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Elastic Deformation of Polycrystalline Metals*

(3rd Report, Influence of Neighbor Grains)

By Takeji ABE**

Elastic deformation of polycrystalline metal was studied analytically. The stress and strain distribution in polycrystals, that is, in an elastic medium having discontinuity of elastic constant were calculated adopting the finite element method. The influence of Young's modulus of the neighbor grains on the deformation mode of a grain is studied with plane square model of grains.

It was found that the deformation mode of the grain changes from constant stress to constant strain with the values of Young's modulus of the neighbor grains. The deformation mode of those grains having very large or very small values of Young's modulus is similar to that of the whole polycrystals. Meanwhile, those grains having Young's modulus nearly equal to the mean value for polycrystals deform rather irregularly owing to the effect of the neighbor grains.

1. Introduction

The purpose of the present study is to analyse factors affecting the elastic deformation of polycrystalline metal by representing it with a simple geometrical model. In the previous report⁽¹⁾⁽²⁾, the influence of grain shape on the distributions of stress, strain and constraint ratio⁽³⁾ during elastic deformation was studied. The influence of the elastic constants of the neighbor grains on the deformation behavior of a grain, that is, on the distributions of stress, strain and constraint ratio is investigated in the present report. The analysis is made by finite element method adopting plane a model of polycrystals.

2. Analysis

The rectangular models of grains shown in Fig. 1(a)~(c) are used in the analysis. Three models, A, B and C are introduced to discuss the effects of neighbor grains from various aspects. Model A is used mainly to study the influence of neighbor grains in the direction of stress axis, while Model B is used to study the influence of adjacent grains perpendicular to the stress axis. Model C supplements the above two models. The values of elastic constants of the grains indicated with + and - signs are fixed in each model. Meanwhile, the values of elastic constants of the grains shown with $\oplus$ and $\ominus$ signs are varied to

Fig. 1 Models of polycrystalline metals

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** Assistant Professor, Faculty of Engineering, Kyoto University, Sakyo-ku, Kyoto.

polycrystals in the stress direction is denoted by ε_0 in the following ($\varepsilon_0 \equiv \varepsilon_x$). The parameter ψ is taken as 0.8 in the most part of the following calculations.

3. Results of calculation and discussion

3.1 Distributions of stress, strain and constraint ratio in grains

The distributions of strain and stress in the grains of Model A are shown in Figs. 3 and 4, respectively. The solid line shows the case of $\theta=0.5$ and the broken line the case of $\theta=-0.5$. The distribution of constraint ratio calculated by substituting the distribution of strain into Eq. (6) is shown with the marks $\circ$ and $\bullet$ in Fig. 5. For the sake of com-

parison, the result for $\theta=1^{(1)}$ is shown with fine solid lines in Fig. 3(a), 4(a) and 5(a) ($\theta=1$ corresponds to the case that only two kinds of grains having Young's moduli $E^{(1)}$ and $E^{(2)}$ are placed reciprocally).

It is seen from the results for $\theta=1$ and $\theta=0.5$ that the strain of the grain 1 having $1.8E_0$ increases and consequently η decreases with an increase in Young's modulus of the upper grain 3 from $0.2E_0$ to $0.6E_0$. On the other hand, the strain or the constraint ratio of grain 2 having $0.2E_0$ scarcely changes when Young's modulus of the upper grain 4 decreases from $1.8E_0$ ($\theta=1$) to $1.4E_0$ ($\theta=0.5$).

This may be interpreted as follows. In the case of $\theta=0.5$, the difference between Young's moduli of the upper and the lower grain is $1.2E_0$ for both the left side grains and for the right side ones (i.e., $\Delta E_L =$

Fig. 3 Distribution of strains ($\psi=0.8$)

Fig. 4 Distribution of stresses ($\psi=0.8$)

$\Delta E_R = 0.6E_0$). The mean value for the right side grains, however, is $\bar{E}_R = 0.8E_0$, and the ratio of the difference ΔE_R of Young's modulus from its mean value $\bar{E}_R$ becomes $\Delta E_R/\bar{E}_R = 0.75$, which is almost equal to 0.8 for the case of $\theta = 1.0$. Meanwhile for the left side grains, $\bar{E}_L = 1.2E_0$ and then $\Delta E_L/\bar{E}_L = 0.5$, which is much smaller than 0.8. Therefore, the distribution of strain or constraint ratio in the right side grains comes close to that of constant strain.

