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The Influence of the Bauschinger Effect on the Subsequent Yield Condition*

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Plastic behavior of metal under complex loading is experimentally investigated by subjecting thin-walled tubular specimens of brass to combined axial load, internal (or external) pressure and torsion. The subsequent yield surfaces after some radial preloadings are established in the axial-circumferential-shear stress space. The yield surfaces which are dependent on the Bauschinger effect are obtained in all quadrants of the two principal stress-space by coaxial stress probes. The effect of the rotation of stress axes on the yield surface is studied by determining the yield stresses with rotating stress probes. Experimental results are represented in the deviatoric plane, and the influence of the Bauschinger effect and the rotation of stress axes on the subsequent yield surface are discussed. It is shown that, when the subsequent yield surfaces after radial preloadings are represented in the deviatoric plane, they are congruent with one another irrespective of the directions in preloading, if they have the same amount of prestrain.

1. Introduction

The subsequent yield surface after preloading is generally represented as a complicated shape in the stress space. An intensive and systematic study of the yield surface for various preloading paths is essential to the research of the theory of plastic anisotropy, namely the theory of the plastic flow which also takes the effect of strain history into consideration.

We already have a considerable number of papers⁽¹⁾⁻⁽⁸⁾ which report the experimental results on subsequent yield surfaces. But, in most of these papers, the yield loci are obtained only in specified planes of the stress space. In these experimental results, we do not find three-dimensional representation of the subsequent yield surface showing the effect of the rotation of stress axes in determining the yield points. Our knowledge on how the three-dimensional subsequent yield surface is represented seems to be still insufficient.

The authors consider it the first step in the study of the theory of plastic anisotropy to investigate the influence of the Bauschinger effect and the rotation of stress axes on the subsequent yield surface.† For this

reason, the authors have performed the following two kinds of experiments on the subsequent yield surface after radial preloading. The first is an experiment to find by coaxial stress probes the yield surface in all quadrants of the same principal stress space as that in preloading. The second is an experiment to obtain by rotating stress probes the yield surface in a different principal stress space from that in preloading.

This paper presents the shape of a three dimensional subsequent yield surface obtained by subjecting thin-walled tubular specimens to combined axial load, internal (or external) pressure and torsion. The results from the foregoing tests are represented in the cylindrical coordinate constituted by the deviatoric plane and the rotation angle of the principal stress axes. The shape of the subsequent yield surface is discussed on the basis of these experimental results.

2. Experimental procedure

Figure 1 shows a combined stress testing machine used in the experiment. The machine is capable of subjecting a thin-walled specimen to combined axial load, internal (or external) pressure and torsion simultaneously. The load capacity is 5 tons in tension and compression, 50 kgm in forward and reverse torsion, and 500 kg/cm² in internal (or external) pressure. In Fig. 1, the axial load is applied to the specimen ① by a hydraulic jack ② and the torsion by rotating a handle ③. The internal pressure is given to the specimen through an inlet ④. A pressure

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† In addition to these problems, the effect of the rotation of the stress axes in preloading is very important. This effect, closely relating to the "law of delay" in plastic deformation, will be discussed in near future by the authors.

chamber ⑤ is used in subjecting the specimen to the external pressure. This chamber is useful not only for subjecting the specimen to the external pressure, but also for preventing the specimen from buckling in the compression test. Both the axial and torsional loads are measured by a load cell ⑥. The transducer of the strain gauge type is used for measuring the internal and external pressures.

The dimensions of the specimen are shown in Fig. 2. It is machined from brass bar stocks (BsBM-2 according to the Japanese standard) which are annealed for 3 hours at 550°C. The nominal wall thickness of the specimen is 1.5 mm. The axial, circumferential and "45 degree" strains are measured by two strain gauges which are attached to the outer surface in the middle part of the specimen so as to be located diametrically opposite to each other. The nominal values of axial, circumferential and shear stresses are calculated on the basis of the original dimensions of the specimen.

