
diameter and air holes. Therefore, we assume the
average pipe friction of the upper and down streams
as the pipe friction of total test length, in all kinds of
flows including the froth flow. Accordingly, in the
case of froth flow, if the results of this experiment
are applied to the passage having a different length
from that used in this experiment, it may bring a large
error. But, for instance, the passages in carburetor
consist of many entrance lengths of single-or two-
phase flow, and therefore the presumption of a pressure
drop in the passages, especially of the two-phase flow,
have been scarcely justified. Accordingly, even if the
empirical equations lead to some errors in some cases,
they will be actually useful. Nondimensional quanti-
ties considered as factors of pressure gradient \((\Delta P/
\Delta Z)_T/\tau_i\), Reynolds number \(Re_{10} \equiv \pi d_1 / u_1\), ratio of kinematic coefficient of viscosity of air
and liquid \(\nu_1/\nu\), ratio of specific weight of air and
liquid \(\tau_1/\tau_i\), ratio of flow rate of air and liquid
\(N \equiv \tau_{10}/\tau_j\).

3. Expressions by the method of L.M

Lockhart and Martinelli \(\text{[3]}\) (abbreviated as L.M
in this paper) have arranged the measured data of
the pipe friction in a two-phase flow in the following
way, in which the situations of two-phase flow are
partitioned depending on \(Re_{10} > 2000, Re_{90} > 2000,
Re_{10} < 1000, \text{ or } Re_{90} < 1000\), and divided into four
combinations, that is, tt, tv, vt, vv (for instance, vt
indicates that \(Re_{90} < 1000 \text{ and } Re_{10} > 2000\); v stands
for viscous, t does for turbulent). The case of vv,
that is, both of liquid and air being smaller than 1000
is the same as in previous paper, and we have found
that the author’s measured data can not be arranged
by the L.M method. Now, instances of the case of
vt (each of them, \(Re_{90} > 2000\)) are shown as follows.

\[ X_{tt} = 18.7 R_{90}^{-0.4} [(1/N)(\tau_i/\tau_2)(v_1/v_2)]^{0.2} \]

\[ X_{tt} = N^{-0.9} (\tau_i/\tau_2)^{0.5} (\nu_1/\nu_2)^{-0.1} \]

\[ \beta \equiv [(\Delta P/\Delta Z)_T - \tau_1^2 + (\Delta P/\Delta Z)_D]^{0.5} \]

Table 1 Degrees of unestablished flow \(\beta = (\Delta P/\Delta Z)_T/((\Delta P/\Delta Z)_D)

<table>
<thead>
<tr>
<th>Tube No.</th>
<th>Tube dia mm</th>
<th>Air hole</th>
<th>Piston flow</th>
<th>Froth flow</th>
<th>Annular flow</th>
<th>Long piston flow</th>
<th>Condition</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6.0</td>
<td>1</td>
<td>1.0~0.6</td>
<td>1.0~0.6</td>
<td>1.0~0.6</td>
<td>Water</td>
<td>Ordinary</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4.2</td>
<td>2</td>
<td>1.0~0.5</td>
<td>1.2~0.8</td>
<td>2.0~0.7</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3.0</td>
<td>4</td>
<td>1.8~0.8</td>
<td>1.0~0.5</td>
<td>1.0~0.7</td>
<td>&quot;</td>
<td>40°C</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>3.0</td>
<td>4</td>
<td>1.0~0.5</td>
<td>1.0~0.7</td>
<td>2.0~0.4</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>3.0</td>
<td>4</td>
<td>1.0~0.5</td>
<td>1.0~0.7</td>
<td>3.0~1.3</td>
<td>Gasoline</td>
<td>Ordinary</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2.15</td>
<td>8</td>
<td>2.0~0.5</td>
<td>1.5~1.0</td>
<td>&quot;</td>
<td>water</td>
<td>&quot;</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1.05</td>
<td>30</td>
<td>3.0~0.6</td>
<td>1.8~0.7</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>4.05</td>
<td>4</td>
<td>1.4~0.8</td>
<td>1.6~0.9</td>
<td>1.0~0.2</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>1.45</td>
<td>4</td>
<td>1.0~0.6</td>
<td>1.3~0.6</td>
<td>5.0~1.5</td>
<td>&quot;</td>
<td>40°C</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>1.45</td>
<td>4</td>
<td>1.0~0.6</td>
<td>1.3~0.6</td>
<td>3.0~1.0</td>
<td>Gasoline</td>
<td>Ordinary</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>1.45</td>
<td>4</td>
<td>1.0~0.6</td>
<td>1.3~0.6</td>
<td>3.0~1.0</td>
<td>Solvent</td>
<td>&quot;</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>1.45</td>
<td>4</td>
<td>1.0~0.6</td>
<td>1.3~0.6</td>
<td>3.0~1.0</td>
<td>LS</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

LS: Nissan Dispanol LS 0.26% Surface active agent solution
\((\Delta P/\Delta Z)_{10}\) is the pipe friction for only the liquid flowing in measured part. The measured part of this experiment is the so-called entrance length. We calculate \((\Delta P/\Delta Z)_{10}\) using the friction factor (the 1st report\(^{(2)}\)) obtained from the pressure drop in liquid flow at that part, and arrange the measured data in two-phase flow by the method of L.M. These results are shown in Fig. 2. Figure 2(a), (b) show the case of vt and tt respectively. The symbols of Fig. 2 follow the standard in Fig. 1. The symbols in the figures hereinafter are also in accordance with this standard unless mentioned otherwise. Tube No. 22 is similar to tubes Nos. 3,20, and has a diameter of 2 mm.