In the case of $\theta = -0.5$, the value of η is greater in the right side grains than in the left side ones as shown in Fig. 6(a). The difference between Young's moduli of the upper and the lower grain is $\Delta E_R = \Delta E_L = 0.4E_0$ in this case, and therefore $\Delta E_L/\bar{E}_L = 0.125$ and $\Delta E_R/\bar{E}_R = 0.5$, for $\bar{E}_L = 1.6E_0$ and $\bar{E}_R = 0.4E_0$, respectively.

As will be mentioned later, the mean strains $\bar{\epsilon}_I$ for the I layer and $\bar{\epsilon}_{II}$ for the II layer are slightly

different from the overall mean strain ϵ_0 for polycrystals [c.f. Fig. 19(a)]. In order to clarify the effect of this difference, the values of constraint ratio are calculated by substituting $\bar{\epsilon}_I$ and $\bar{\epsilon}_{II}$ into Eq. (6) instead of ϵ_0 , which is shown with the marks $\triangle$ and $\blacktriangle$ in Figs. 5 and 6. The distribution of constraint ratio thus obtained is almost symmetric with respect to the central grain boundary.

η becomes larger than 1 or smaller than 0 as is seen in Fig. 5(b) or 6(b), though it is usually expected that $0 \leq \eta \leq 1$ from the definition of $\eta^{(3)}$. The constraint ratios of the grain 1 and 2 in Figs. 5 and 6 having respectively the largest and the smallest value of Young's modulus are surely in the range of $0 \leq \eta \leq 1$. On the other hand, the variation of η is large in those grains having small values of $|\Delta E_\epsilon|$, for η defined by Eq. (6) includes the term $\Delta \epsilon/\Delta E_\epsilon$. It is expected, for instance, that the grain whose value of Young's modulus is equal to the overall mean E_0 for polycrystals ($\Delta E_\epsilon = 0$) deforms just like that of the polycrystals. The deformation of the grain, however, is affected by the neighbor grains having large values of $|\Delta E_\epsilon|$ and the value of η of the grain is different from the mean value for polycrystals as the result.

Let us examine the case of $\eta > 1$ in detail. Among

Fig. 5 Distribution of constraint ratios ($\psi=0.8$)

Fig. 6 Distribution of constraint ratios ($\psi=0.8$)

the left side grains in Fig. 5 the upper grain 3 which has $0.6E_0$ is largely strained because the lower grain 1 has a large value $1.8E_0$. Among the right side grains, the upper grain 4 which has $1.4E_0$ is hardly elongated, for the lower grain 2 has a small value $0.2E_0$. Therefore, it is concluded that $\eta > 1$ corresponds to the fact that a greater difference $\Delta\epsilon$ in strain than that expected from the difference ΔE_e in Young's modulus occurs, owing to the influence of neighbor grains.

Next, let us consider the case of $\eta < 0$ in Fig. 6. Among the left side grains, the upper grain 3 which has a large value $1.4E_0$ of Young's modulus is strained more than the mean strain ϵ_0 of polycrystals [the broken line in Fig. 3(b)], because the lower grain 1 has a greater value $1.8E_0$. So, the signs of ϵ and ΔE_e are the same and η is negative. Meanwhile, in the right side, the upper grain 4 which has a small value $0.6E_0$ of Young's modulus is strained less than the mean strain ϵ_0 , because the lower grain has a smaller value $0.2E_0$. η becomes negative here again. Consequently, it can be said that the negative value of η results from the fact that the signs of ΔE_e and $\Delta\epsilon$ are the same, owing to the influence of the neighbor grains, in contrast with the usual case where the signs are different. The deformation behavior of a grain in polycrystals is largely affected by the values of Young's modulus of the adjacent grains in the direction of stress axis.