The prestrains are given to the specimen by radial preloading under combined axial, circumferential and shear stresses. The amounts of the prestrain are chosen as 0.8%, 1.8%, 3.4% and 5.1%. The directions of the preloading paths are shown in Fig. 3. The axial, circumferential and shear stresses are denoted by σ_x , σ_y and τ_{xy} , respectively, in Fig. 3. The ratios among σ_x , σ_y and τ_{xy} are indicated in the parentheses in Fig. 3.

The subsequent yield point is determined by the reloading path of a straight line like O'A in Fig. 5(b). The point O' is chosen within the yield surface in such a way that the elastic behavior of the specimen is

expected in the neighborhood of this point. The yield point is defined by the effective proof strain of 0.02%.

3. Experimental results

Some of the experimental results on the initial and the subsequent yield surfaces are shown in Figs. 4 to 6. Figures 4(a) and (b) show the subsequent yield loci established in the σ_x - σ_y plane for preloading along the paths (1) and (2) in Fig. 3. The subsequent yield loci after preloading along the paths (3) and (4) in Fig. 3 are shown in Figs. 4(c) and (d). The principal stresses in the radial preloading by combined σ_x , σ_y and τ_{xy} are denoted by σ_1 and σ_2 in Figs. 4(c) and (d). The yield points in the yield loci of Figs. 4(c) and (d) are determined by the coaxial stress probes in which principal stress axes do not rotate with respect to those in preloading. Figures 4(e) and (f) show the subsequent yield loci determined in the σ_x - τ_{xy} plane for preloading along the paths (5) and (1), respectively, in Fig. 3. The subsequent yield surfaces established three-dimensionally by combined σ_x , σ_y and τ_{xy} are shown in Figs. 5(a) and (b). The preloading directions in these figures are taken along the paths (1) and (5) in Fig. 3. These yield surfaces of Fig. 5 are illustrated in the σ_x - σ_y and σ_x - τ_{xy} plane, respectively, by projecting the contour lines for the constant values of τ_{xy} and σ_y . The results of Figs. 5(a) and (b) represent the shape of the subsequent yield surface dependent on the rotation of stress axes in determining the yield point as well as on the Bauschinger effect.

From the experimental results shown in Figs. 4 and 5, the following facts can be obtained on the behavior of the yield surface. The initial yield surface coincides with the Mises condition approximately as

Fig. 1 Testing machine

Fig. 2 Thin-walled tubular specimen

Fig. 3 Directions of preloading paths

(a) For preloading along the path (1) in Fig. 3

(b) For preloading along the path (2) in Fig. 3

(c) For preloading along the path (3) in Fig. 3

(d) For preloading along the path (4) in Fig. 3

(e) For preloading along the path (5) in Fig. 3

(f) For preloading along the path (1) in Fig. 3

Fig. 4 Experimental results of the subsequent yield loci

shown in Figs. 4(a), (b) and (c). The subsequent yield locus does not expand isotropically. A translation of the yield surface is observed in the preloading direction. The subsequent yield surface has a rounded nose of large curvature at the prestress point. A flattening of the yield surface is observed at the opposite side of the prestress point.

On comparing Fig. 4(a) with Fig. 4(c), and Fig. 4(b) with Fig. 4(d), it will be found that the subsequent yield loci which are determined by coaxial stress probes for the radial preloading to the same magnitude have shapes similar to one another if the principal stress ratios in the preloading are so chosen as to have the same value, even though the preloading is conducted by any combination of σ_x , σ_y and τ_{xy} .