The measured values arranged by L.M. method are almost corresponding with the curves of L.M. method in both of vt and tt, but \(X_{vt}, X_{tt}\) have not been such as shown in the author's previous paper. The equation of L.M. method has been derived by the use of the mean depth of flow, so it may be said that L.M. method is based on annular flow. It is generally said in many papers that tt curve of L.M. method especially coincides well with the other's data. Nevertheless, L.M. method has been based on the experiments in the horizontal tube, and has not been examined about the correlations between pipe frictions and flow patterns. It is important matters that the range of \(R_{sp}>2000\) occupies only a part of the range of annular flow in the author's experiments, and the greater parts of the author's experiments are \(R_{sp}<1000\), and, as shown in his previous paper, L.M curve can't be adopted for his measured values in vit range. The reasons for these differences may be considered that the passages of author's experiments are in the range of entrance length, and the tubes used in his experiments are small vertical ones. Therefore, author shall examine the measured data in each flow pattern.

4. Piston flow

So far the above Eq. (1) has generally been used as a formula of defining the pipe friction of two-phase flow \((\Delta P/\Delta Z)_T\), independency of flow patterns. But, in a piston flow, as the value of \((\Delta P/\Delta Z)_T\) is small in a large diameter's tube, the values of \((\Delta P/\Delta Z)_T\) defined by Eq. (1) have frequently taken minus sign in some papers\(^{(4)}\)-\(^{(7)}\). The minus values of pipe friction imply the inappropriateness in the expression of the pipe friction, so the possibility of minus value of pipe friction defined by Eq. (1) should be examined. The results of examination will be useful to derive the empirical equations of pipe friction.

In the model of piston flow of Fig. 3, provided that all points in one section have an identical pressure at the same section 1,2,3 respectively, that all points in one bubble have also an identical pressure, the radius of curvature of the bubble's head is assumed to be \(d_b/2\), that the bottom of the bubble is postulated to be flat, and that the dynamic pressure at the head of bubble is negligible, the correlations of all points in Fig. 3 are shown as follows.

\[ P_2-P_1=4\sigma/d_1 \]

\[ P_3-P_2=\tau(L-L_b)+P_b+P_l \]

The numbers of subscripts indicate respectively the
places in Fig. 3. \( P_R \) is a pressure increase which is generated on flowing of the liquid of 1s part into 1c part, and \( P_f \) is a pipe friction of 1c part. If the mean velocities of the liquid of 1s, 1c part are put respectively as \( v_{1s}, v_{1c} \), velocity of bubble as \( v_B \), and cross-sectional area of 1s part as \( A_{1s} \), then

\[
\text{quantity of liquid flowing into 1c par per second} = A_{1c}(v_B - v_{1c})
\]

momentum which this liquid has had before the flowing into 1c part

\[
= (\gamma_B/g)A_{1s}(v_B - v_{1s})(v_{1s} - v_{1c})
\]

momentum after the flowing into 1c part

\[
= (\gamma_B/g)A_{1c}(v_B - v_{1c})v_{1c}
\]

As the increase of momentum is equal to the pressure increase

\[
P_R = (\gamma_B/g)(A_{1s}/A)(v_B - v_{1s})(v_{1s} - v_{1c}) \quad \cdots \quad (5)
\]

from Eqs. (3) \( \sim \) (5)

\[
P_S - P_f = 4\sigma/d_1 + \gamma_B(L - L_B) + (\gamma_B/g)\]
\[
\times (A_{1s}/A)(v_B - v_{1s})(v_{1s} - v_{1c}) + P_f
\]

Dividing both sides by \( L \), since \((P_S - P_f)/L = (\Delta P/\Delta Z)_T\),

we get

\[
(\Delta P/\Delta Z)_T = 4\sigma/Ld_1 + \gamma_B(1 - L_B/L) + (\gamma_B/g)\]
\[
\times (A_{1s}/AL)(v_B - v_{1s})(v_{1s} - v_{1c}) + P_f/L \quad \cdots \quad (6)
\]

and substituting Eqs. (6), (7) into Eq. (1), we get

\[
(\Delta P/\Delta Z)_Tv_{1s} = 4\sigma/(\gamma_B/Ld_1) + (1/g)(A_{1s}/AL)\]
\[
\times (v_B - v_{1s})(v_{1s} - v_{1c}) + P_f/(\gamma_BL)\]
\[-(A_{1s}/AL)(L_B/L) \quad \cdots \quad (8)
\]

Now, putting \( d = 0.18 \text{\( \phi \)}, v_{10} = 0.2 \text{ m/sec}, v_{20} = 1 \text{ m/sec}, \) and supposing each value as follows from the papers of Akagawa and others\(^{(9)}\), we will make approximations of terms in Eq. (8).