The constraint ratio based on the mean values $\bar{E}_L$ and $\bar{E}_R$ of Young's modulus for the left side and the right side grains is calculated to make this point clear. First of all, it is necessary to study the relation between constraint ratios calculated from different mean values $\bar{\epsilon}$, $\bar{E}$ and $\bar{\epsilon}'$, $\bar{E}'$. The constraint ratio η' is represented in terms of $\bar{\epsilon}'$ and $\bar{E}'$ in the following, where the value of η of the same grain has been obtained in terms of $\bar{\epsilon}$ and $\bar{E}$. From the definition of constraint ratio, we have

$$\epsilon = \bar{\epsilon} + \Delta\epsilon = \bar{\epsilon}' + \Delta\epsilon' \quad \dots\dots\dots (7)$$

$$\eta = -\frac{\Delta\epsilon}{\bar{\epsilon}} \frac{\bar{E}}{\Delta E}, \quad \eta' = -\frac{\Delta\epsilon'}{\bar{\epsilon}'} \frac{\bar{E}'}{\Delta E'} \quad \dots\dots\dots (8)$$

From Eqs. (7) and (8),

$$\eta' = h + k\eta \quad \dots\dots\dots (9)$$

$$h \equiv \frac{-\bar{E}'}{\Delta E'} \left(\frac{\bar{\epsilon}}{\bar{\epsilon}'} - 1 \right), \quad k \equiv \frac{\bar{E}'}{\Delta E'} \frac{\Delta E}{\bar{E}} \frac{\bar{\epsilon}}{\bar{\epsilon}'}$$

When the mean strains $\bar{\epsilon}$ and $\bar{\epsilon}'$ are equal, Eq. (9) reduces to

$$h = 0, \quad k = (\bar{E}' \Delta E) / (\bar{E} \Delta E') \quad \dots\dots\dots (10)$$

Now, it is possible to calculate the constraint ratio based on $\bar{E}_L$ and $\bar{E}_R$. When $\psi = 0.8$ and $\theta = 0.5$ in Model A, $\bar{E}'$ and $\Delta E'$ in Eq. (10) for the left and the right side grains are given as follows.

$$\bar{E}_L = 1.2E_0, \quad \Delta E_L = 0.6E_0$$

$$\bar{E}_R = 0.8E_0, \quad \Delta E_R = 0.6E_0$$

If we denote the value of k of grain i by k_i ($i = 1 \sim 4$),

substituting $\bar{E} = \bar{E}_e$ and $\Delta E = \Delta E_e$ into Eq. (10), we have

$$k_1 = 1.6, \quad k_2 = 1.07, \quad k_3 = 0.8, \quad k_4 = 0.53 \quad \dots\dots\dots (11)$$

Similarly, for the case of $\theta = -0.5$ of Model A, we have

$$k_1 = 6.4, \quad k_2 = 1.6, \quad k_3 = -3.2, \quad k_4 = -0.8 \quad \dots\dots\dots (12)$$

The distribution of η' is obtained by multiplying the values of η shown in Figs. 5(b) and 6(b) with the values of k_i in Eqs. (11) and (12). The distributions along the lines $x/a = 1/20$ and $x/a = 19/20$ are shown in Fig. 7. It is seen from the figure that the distribution of η' for $\theta = 0.5$ is nearly equal to that for $\theta = -0.5$. It may be concluded from the above that (1) the value of constraint ratio η sometimes becomes larger than 1 or smaller than 0 when it is calculated from the overall mean Young's modulus E_0 and that (2) the distribution of constraint ratios or the deformation behavior of grains is similar irrespective of the value of θ especially inside the grains, so far as the constraint ratio is calculated based on the mean Young's modulus $\bar{E}_L$ or $\bar{E}_R$ of the grains adjacent in the direction of stress axis.