Figures 6(a) to (c) show the effect of varying the definition of the yield point on the shape of the subsequent yield loci. These results are illustrated in the non-dimensional stress plane by dividing the

yield stress with the initial uniaxial yield stress (σ_0') or the initial shear one (τ_0') determined for each definition of yield point. In the result shown in Figs. 6(a) to (c), the prestrains of 5.1% are given by preloading along the paths (1), (2) and (5) in

(a) For preloading along the path (1) in Fig. 3

(b) For preloading along the path (5) in Fig. 3
Fig. 5 Experimental results of the subsequent yield surface

(a) For uniaxial tensile preloading

(b) For pure shear preloading

(c) For simple shear preloading

Fig. 6 Effect of the definition of the yield point on the yield locus after 5.1% prestrain

Fig. 3. The proof strains employed for the definition of the yield point are indicated by the percentage values in Fig. 6. For the proof strain increment of the same value, the yield stress increment of the initial yielding is observed to be larger than that of the subsequent yielding in the preloading direction. This causes the subsequent non-dimensional yield point in the preloading direction for a larger proof strain to be smaller than that for a smaller proof strain in Figs. 6(a) to (c). The subsequent yield locus expands with an increase in the amount of the proof strain.* The expansion is remarkable in the opposite side to the prestress point.

4. Discussion on the shape of the subsequent yield surface

4.1 Theoretical consideration

The yield locus is usually plotted in the Cartesian coordinate constituted by two or three stresses used in the test as shown in Figs. 4 and 5. In such a representation, the variation of the shape of the yield surface is extremely complex depending on the magnitude and direction of preloading. This makes it difficult to draw a definite conclusion about the effect of preloading on the yield locus from the experimental results of the subsequent yield loci after different radial preloadings. The authors represent the yield surface in the deviatoric plane⁽⁹⁾ in consideration of the fact that the plastic deformation is characterized by the incompressibility and the independence of the mean hydrostatic stress under the ordinary stress state.

The stress space as shown in Fig. 7 is used for representing the yield surface. This space will hereafter be referred to as the "deviatoric stress space." The base plane in this stress space, denoted as F_0 in Fig. 7, corresponds to the deviatoric plane in the principal stress space whose axes coincide with the axial, circumferential and radial directions of the specimen. The vertical axis α in relation to the base plane of Fig. 7 indicates the rotation angle of the principal stress axes with respect to the radial direction of the specimen, which can be calculated by

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (|\alpha| \leq 45^\circ) \quad \dots\dots\dots (1)$$

Then the deviatoric plane corresponding to the principal stress space having a rotation angle α is represented as the plane at a vertical distance of α from the origin $\bar{O}$ in Fig. 7. The following indices are used for representing the magnitude and the direction of the deviatoric stress vector⁽⁹⁾:

* The initial yield locus in such a non-dimensional stress plane as shown in Fig. 6 intersects the points $(\pm 1, 0)$ and $(0, \pm 1)$ independently of the amount of the proof strain.

$$\xi = \frac{1}{\sqrt{2} \sigma_0} \{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{1/2} \quad \dots\dots\dots (2)$$

$$\zeta = \frac{6}{\pi} \arctan \frac{(\sigma_2 - \sigma_3)\sqrt{3}}{2\sigma_1 - \sigma_2 - \sigma_3} + 3 \left(1 - \frac{\sigma_2 - \sigma_3}{|\sigma_2 - \sigma_3|}\right) \quad \dots\dots\dots (3)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses, and σ_0 is the initial yield stress under uniaxial loading. Figure 7 shows also a correspondence between the yield locus in the ordinary stress space and that in the deviatoric stress space. Let the locus PA in Fig. 7(a) be determined for the preloading OP by combined σ_x and σ_y . In such stress probes, the principal axis does not rotate. Hence the locus PA is represented as the locus P'A' in the base plane of the deviatoric stress space. On the other hand, the locus QB which is determined by combined σ_x and τ_{xy} does not lie in a plane in which the value of α is constant. It is represented as the curved space line Q'B' in the deviatoric stress space, since the yield points in the locus QB have different rotation angles of the principal stress axis. In the deviatoric stress space, a yield locus determined by coaxial stress probes is represented as a closed curve in a plane with a constant value of α and one determined by rotating stress probes is represented as a curved space line.