\[
A_{1s}/A = 0.35, \quad v_B = 1.25(v_{10} + v_{20}) = 1.5 \text{ m/sec}
\]
\[
v_{1s} = v_{10} + v_{20} = 1.2 \text{ m/sec}, \quad L_B = 0.53 \text{ m}
\]
\[
L = 0.75 \text{ m}
\]

as for \( v_{1s} \), from the equation of continuity,

\[
A_{1s}v_{1s} = (A - A_{1s})v_B + A_{1c}v_{1c}
\]

therefore

\[
v_{1s} = \frac{(A_{1s}v_B - (A - A_{1s})v_B)}{A_{1s}} = 0.54(v_{10} + v_{20})
\]

\[
\cdots \cdots \cdot (9)
\]

then we get \( v_{1s} = 0.68 \text{ m/sec} \)

As for \( P_f \), if it is assumed that the distribution of velocity of liquid slug part is similar to that of the established flow, and \( \lambda = 0.02 \) is put provisionally,

\[
P_f = (\lambda/D)(L - L_B)(\gamma_B/2g)v_{1s}^2 = 11.5 \text{ kg/m}^2
\]

Assuming that \( d_1 \) is equal to the average diameter of bubble, from \( \sigma = 7.4 \times 10^{-3} \text{ kg/m}, \)

\[
\frac{4\sigma}{\gamma_BLd_1} = 1.76 \times 10^{-3}
\]

After all, each term of Eq. (8) takes respectively the following value: that is, 1st term = 1.76 \times 10^{-3}, 2nd term = 21.8 \times 10^{-4}, 3rd term = 15.4 \times 10^{-4}, 4th term = 250 \times 10^{-4}. Even if the third term \( P_f \) is still larger actually, since the fourth term is much larger than the other terms, the right hand side of Eq. (8) takes minus value. When the diameter of tube is smaller, the first term and second term of Eq. (8) become larger, and so \((\Delta P/\Delta Z)_T\) has a tendency of plus. Figure 4 is obtained from the graphs and the tables in the papers of experiments by Govier\(^{(4)}\), Sasaki\(^{(6)}\) and Ueda\(^{(9)}\), and shows the region in which \((\Delta P/\Delta Z)_T - \gamma_Bf_1\) takes minus values.

From a different viewpoint, that \((\Delta P/\Delta Z)_T\) defined by Eq. (1) takes minus value in some cases implies that the liquid at 1s part doesn’t fully display the conservative force for head term. Namely, the question is the reason why the pressure difference is so small between parts 1 and 2 in Fig. 3. This will be because the liquid of 1s part undergoes upward shearing stress from the tube wall at the time of flowing downward, and consumes the energy of liquid head. In the example of the above literature, the liquid of 1s part of piston flow will have plus value from Eq. (9).

![Fig. 3 Model of piston flow](image)

![Fig. 4 The region of plus or minus values of (\Delta P/\Delta Z)_T](image)
and flow upward. But, observations of \( v_{ia} \) in Akagawa and other's experiment have shown that the region of downward flow of 1s part is wide, which is indicated by a dotted line in Fig. 4. It may be considered that the distribution of velocity is not so simple, and downward flow occurs close to the wall, even if the average velocity is upward. From these phenomena, it may be found that the pipe friction defined by Eq. (1) is irrational.

In Eq. (6), terms which can be considered as head terms are (a): \( \tau_i(1-L_B/L) \), and (b): \( 4a/Ld_i+(\tau_i/g)(A_{is}/AL)(v_B-v_{ia})(v_{ia}-v_{is}) \). Part (a) is the weight of the liquid of 1c part, but part (b) is only one part of the weight of that of 1s part. Quantity \( 1-L_B/L \) is shown in Fig. 5 from the experiments in 3 mm dia tubes. The next equation is an approximate empirical equation.

\[
1-L_B/L \approx 1/(N+1)^{1/4}
\]

(10)

Since, for the above reason, the effect of head term of liquid of 1s part is a little, we define pipe friction as \( (\Delta P/\Delta Z)_{k_1}-\tau_i/(N+1) \) using \( 1/(N+1) \) which is an intermediate value between liquid holdup and the value obtained from Eq. (10), and we will examine the nondimensional empirical equation.

In 6 mm dia tubes, piston flow generally hardly occurs, but, in case of tubes Nos. 1~3 and 6 which have large air holes, piston flow occurs (cf. Table 1). In 6 and 3 mm dia tubes, \( (\Delta P/\Delta Z)_{k_1}-\tau_i/(N+1) \) can be expressed in terms of \( F_r \), \( K_{10} \) and \( N+1 \), and the following equation is obtained.

\[
Y_F = (1.15(N+1)^{0.04})
\]

where

\[
Y_F = \left( \frac{\Delta P/\Delta Z}{\tau_i} \right)_{k_1}/(N+1)
\]

(11)

Figure 6(a) shows, using standard symbols, the measured values of Nos. 1,3,6 (6 mm dia's tubes), and Fig. 6(b) shows those of Nos. 20,21 (both of them are 3 mm dia's tubes, and No. 21 has 0.86 mm dia x 4 of air holes) using tap water.