The distribution of constraint ratio for $\theta = 0.5$ of Model B is shown in Fig. 8. The distributions of constraint ratios, strains and stresses for $\theta = -0.5$ are shown in Figs. 9, 10 and 11, respectively. The value of η near the center of the left side grain in Fig. 9 is almost equal to that of the right side grain. The value of η near grain boundary, however, increases at one side of the grain boundary, though it usually decreases to 0 at the boundary⁽³⁾. This is interpreted as follows. When $\theta < 0$, the difference $\Delta E_R = \theta\psi$ in Young's modulus for the right side grains is smaller than the difference $\Delta E_L = \psi$ for the left side grains, and therefore $\Delta\epsilon_R < \Delta\epsilon_L$ (Fig. 10). The value of Young's modulus of the lower grain, however, is larger than that of the upper grain in both sides. So the strain $\Delta\epsilon$ or η in the right side grain increases near the grain boundary owing to the relatively large

Fig. 7 Distribution of modified constraint ratios calculated from $\bar{E}_L$ or $\bar{E}_R$ ($\psi = 0.8$)

Fig. 8 Distribution of constraint ratios

Fig. 9 Distribution of constraint ratios

Fig. 10 Distribution of strains ($\psi=0.8$) Fig. 11 Distribution of stresses ($\psi=0.8$)

strain in the left side grain. It is said from the above that, (1) the increase of η near the grain boundary is caused by the influence of the grain adjacent in the perpendicular direction to the stress axis, and that (2) this phenomenon is peculiar to the case where the values of Young's modulus of the upper and the lower grain are in the same order in the right and in the left side.

For comparison, the constraint ratio is calculated based on the mean Young's modulus $\bar{E}_I$ or $\bar{E}_{II}$ instead of E_0 and the mean strain $\bar{\epsilon}_I$ or $\bar{\epsilon}_{II}$ for each layer. The constraint ratio shown with the mark ∇ in the figure decreases to almost zero at the grain boundary, as is usually expected near the grain boundary parallel to the stress axis⁽³⁾. It is concluded from the above that the deformation behavior of the region near the grain boundary parallel to the stress axis is fairly affected by the relative value of Young's modulus in the both side grains adjacent in the perpendicular direction to the boundary or the mean value $\bar{E}_I$ or $\bar{E}_{II}$ of them. The results of calculation for $\phi=0.8$ and $\phi=0.4$ are shown respectively with the solid line and the broken line in Figs. 8 and 9. The influence of ϕ is comparatively small as is seen from the figure.

The distribution of stresses for $\theta=-0.5$ (Fig. 11) is nearly constant, especially in the interior of grains.

Figures 12 and 13 show the distributions of constraint ratios for $\theta=0.5$ and $\theta=-0.5$ of Model C, respectively. η greater than 1 or smaller than 0 in Figs. 12 and 13 results from the same cause as in the case of Model A mentioned above. It is also possible in Model C to calculate the modified constraint ratio η' based on $\bar{E}_L$ or $\bar{E}_R$. The value of

Fig. 12 Distribution of constraint ratios ($\phi=0.8$)

k_i in Eq. (10) is given for $\theta=0.5$ as follows.

$$k_1=6.4, \quad k_2=-0.8, \quad k_3=-3.2, \quad k_4=1.6 \quad \dots\dots\dots(13)$$

Similarly, for $\theta=-0.5$, we have

$$k_1=1.6, \quad k_2=0.53, \quad k_3=0.8, \quad k_4=1.07 \quad \dots\dots\dots(14)$$

The distribution of η' for Model C calculated with those values is shown in Fig. 14. The values of η' near the center of grain are similar for $\theta=0.5$ and $\theta=-0.5$, though they increase near the grain boundary in the left side grain.

It is obvious that $\Delta E_R > 0$, $\Delta E_L > 0$ and $\bar{E}_L \geq \bar{E}_R$ for the lower grains of Model C, so $\Delta E_L / \bar{E}_L < \Delta E_R / \bar{E}_R$. Therefore, η increases near the grain boundary in the left side grain, similar to the case of Model B shown in Fig. 9. In the case of $\theta=-0.5$ the increase of η near the grain boundary is small, because the difference between $\bar{E}_L$ and $\bar{E}_R$ is small.