Figure 8 shows the comparison of the shape of the initial yield surface as represented by the Mises condition in the $\sigma_x - \sigma_y - \tau_{xy}$ stress space and in the deviatoric stress space. The yield surface represented

Fig. 7 Comparison between the yield locus in the $\sigma_x - \sigma_y - \tau_{xy}$ stress space and that in the deviatoric stress space

as an ellipsoid in the $\sigma_x - \sigma_y - \tau_{xy}$ stress space takes the shape of a cylinder in the deviatoric stress space. For example, the initial yield locus AGECHFA in the $\sigma_x - \sigma_y$ plane is represented as a circle in the base plane of $\alpha = 0^\circ$, and the initial yield locus DABCD in the $\sigma_x - \tau_{xy}$ plane as two curved space lines in the curved surface with hatching lines. The yield locus in the plane of a vertical distance α of the deviatoric stress space corresponds to the yield locus in the plane,

$$\sigma_x - \sigma_y - (2\tau_{xy})/(\tan 2\alpha) = 0$$

of the $\sigma_x - \sigma_y - \tau_{xy}$ stress space.

Figures 9(a) and (b) show the correlation between the shapes of the subsequent yield surfaces in the foregoing two stress-spaces. The subsequent yield surfaces after uniaxial ($\zeta = 0$, $\alpha = 0^\circ$) preloading are shown in Fig. 9(a), where the loci S_0 in the plane $\tau_{xy} = 0$ and S_0 in the plane $\sigma_y = 0$ correspond to the loci $\bar{S}_0$ in the plane $\alpha = 0^\circ$ and $\bar{S}_0$ in the curved surface with hatching lines, respectively. The subsequent yield surfaces after simple shear preloading are shown in Fig. 9(b)*, where the loci S_1 in the plane $\sigma_x - \sigma_y = 0$ and S_b in the plane $\sigma_y = 0$ are represented as the loci $\bar{S}_1$ in the plane $\alpha = 45^\circ$ and $\bar{S}_b$ in the curved surface with hatching lines, respectively.

We will now discuss the shape of the subsequent yield surface in the deviatoric stress space. The sub-

sequent yield surface after radial preloading is expected to be symmetric with respect to the plane constituted by the vertical axis of α and the preloading path since the initial yield surface takes a cylindrical shape. For example, the subsequent yield surfaces shown in

(a) For uniaxial tensile preloading

Fig. 8 Initial yield surface in the deviatoric stress space

* When the yield surface is divided into two parts by the plane $\sigma_x - \sigma_y = 0$, one part will lie in the space $\alpha > 0^\circ$ and the other in $\alpha < 0^\circ$. Only the yield surface in the space $\alpha > 0^\circ$ is illustrated in Fig. 9(b).

(b) For simple shear preloading

Fig. 9 Subsequent yield surface in the deviatoric stress space

Figs. 9(a) and (b) will be symmetric with respect to the planes $\bar{S}_R$ and $\bar{S}_T$, respectively. Although the yield surfaces for radial preloading in the planes where $\alpha=0^\circ$ and $\alpha=45^\circ$ are also symmetric with respect to the plane $\alpha=0^\circ$, this symmetric configuration is not observed in the subsequent yield surfaces in other cases of radial preloading. Because such yield surfaces have the shape of a surface whose upper or lower portion is cut by the plane where $\alpha=45^\circ$ or $\alpha=-45^\circ$ according to the α value of the preloading plane.

Now, let f_1 and f_2 in Fig. 10 be two subsequent yield loci after radial preloading in the same plane with respect to α . Then, the locus f_1 will be congruent with the locus f_2 irrespective of their preloading directions, if they have the same amount of equivalent prestrain. This congruency will also be able to be seen between the yield loci in different planes where

Fig. 10 Subsequent yield loci in the same preloading plane

Fig. 11 Subsequent yield loci in the different preloading plane

* In Fig. 9(b) the two surfaces divided by the plane $\sigma_x - \sigma_y = 0$ will be symmetrically situated with respect to the plane $\alpha=0^\circ$.