5. Froth flow

As mentioned above, froth flow has a strong tendency of unestablished flow, and not only the differences of pipe friction between up and down stream are large, but the pressure variance with time is also remarkable. As described in the 1st report, the region of froth flow varies with the geometric conditions of air holes and tube diameter, and contains generally that of about \( v_{10}>0.15 \text{ m/sec} \) and \( 0.2<v_{20}<0.7 \text{ m/sec} \). Figure 7 shows the measured values.
(ΔP/ΔZ)τ/τ₁ obtained with \( v_θ = 0.71 \) m/sec and changing \( v_θ \) in 6 mm dia. tubes Nos. 1~5. In tubes Nos. 3, 4, 5, if \( v_θ \) is increased slowly, fish scale type slug flow occurs, and suddenly it changes to a froth flow, shifting from A to B, and under some conditions of the passage, relaxation oscillation occurs which moves back and forth between A and B. This means that the flow rate is unstable between A and B. The flow between B and C is a stable froth flow (of course, the tendency of unstable flow is strong), and one over point C is an annular flow. In tubes Nos. 3, 4, 5, when the average flow rate of \( v_θ \) exists between A and B, the relaxation oscillation is likely to occur. In case of low frequency, a fish scale type slug flow and a froth flow occur alternately, but in case of high frequency a distinct fish scale type slug flow doesn’t occur. In case of large air holes such as Nos. 1, 2 (No. 6 is the same, though not presented here), a fish scale type slug flow hardly occurs, and, as shown in Fig. 7, a stable froth flow occurs even between A and B, for instance, at \( v_θ = 0.71 \) m/sec. Therefore, there are two kinds of froth flows, whose stable region is wide in one and narrow in the other. But even if the flow is a stable froth flow, it has a strong tendency of unestablished flow, so the degree of unestablishedness varies in complexity, and the nondimensional empirical equation of pipe friction can’t be obtained with a high accuracy. The pipe friction is expressed nondimensionally as follows in terms of \( F_r, R_0, v_θ, \nu_θ/\nu, N \). The pipe friction may be related with \( \sigma \) or \( \tau_θ/\tau_1 \) also, but separation of them is difficult, so they are neglected.

\[
Y_F = 8.5N^{0.20}
\]

where

\[
Y_F = \frac{(ΔP/ΔZ)τ - \tau_1/(N+1) - 0.2τ_1}{\tau_1(v_θ^3/gd)^{0.87}(v_θ/νd)^{0.37}(v_θ/ν_θ)^{0.5}}. \]

Friction term is \((ΔP/ΔZ)τ - \tau_1/(N+1)\) in this equation.

The average pipe frictions of up and downstream are shown in Fig. 8. Figure 8(a) shows the cases of tubes Nos. 3, 7 at ordinary temperature and 40°C, Fig. 8(b) shows those of Nos. 1, 2, 6 and Fig. 8(c) shows those of Nos. 4, 5, 8, 9. There is little scattering of data in tubes Nos. 1, 2, 6 in which a fish scale type slug flow hardly occurs. Figure 8(d) shows the case of 3 mm dia tubes with tap water at ordinary temperature and 40°C, and Fig. 8(e) shows the case of tube No. 20 (3 mm dia) with gasoline, solvent and surface active agent solution. In each figure, Eq. (12) is indicated with solid line. It can be found that Eq. (12) is adequate.

6. Annular flow

By photograph, it is observed that the interface between air and liquid is rough in case of gasoline and solvent, but the interface is not so rough in case of surface active agent solution and water. It may be related with the surface tension, but the separation of specific weight and surface tension in solvent, gasoline and water may be hard because the two physical quantities have a similar tendency. Surface active agent solution which has the same specific weight as water is a matter of great concern for its own surface tension as mentioned above. But, using the little difference in the former three liquids between the
tendencies of specific weight and surface tension, we examine the effects of \( \sigma \) and \( \gamma_1 \) of these liquids. It may be said from this examination that pipe friction has some correlations with \( \gamma_1 \). When we designate \((v_0^2/g) (\gamma_1 \sigma / \sigma)\) as actual Weber number \((v_0: \text{relative velocity of air and liquid})\), the actual Weber number is about 40 in case of \( v_0 = 1 \text{ m/sec} \), \( d = 3 \times 10^{-3} \text{ m} \) with water, so it may be considered that there is no great effect of \( \sigma \). In consideration of these matters and other investigations, we will examine a nondimensional equation without \( \sigma \). As the column of air passes completely through the center of tube in an annular flow, and the liquid film is thin, the case that only air flows in tube may be the base of the examination. But, it must be considered that the friction at the interface of air and liquid may be larger than that of tube, and that this flow is in an entrance length. If \((\Delta P/\Delta Z)_{T} \) is expressed in terms of \( \gamma_2, v_0 \sigma^2 / gd, v_0 \sigma/d, N \), the empirical equation is as follows.

\[
(\Delta P/\Delta Z)_{T} = \gamma_2 \left[170 + 74(v_0 \sigma^2 / gd) \times (v_0 / v_0 \sigma/d)^{0.12} N^{-0.9}\right] \quad (13)
\]

If \( \gamma_1 \) is used as in other flow patterns, Eq. (13) can be transformed into next equation.

If we set

\[
Y_A = \frac{(\Delta P/\Delta Z)_{T} - 0.2 \gamma_1}{\gamma_2 (v_0 \sigma^2 / gd) (v_0 / v_0 \sigma/d)^{0.12} (v_0 / \nu_0)^{0.25}}
\]

then

\[
Y_A = 74N^{0.05}
\]

expressing in terms of \( N + 1 \), then

\[
Y_A = 62(N + 1)^{0.99} \quad \ldots (15)
\]

Quantity 0.2 \( \gamma_1 \) in the above definition of \( Y_A \) is a part of empirical equation, and the pipe friction is about equal to \((\Delta P/\Delta Z)_{T}\). The results of arranging data are shown in Fig. 9. Figure 9(a) shows the comparison of tap water (at ordinary temperature and 40°C) with gasoline (at ordinary temperature) in Nos. 3, 7 and the solid line indicates Eq. (15). Figure 9(b) shows the comparison of the size of air holes in tube of 6 mm dia., and there are little differences between Nos. 1, 2, 6 having large air holes and Nos. 4, 5, 8, having small air holes. Figure 9(c) shows the case of Nos. 20, 21. Figure 9(d) shows the case of some kinds of liquid with 3 mm dia tubes and the case of tap water with 2 mm dia tubes. Equation (15) is adequate in each case.