3.2 Mean strain and mean constraint ratio of grain

The main interest concerning the influence of the neighbor grain in the deformation behavior of polycrystals lies in the problem of how much the deformation behavior of a grain is affected by its

Fig. 13 Distribution of constraint ratios ($\phi=0.8$)

Fig. 14 Distribution of modified constraint ratios calculated from $\bar{E}_L$ or $\bar{E}_R$ ($\phi=0.8$)

neighbor grains. The changes of the mean constraint ratio and the mean strain for each grain versus θ in Eq. (2) are calculated in the following to investigate this point.

Figure 15 shows the distribution of constraint ratios along $x/a=1/20$ of Model A for various values of θ . The value of η decreases gradually with a decrease of θ from 1.0 to -1.0 . Figures 16(a)~(c) show the relation between the mean value $\bar{\eta}_i$ of constraint ratio for each grain ($i=1\sim 4$) and θ . The values of $\bar{\eta}_1$ and $\bar{\eta}_2$ change monotonously with the value of θ in Fig. 16(a), while $\bar{\eta}_3$ and $\bar{\eta}_4$ do not, especially at $\theta=0$. This may be explained as follows. The grains 1 and 3 in Fig. 2 or the grains 2 and 4 are adjacent in the stress direction. Then, it holds that $\bar{\epsilon}_1 + \bar{\epsilon}_3 = 2\epsilon_0$ and $\bar{\epsilon}_2 + \bar{\epsilon}_4 = 2\epsilon_0$. Therefore, $\Delta\bar{\epsilon}_3 = -\Delta\bar{\epsilon}_1$ (where $\Delta\bar{\epsilon}_i = \bar{\epsilon}_i - \epsilon_0$) and the following relation is obtained from Eqs. (1), (2) and (6).

$$\bar{\eta}_3 = \bar{\eta}_1/\theta, \quad \bar{\eta}_4 = \bar{\eta}_2/\theta \quad (15)$$

It is obvious from Eq. (15) that the values of $\bar{\eta}_3$ and $\bar{\eta}_4$ become large when θ comes close to zero [It is noted, however, that Eq. (15) is satisfied only for Model A where the polycrystals is composed of four kinds of grains having same grain size.].

In the case of Model B shown in Fig. 16(b), the variation of $\bar{\eta}_i$ is small as compared with those shown in Figs. 16(a) and (c). The slight deviation of the value shown with the mark $\bullet$ in the figure corresponds to the increase of η in the grains 2 and 4 near the grain boundary parallel to the stress axis of Model B shown in Figs. 8 and 9. Similarly, the following relation is derived for Model B,

$$\bar{\eta}_3 = \bar{\eta}_1, \quad \bar{\eta}_4 = \bar{\eta}_2 \quad (16)$$

In the same way for Model C, we have,

$$\bar{\eta}_3 = -\bar{\eta}_1/\theta, \quad \bar{\eta}_4 = -\bar{\eta}_2/\theta \quad (17)$$

It is seen from Figs. 16(a)~(c) that $\bar{\eta}_i$ changes monotonously in those grains having the largest and the smallest values of Young's modulus E_i . If the mean value of constraint ratio η for those two grains having the largest and the smallest value of E_i is

denoted by $\bar{\eta}^*$, it is represented by the following relation for each model.

$$\left. \begin{aligned} \text{A: } \bar{\eta}^* &= (\eta_1 + \eta_2)/2 \\ \text{B: } \bar{\eta}^* &= (\eta_1 + \eta_3)/2 \\ \text{C: } \bar{\eta}^* &= (\eta_1 + \eta_4)/2 \end{aligned} \right\} \dots\dots\dots (18)$$

The change of $\bar{\eta}^*$ with θ is shown in Fig. 17.