α is constant. In Fig. 11, the locus g_1 in the α_1 -plane will be congruent to the locus g_2 in the α_2 -plane, if they have the same magnitude of preloading.

As the value of α is limited to within $\pm 45^\circ$, the upper or lower part of the subsequent yield surface represented in the deviatoric stress space will be cut by the plane where $\alpha=+45^\circ$ or $\alpha=-45^\circ$, depending on the α -value of the preloading path. The congruency among the subsequent yield surfaces having different preloading paths with respect to α will hold good for the common parts of the yield surfaces obtained by translating vertically their preloading planes to the base plane. For example, the yield surface in Fig. 9(b) will be made to coincide with the lower part of the yield surface in Fig. 9(a) by translating the former vertically to the plane $\alpha=0^\circ$ and then rotating its preloading direction to $\zeta=0$.

4.2 Presentation of the experimental results

The initial yield points determined by radial loading in various directions of α are shown in Fig. 12(a)

(a) Initial yield surface

(b) Subsequent yield loci after uniaxial tensile preloading

Fig. 12 Representation of experimental results in the deviatoric stress space

by projecting the yield points in the plane $\alpha \neq 0^\circ$ into the plane $\alpha = 0^\circ$. The initial yield points in different α -planes are found to fall on a circle approximately. This shows that the initial yield surface takes a cylindrical shape in the deviatoric stress space (cf. Fig. 8).

The subsequent yield loci of Fig. 4(a) are replotted in the deviatoric stress space as shown in Fig. 12(b). The loci obtained seem to be symmetric with respect to the preloading direction $\zeta = 0$.

The congruency of the subsequent yield loci after radial preloading is examined in Fig. 13* by replottting the third subsequent yield loci of Figs. 4(a) to (d) in the deviatoric stress space.

Figure 14 shows the congruency of the three-dimensional yield surface by representing two subsequent yield surfaces of Figs. 5(a) and (b) in the deviatoric stress space. In Fig. 14 the preloading plane and direction of the yield surface of Fig. 5(b) are translated to the plane $\alpha = 0^\circ$ and then rotated

Fig. 13 Symmetry and congruency of the subsequent yield loci after radial preloading

Fig. 14 Symmetry and congruency of the subsequent yield surface after radial preloading

* ξ in Fig. 13 indicates the non-dimensional value of the subsequent yield stress divided by the value of the prestress.

to the direction $\zeta = 0$, respectively. The shape of the coupled surface obtained by the foregoing translation and rotation is shown in Fig. 14 by drawing the contour lines of $\alpha = 0^\circ, 20^\circ$ and 30° .

The effect of varying the yield point definition on the symmetry and congruency of the subsequent yield loci is investigated in Fig. 15 by using the results of Figs. 6(a) and (b). The results show that the symmetry and congruency of the yield loci will be independent of the definition of the yield point.

5. Conclusions

The subsequent yield surface after radial preloading is experimentally investigated by subjecting a thin-walled tubular specimen of brass to combined axial load, internal (or external) pressure and torsion. The subsequent yield loci dependent on the Bauschinger effect are obtained in all of the four quadrants of the principal stress plane. The shape of the three-dimensional subsequent yield surface is determined in the axial-circumferential-shear stress space in order to show the effect of the rotation of stress axes on the subsequent yield condition.

It has been shown that, for a given preloading, the correlation between the yield locus determined in the $\sigma_x - \sigma_y$ plane and that in the $\sigma_x - \tau_{xy}$ plane can be obtained by representing the yield locus in the deviatoric stress space.

About the shape of the subsequent yield surface, the following results have been obtained.

(1) The subsequent yield surface after radial preloading has a rounded nose of large curvature in the vicinity of the prestress point. A flattening of the surface is observed on the opposite side of the prestress point.

(2) The effect of varying the definition of the yield point on the shape of the yield surface is more

Fig. 15 Effect of the definition of the yield point on the symmetry and congruency of the yield loci

remarkable on the opposite side of the prestress point than in the vicinity of this point.