![Fig. 9 Friction of annular flow](image-url)
7. Long piston flow

Long piston flow occurs when $v_0$ is small and $v_{90}$ is relatively large and the pipe friction is as small as that of piston flow. The nondimensional equations are as follows. If we set

$$ Y_L = \frac{(\Delta P/\Delta Z)_T - \tau_1/(N+1)}{\tau_1(v_{90^2}/gd)^{0.35}(\nu_1/\nu_9d)^{0.25}} $$

then

$$ Y_L = 3.5 $$

It should be noted that $Y_L$ is not affected by $v_{90}$ in the region of this flow pattern. The cases of Nos. 3,7 are shown in Fig. 10. In a long piston flow, the variation of pressure and the scattering of data are large. The other tubes of 6 mm and 3 mm dia show the same results which are not presented here. In gasoline and solvent, this flow hardly occurs.

8. General pipe friction covering all regions

For the pipe frictions in each flow pattern, the nondimensional empirical equations have been expressed. Nevertheless, when we want to calculate the pressure drop of two-phase flow in carburetor, the equation which covers all the regions of each flow pattern is convenient for the estimation of it. Therefore, in consideration of the empirical equations of each flow pattern, we obtain the following equation.

$$ Y_T = 8.2(Nv_{90d}/\nu_1)^{0.4} $$

where

$$ Y_T = \frac{(\Delta P/\Delta Z)_T - \tau_1/(N+1)}{\tau_1(v_{90^2}/gd)^{0.35}(\nu_1/\nu_9d)^{0.25}} $$

This is shown in Fig. 11. Figure 11(a) shows the case of No. 3 with tap water and gasoline. The abscissa $Nv_{90d}$ is $v_{90}/\nu_1$, and some discontinuities occur at $Nv_{90}=1.4 \times 10^4$ with tap water of $v_{90}=0.35$ m/sec at ordinary temperature. This point is the border between fish scale type slug flow and froth flow. But, this discontinuity doesn't appear at 40°C or $v_{90}<0.18$ m/sec of the ordinary temperature, since a fish scale type slug flow doesn't occur under these conditions. Figure 11(b) shows examples in tubes of 3 and 2 mm diameters. A discontinuity also occurs at about $Nv_{90}=10^4$ in case of large $v_{90}$. The scattering of data arranged by Eq. (17) is larger than that by the equations of each flow pattern. The scattering has a maximum at about $Nv_{90}=10^4$, and the range of scattering reaches ±50%. Nevertheless, Eq. (17) will be very useful to make an approximation of $(\Delta P/\Delta Z)_T$.

9. Conclusions

The examinations of pipe friction of two-phase flow in a vertical small tube immediately behind the confluence are finished. The pressure gradient of flow has a strong correlation with the flow patterns. The tendency of unestablished flow is very conspicuous in a froth flow, and somewhat in an annular flow, piston flow and long piston flow. Therefore it may be said that the two-phase flow having a large flow rate ratio of air and liquid $N$ has a tendency of unestablished flow. The pressure gradients of two-phase flow are examined with the average values of the measured part respectively in each flow pattern.

The formula of defining pipe friction $(\Delta P/\Delta Z)_T - \tau_1/\nu_1$ which has been used in general is likely to take minus value in a piston flow, but this matter is considered irrational. Therefore according to the formulas assumed to be adequate for the definition of pipe friction respectively in each flow pattern, nondimensional quantities of $(\Delta P/\Delta Z)_T/\tau_1$, $v_{90^2}/gd$, $\nu_1/\nu_9$, $N$ are examined.

(1) In piston flow, the choice of various formulas defining the pipe friction is a matter of importance,
and the pipe friction is defined as $(\Delta P/\Delta Z)_{T-T_1}/(N+1)$. An empirical equation (11) is obtained.

(2) In froth flow, the tendency of unestablished flow is strong and the pressure variations and scattering of data are large. An empirical equation (12) is obtained.

(3) In annular flow, empirical equations (13), (14), (15) are obtained.

(4) In long piston flow, the pipe friction is hardly affected by $v_p$. An empirical equation (16) is obtained.

(5) A general empirical equation (17) covering all kinds of flow patterns is obtained. Though this includes some errors, it will be of practical use.

References


Discussion

M. Arie (Hokkaido University):

(1) What is the implication of $\gamma$ sign, which is used at several places of the present paper.

(2) The discussor wishes to know the details of the piezometric hole provided for the measurement of the pressure gradient along the test pipe.

(3) Is there not any difficulty in physically understanding the technique of expressing $Y_F$ and $Y_A$ in one equation, since the numerator of $Y_F$ for froth flow and that of $Y_A$ for annular flow are essentially different in their forms?

(4) The significance of $1/(N+1)$ defined as the mid-value of $f_1$ for a piston flow at the 21 st line of the left column on page 1538 is not clear.

K. Akagawa (Kobe University):

(5) The measured part is described to be “the length of 11.5$d$ from 3.5$d$ to 15$d$”. If the purpose of this investigation is to make clear the characteristics of unestablished flow, the measurement of pressure distribution along the tube is necessary as the entrance length varies with the flow rate of air and liquid. So, it is questionable to measure the pressure drop of the fixed length. The average values of up and down stream parts used in this paper are equal to the average values of whole length, therefore the meaning of $\beta$ will be lost. Furthermore, these are not compared with those in the region of the established flow.

(6) Concerning the fact that the pipe friction defined by Eq. (1) has a possibility of taking minus values under some conditions of piston flow, it is stated that “the definition of pipe friction by Eq. (1) is irrational”. But, as discussed in our paper,*1 we have considered that to take the minus values depends on merely the definition, and so this is not irrational. Meanwhile, why is $(\Delta P/\Delta Z)_{T-T_1}/(N+1)$ defined newly in this paper regarded as the pipe friction? It seems to me that the physical meaning is not clear.

(7) $(\Delta P/\Delta Z)_{T-T_1}/(N+1)-0.271$ or $(\Delta P/\Delta Z)_{T-T_1}/0.271$ are used in Eq. (12) or (14), but the physical meaning of these equations is not clear for the pipe friction. If these equations are presented as the empirical equations of the pressure losses, they are useful in their own way, but from the view point of pipe friction, they are disunited theoretically.

(8) I wish to explain about our investigation** referred to in page 1538. The $v_B$ in the end of page 1537 is obtained by using the diameter of assistive cylinder of bubble calculated from the volume and the length of bubbles, and so, $v_B$ is the average liquid velocity around bubble. On the other hand, the $v_B$ in the top of page 1538 is the velocity in the lower part of 1s, therefore, even if the film of liquid of upper part of 1s flows upwards, the film is likely to flow downward in the lower part of 1s. I can’t agree with the explanation that “the distribution of velocity is not so simple and downward flow occurs close to the wall, even if the average velocity is upward”. In our experiments these phenomena were not observed.

(9) The author said that, in froth flow, the variations are very strong. We wish to know the degree of variation and the measuring method of the average values of the changing pressure.

K. Okuda (Muroran Institute of Technology):

(10) The author says, “as shown in Table 1, in case of large $N$,...” in 17 th line of left column
at page 1535. Questioner wishes to know the definite values of \( N \).

(11) The meanings of \( X \), \( X_{st} \), and \( X_{th} \) in Eq. (2) should be described in addition.

(12) The sentence “This will be because the liquid of 1s part undergoes upward shearing stress... and downward flow occurs close to the wall...” is given in 29th line of light column at page 1337. Have these phenomena been confirmed experimentally?

(13) In what way are the exponents of \( F_{st} \equiv \varepsilon_{st} / \varepsilon_{gd}, R_{st} \equiv \varepsilon_{st} / \nu_{st}, \nu_{st} / \nu_1 \) and 0.2 of \( 2\gamma_1 \) derived in Eq. (11)~(14), (16), (17)?

Y. Hôšho (Hitachi Ltd.):

(14) What are the relations between \( \beta \) and flow rate variation? Is the Eq. (17) applicable to the unsteady flow?

(15) There are a bend, an orifice, a reducing tube, an expansion tube and so on in the upstream or downstream of air bleed part in the fuel passage of carburetor. If these losses and the pressure drop of two-phase flow obtained from this investigation are summed up, can the total losses be estimated?

**Author's closure**

1) They imply that the formulas are those of definitions.

2) The pressure drop along the test pipe is measured with manometers. The holes for measuring are set with pitches of about 2.5d. The holes for measuring are shown in Append.-Fig. 1. When bubbles fill the holes whose diameters are small, large errors in manometer’s readings (the maximum reaches 50 mmHg) come out. The lengths of the 0.7 mm dia’s holes are made as small as possible, and transparent hard vinyl chloride resin pipes of 5 mm of inner diameters are connected to holes. These pipes lead to manometers. Therefore, even if small bubbles pass through the small hole to enter the 5 mm dia’s pipe, the bubbles are floating in the upper part of this 5 mm pipe and do not fill the pipe so far as the volume of the air is small. When the quantity of the air increases to fill the whole cross section of pipe and interferes the measurement, a little water is poured into the driving out air tube and the bubble is driven out through the 0.7 mm dia’s hole.

In case of a large variance of pressure, as in a froth flow, we measure the pressure with the constricted passages connecting to manometers.

3) Equation (17) is a general equation covering all kinds of flow patterns. As mentioned in 20th line of column at page 1541, this equation is an empirical equation obtained regardless of flow patterns and it is of practical use but has no physical meaning.