(3) In the deviatoric stress space, the subsequent yield surface has symmetry with respect to a plane containing the preloading path.

(4) The shapes of the subsequent yield loci after radial preloading determined by coaxial stress probes are mutually congruent in the deviatoric stress space irrespective of their preloading directions, so long as they have the same amount of equivalent prestrain.

(5) The three dimensional yield surface for simple shear preloading is congruent with the lower (or upper) part of the yield surface for uniaxial preloading with respect to the preloading plane.

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Discussion

K. SAITO (University of Osaka Prefecture):

(1) What definition of yield stress was used to determine the subsequent yield surface experimentally?

H. FUKUOKA (Osaka University):

(2) If the effective plastic strain can be defined such that the effective stress may be a function of plastic working by assuming a yield function including some suitable anisotropic parameters, the intensive experimental results of this paper reported will be more significant as the foundation to the theory of plasticity.

T. INOUE (Kyoto University):

(3) It seems that the radial loading of the specimen is conducted by the minute step loading. Was there any trouble in determining the effective proof strain of 0.02 per cent?

K. KAWASHIMA (Nagoya University):

(4) In determining a subsequent yield surface by using only one specimen, the shape of yield surface will be influenced by the sequence of determining the yield points.

A. SHINDO (Kobe University):

(5) Show the composition, the grain size and the formation of the material used.

(6) In determining the yield points under the combined axial load, internal (or external) pressure and torsion, was the value of α held constant?

(7) Explain the drawing-method of drawing the thin-lines indicating the yield surfaces for less experimental points.

(8) What difference is there between the prestress point and the subsequent yield point in the preloading direction?

(9) The absolute value of α is not essential

so long as the initial isotropy is assumed and if the yield surfaces are represented in the area of $\alpha > 45^\circ$ in Fig. 9(b), both subsequent yield surfaces shown in Figs. 9(a) and (b) will be mutually congruent.

(10) Is the normality of the plastic strain increment to the yield surface observed?

S. KUMAKURA (Yamagata University):

(11) Show the variations in the initial yield stress of the material used for the specimen if a yield surface is determined by using different specimens for each point on it.

(12) When the subsequent yield surface is represented in the deviatoric stress space, does it become a closed surface with respect to α axis as shown in Figs. 9(a) and (b)?

(13) It is natural that a congruence among the subsequent yield surfaces in the different planes with respect to α is observed with the isotropic material.

H. IGAKI (University of Osaka Prefecture):

(14) In the case of the brass specimen, the straight part of the stress-strain curve in unloading along the same path as the preloading is occasionally very short. Therefore the origin of reloading path to determine the subsequent yield points should be chosen at the preloading point rather than at the point O' as shown in Fig. 5(b).

K. MOTOIE (Hiroshima Electrical and Mechanical University):

(15) If the normality of plastic strain increment to the yield surface is assumed, it will be difficult to determine the strain increment at the corner in the preloading direction of the subsequent yield surface, as observed in the results of Figs. 4(c) and (d).

Authors' closure

(1) and (2) The yield point was determined by use of the effective plastic strain defined as follows,

$$\bar{\epsilon} = (2/\sqrt{3})(\epsilon_x^2 + \epsilon_x\epsilon_y + \epsilon_y^2 + \gamma_{xy}^2)^{1/2}$$

where ϵ_x , ϵ_y and γ_{xy} are respectively tensile, circumferential and shear plastic strains. The effective plastic strain including the strain history must be adopted, in strict sense, to determine the subsequent yield point. But, the authors do not know the theory of plasticity including the strain history. Hence the effective plastic strain above mentioned is used as the first approximation.

(3) The combined stress testing machine used in the experiment is hand-operated, so a strict radial loading is difficult. But, there is no trouble to obtain yield points, as the behavior of the specimen in deter-

mining yield points is nearly elastic.