4) In such a continuous flow as bubble or annular flow, the liquid has a conservative energy of height, but in such a discontinuous flow as a piston or long piston flow, this does not hold. Since the whole weight of liquid of the 1c part fully gives pressure to the head of bubble, the liquid energy of head term is completely conserved. The liquid velocity of 1c part decreases from \( \nu_{te} \) when it enters the 1s part, and decreases still more with gravity in the 1s part. Therefore, even if the velocity is upward at the upper 1s part, it is likely to become downward in the lower 1s part. In the downward flow, one part of the liquid energy of head term of the 1s part is consumed as heat with the shearing force at the wall of tube. Moreover, when the liquid of the 1s part flows into the 1c part, not all the momentums of the 1s part are changed into the pressure, but one part is changed into heat energy. Therefore, in piston flow, all of the weight of the liquid of holdup \( f_1 \) is not conservative quantity. In other flow patterns, the head term of liquid is dealt with as the conservative quantity. If piston flow is dealt with in the same way, all of the 1s part must not be considered as the head term of liquid. Consequently, the conservative quantity may be the middle value between \( f_1 \) and \( 1 - L_B/L \) from Eq. (7).

\[ f_1 = (N + 6)/(N + 1) \]

from this equation and Eq. (10), the following inequality is deduced.

\[ (N + 6)/(N + 1) > 1/(N + 1) > 1/(N + 1)^{1.4} \]

so, \( 1/(N + 1) \) is considered as conservative quantity provisionally. Using \( 1/(N + 1) \), we can arrange well the data by Eq. (11). Consequently, \( 1/(N + 1) \) is a semi-empirical equation containing the above theoretical consideration.

5) As you mention, the length of unestablished flow varies with the change of the flow rate of air and liquid. As shown in Append.-Fig. 2,3 of the 3rd report which indicates examples of experiments with vertical small tubes, the length varies from 20d to 100d in the range of these experiments. But, as described from the 29th line of left column on page 1526 in the 3rd report, whether the measured parts are in the range of unestablished flow or not, when \( L_1/d \) and \( L_2/d \) (distance of both ends of measuring part from the confluence point) are constant, the pressure gradient of that part \( \Delta P/\Delta Z \gamma_1/\gamma_1 \) is a function of \( R_{st}, \varepsilon_{st}/\varepsilon_{gd}, N, \gamma_1 d^2/\sigma, \nu_{st}/\nu_1, \gamma_{st}/\gamma_1 \). It will be the most desirable that \( \Delta P/\Delta Z \gamma_1/\gamma_1 \) is expressed in terms of distance \( L \). But as shown in Append.-Fig. 3 of the 3rd report, the pipe friction is too complicated to do so. The purpose of this investigation is to apply these results to the two-phase flow in the carburetor, so these experiments are made in constant length of passage.

Therefore, in \( 6 \sim 2 \) mm dia’s passage having dimensional similarity to those used in this investiga-
tion, these equations may be applied. Furthermore, when $\beta=1$, these equations may be applied even if $U/d$ differs a little from these experiments. When $\beta$ departs from unity, it will produce large errors to apply these equations to the part beyond the part of $3.5d \sim 15d$. We take $\beta$ as a standard of appropriateness in applying these equations. As shown in Appendix Fig. 3 in the 3rd report, $(\Delta P/\Delta Z)T/\rho_1$ has generally a tendency of decreasing as the flow goes downward. The comparison with the investigations of established flow may be obtained from the L.M method of Fig. 2 in this report and of Fig. 5 in the 3rd report. Since, in these figures, the denominator of the ordinate, $(\Delta P/\Delta Z)_{Tn}$, is the pressure gradient of the same part in liquid flow, if $(\Delta P/\Delta Z)_{Tn}$ of established flow of liquid is used as the denominator, the position of the data will move upwards, and the differences from L.M curve will be increased further more.

(6) "The definition of the pipe friction by Eq. (1) is irrational" means that Eq. (1) is irrational for the definition of the pipe friction, and not that the phenomenon of taking minus values in Eq. (1) is irrational. As you mention, this is entirely a matter of definition. But, the conception that the language of friction expresses the phenomenon generating heat is commonly accepted. If we are to be faithful to this conception, we can't admit the friction taking minus values. About the physical meaning of $(\Delta P / \Delta Z)T/\rho_1-1/(N+1)$, I wish you refer to the answer (4) to Prof. Arie.

(7) In piston, slug and bubble flow respectively, we can express nondimensionally the pipe friction as a monomial formula using suitable head terms of liquid. In froth, annular and long piston flow, the holdup of liquid $f_1$ is small, and so the importance of head term of liquid decreases. Therefore, we take the head term of liquid in froth and long piston flow as $1/(N+1)$, and neglect it in annular flow. But, since in froth and annular flow we can't fully express them in monomial formulas we need a constant, i.e., 0.2. The formulas defining the pipe friction of each flow pattern are shown in Appendix Table 1.

(8) The author agrees with you. It may be possible to flow downwards at only the laminar boundary layer of $1$s part, but this is an unnecessary estimation, and such estimation as in this text must not be made. Therefore, the sentence "It may be considered...close to the wall." from 4th line of left column at page 1538 should be corrected to the following expression. That is," This is because the velocity of the liquid part is decreases by the gravity, and in some cases $v_{le}$ becomes downwards at the lower part of $1$s. That a region of minus values of $(\Delta P/\Delta Z)T-1/\rho_1f_1$ exists within that of downwards velocity will support the estimation that the downwards the liquid must receive the upward shearing stress at the wall of tube."

(9) As mentioned in the text, there are two kinds of froth flows, one being stable and the other unstable. The amplitude of pressure oscillation in the unstable froth flow shall be described in 5th report. The pressure variations of pipe friction in stable froth flow have not been measured yet quantitatively, but from the following phenomena, large variations can be estimated.

(a) For example, the liquid chamber pressure is measured at the entrance part of vertical tube, which has 3 mm diameter and 45 mm length after confluence, with the strain meter type pressure gauge. The frequency of pressure oscillation is about 80 Hz, and the maximum of pressure amplitude is about 25 kg/m² in case of $v_{le}=0.7$ m/sec, $v_{le}=6$ m/sec. But, the variances of the pressure drop at the confluence are contained in this variance of the chamber's.

(b) The velocity of the liquid slug part of froth flow is measured with a high speed movie camera. It can be observed how the velocity varies very much with time.

(c) From the observation with a high speed camera, we can find by the Schlieren effect of projecting light that strong turbulences occur in the passages to the manometers affected with the turbulence of froth flow through the small hole for measuring the pressure.

The central values of varying pressure are obtained by narrowing the passage to the manometers.

(10) This means concretely froth, annular and long piston flow. Though there are some differences depending on the conditions of the diameter of tube or air holes, these flow patterns occur in about $N>5$. 

Appendix Table 1: Formular defining pipe friction

<table>
<thead>
<tr>
<th>Flow pattern</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble flow</td>
<td>$(\Delta P/\Delta Z)T/(\rho_1-1/(N+1))$</td>
</tr>
<tr>
<td>Slug flow</td>
<td>$(\Delta P/\Delta Z)T/(\rho_1-(N+6)/6(N+1))$</td>
</tr>
<tr>
<td>Piston flow</td>
<td>$(\Delta P/\Delta Z)T/(\rho_1-1/(N+1))$</td>
</tr>
<tr>
<td>Froth flow</td>
<td>$(\Delta P/\Delta Z)T/(\rho_1-1/(N+1))$</td>
</tr>
<tr>
<td>Annular flow</td>
<td>$(\Delta P/\Delta Z)T/\rho_1$</td>
</tr>
<tr>
<td>Long piston flow</td>
<td>$(\Delta P/\Delta Z)T/(\rho_1-1/(N+1))$</td>
</tr>
</tbody>
</table>

Appendix Fig. 1: Sketch of a section for measuring pressure
(11) The L.M method expresses the correlation of the abscissa X and the ordinate $\phi$ with four curves. The definition of the abscissa $X$ is changed with such condition as Reynolds number of liquid or air being below 1,000 (e) or over 2,000 (t). Out of the four combinations, $X_{st}$, $X_{at}$ are shown in Eq. (2). $X_{st}$, $X_{at}$ are shown as follows:

$X_{st} = \frac{(\frac{\gamma}{\gamma'})^2 (\nu_1 / \nu_2) (1/N)^{0.5}}{5.36 \times 10^{-2} R_{in}^{0.4} (1/N) (\gamma / \gamma') (\nu_1 / \nu_2)^{0.5}}$

(12) These phenomena are not confirmed experimentally. I wish you refer to the answer (8) for Prof. Akagawa.

(13) If, at first $(\Delta P / \Delta Z)_T = f(\nu_0, \nu_0, d, \nu_1, \nu_2, \gamma_0, \gamma_0, \sigma)$ is presumed, this is converted as follows by the dimensional analysis:

$(\Delta P / \Delta Z)_T / \gamma_0 = \frac{((\Delta P / \Delta Z)_T - \text{head term of liquid}) / \gamma_0}{(\Delta P / \Delta Z)_T / \gamma_0}$

As the experiments are made generally at nearly constant $\nu_0$, the exponents $m$ and $n$ of the following equation can be obtained from the graph of the abscissa $(\Delta P / \Delta Z)_T = \text{head term of liquid} / \gamma_0$ and the ordinate $\nu_0$ (the parameter is approximate values of $\nu_0$).

$(\Delta P / \Delta Z)_T / \gamma_0 \propto \nu_0^m d^n$

The measured values of $(\Delta P / \Delta Z)_T / \gamma_0$ are divided by $\nu_0^m d^n$ whose $\nu_0$ and $d$ are actual values, and the results are plotted in a graph whose abscissa is $N$ or $N+1$, and the appropriateness of the estimation is examined. At that time, which is to be adopted $N$ or $N+1$ is decided according as the ordinate can be expressed in the mononial expression of the abscissa or not. Nevertheless, in bubble flow, the equation is derived from theoretical formula, and in froth and annular flow, a constant term is added to it since we can’t derive a monomial expression without the constant term. Then, the influences of $\nu_1$, $\nu_2$ are examined from the data at different temperatures and those of the different kinds of liquid. In the flow patterns in which the influences of $\gamma_0$, $\nu_0$, $\sigma$ are not so clear, they are neglected. From the exponents of $\nu_0$, $d$, $\nu_1$ and so on, the exponents of nondimensional quantities are obtained.

(14) In froth flow, $\beta$ does not only depart from unity but also the pressure variance is large. The author considers that there is no correlation between $\beta$ and flow rate variance. Since the author has not yet made any examination about the unsteady flow, he can’t assert it, but considers that it may be possible to apply these results to the unsteady flow, as Eq. (17) is a general equation having no relation with each flow pattern, though its accuracy is not satisfactory.

(15) The one part of your questions is answered in the author’s paper. The pressure recovery after main-jet and the effect of main-jet on the two-phase flow patterns must be considered. When there are orifices and so on which cause turbulence in the upstream of the confluence point or particularly in the two-phase flow, the flow is turbulent and any fish scale type slug flow hardly occurs. In those cases, $\beta$ will also vary, and it may not be always possible to apply the equation obtained in each flow pattern to them. But, the approximation can be made by Eq. (17).