(4) Append.-Fig. 1 shows two yield surfaces determined under the same prestraining by using two different sequences. A slight effect on the shape of subsequent yield surface of the sequence of determining the yield point is found in the reverse direction of preloading as shown in Append.-Fig. 1. To reduce this effect, the sequence of obtaining the yield point was started from the opposite side of the prestress point.

(5) Append.-Table 1 shows the compositions of the material. Append.-Figs. 2 to 5 are microphot-

Append.-Table 1 Chemical analysis of brass

Element	Cu	Zn	Pb	Fe	Sn
Per cent	58.60	38.23	2.45	0.23	0.49

Append.-Fig. 1 Effect of the sequence of determining the yield points on the subsequent yield surface (after 3.4% tensile prestraining)

Append.-Fig. 2 Microstructure of brass after heat-treatment (x240)

Append.-Fig. 3 Microstructure of brass after heat-treatment (x60)

Append.-Fig. 4 Microstructure of brass after heat-treatment (x240)

Append.-Fig. 5 Microstructure of brass after heat-treatment (x60)

graphs of the vertical section (section: R) and parallel section (section: L) to longitudinal direction of the specimen, respectively. No anisotropy was found in both sections from these microphotographs. The number of grain size according to ASTM was 8.04~8.34 in the section R, 7.97~8.40 in the section L.

(6) The value of α was held constant in determining the yield points in a principal stress plane such as shown in Figs. 4(a)~(d). But, the values of α in the reloading path of Figs. 5(a) and (b) were not always constant, as the reloadings in those cases were conducted by the stress path of straight line from the reference point within the yield surface.

(7) The thin-lines indicating yield surfaces are drawn on the assumption that the direction of plastic strain increment is normal to the yield surface.

(8) The prestress point coincides with the subsequent yield point in the preloading direction for a smaller prestrain, but for a larger prestrain a difference is found between these two values.

(9) If the subsequent yield surfaces after torsional preloading are represented in the range of $0^\circ \leq \alpha \leq 90^\circ$ (as shown in Append.-Fig. 6), they will be congruent to those after tensile preloading represented in the range of $|\alpha| \leq 45^\circ$. As it is unreasonable to change the range of α depending on preloading direction, all the subsequent yield surfaces are represented in the range of $|\alpha| \leq 45^\circ$.

(10) The normality of plastic strain increment to the yield surface is found as shown in Append.-Fig. 8.

(11) For example, the variation of the specimen used in the initial yield stress can be known from Fig. 12(a).

(12) If the subsequent yield surface represented in the stress space ($\sigma_x, \sigma_y, \tau_{xy}$) is not cut by the plane of $\sigma_x/\sigma_y=1$, this yield surface becomes a closed surface in the direction of α in the deviatoric stress space.

(13) The congruency among the subsequent yield surface in different planes with respect to α can not be investigated in the ordinary stress space ($\sigma_x, \sigma_y, \tau_{xy}$).

Append.-Fig. 7 Subsequent yield surface after uniaxial tensile preloading in the deviatoric stress space

Append.-Fig. 6 Subsequent yield surface after simple shear preloading in the deviatoric stress space

Append.-Fig. 8 Directions of the plastic strain increments on a yield locus (after 1.8% tensile pretraining)

Append.-Fig. 9 Bauschinger curve after 5.1% tensile prestraining

(14) As to the material used in this experiment, the stress-strain curve in unloading has a straight line part enough for determining the origin O' of reloading as shown in Append.-Fig. 9. If the prestress point is adopted as the origin to determine the yield points, it will be difficult to determine the yield points in the neighborhood of the prestress point. The prestress point is not necessarily in the subsequent yield surface. The stress value of the point O' in Fig. 5(b) is set at about 85 per cent of the prestress value.

(15) The more intensive investigations will be necessary about the existence of the corner in the subsequent yield surface. All the results in this paper were determined experimentally. Though the direction of the strain increment at the corner of the yield surface can not be established theoretically, it can be determined experimentally.