When $\theta = -1.0$, Model A represents the laminated composite parallel to the stress axis as is seen from Fig. 2, while Model B dose the laminated composite perpendicular to the stress axis. These laminates are expected to deform under the constant strain ($\eta=0$) and the constant stress condition ($\eta=1$), respectively. The value of $\bar{\eta}^*$ shown in Fig. 17(a) changes almost linearly from 1 to 0 with θ for Model B and Model A in Fig. 17(a) and for Model C shown in Fig. 17(b). As mentioned before, Model A represents mainly the influence of the neighbor grain adjacent in the direction of stress axis, while Model B represents mainly the influence of the neighbor grain perpendicular to the stress axis. It may be concluded that the influence of Young's modulus of the neighbor grain

Fig. 15 Distribution of constraint ratios ($\psi=0.8$)

Fig. 16 Relation between the averaged constraint ratio $\bar{\eta}_i$ for each grain and θ

adjacent in the direction of stress axis is relatively large for the plane square grain model, because the value of $\bar{\eta}^*$ at the cross point of Model A and Model B is $\bar{\eta}^*=0.756$ and the range of variations of $\bar{\eta}^*$ for Model A is greater than that for Model B.

The value of $\bar{\eta}^*$ at $\theta=-1.0$ of Model B or C shown in Fig. 17 is not equal to 1.0, which results from the boundary condition of the present analysis that the displacement in the y direction is restricted with Poisson's ratio ν_0 . The value of $\bar{\eta}^*$ at $\theta=-1.0$ is 0.91 for the calculation under the plane stress as shown with the mark $\odot$ in Fig. 17(a), while for the plane strain it is 0.87. The difference between the results for the plane stress and for the plane strain in the two-dimensional analysis is thought to result, from the secondary effect of the restriction of the strain in z -direction i.e., the direction perpendicular to the plane surface. Therefore, the difference is largest at $\theta=-1.0$ of Model B or Model C, where the deformation is near to that of the constant stress condition. The difference is smaller for the other cases.

Next, the change of the mean strain $\bar{\epsilon}_i$ for each grain versus θ is calculated and the result is shown in Fig. 18. In the case of Model B shown in Fig. 18(b), the mean strain $\bar{\epsilon}_i$ changes almost linearly with θ and is almost inversely proportional to Young's modulus E_i . At $\theta=0$ (where $E_2=E_4=E_0$), however, the mean strains $\bar{\epsilon}_2$ and $\bar{\epsilon}_4$ deviate from ϵ_0 by δ' and δ , respectively, owing to the influence of neighbor grains.

The change for Model C shown in Fig. 18(c) resembles that for Model A shown in Fig. 18(a). In this case, the mean strains $\bar{\epsilon}_1$ and $\bar{\epsilon}_4$ are greater than $\bar{\epsilon}_2$ and $\bar{\epsilon}_3$, respectively. ($\bar{\epsilon}_1+\bar{\epsilon}_3=2\epsilon_0$ and $\bar{\epsilon}_2+$

$\bar{\epsilon}_4=2\epsilon_0$ for all models).

Figures 19(a)~(c) show the change in the mean strains $\bar{\epsilon}_I$ and $\bar{\epsilon}_{II}$ for respective layers. The mean strain of I layer of Model A is greater than that of II layer as shown in Fig. 19(a), though the mean Young's moduli are equal, i.e., $\bar{E}_I=\bar{E}_{II}=E_0$ as shown in Eq. (5). The difference between $\bar{\epsilon}_I$ and $\bar{\epsilon}_{II}$ is large for the range $\theta=0\sim-0.25$, and increases with an increase in ψ , for $\bar{\epsilon}_I$ and $\bar{\epsilon}_2$ in Fig. 18(a) are greater than $\bar{\epsilon}_4$ and $\bar{\epsilon}_3$, respectively. Although the difference in Young's moduli of the upper and the lower grain is equal in the left and the right side grains of Model A, the influence of the difference of Young's modulus is larger in the right side grains, for the relative difference is greater at the right, i.e., $\Delta E_R/\bar{E}_R > \Delta E_L/\bar{E}_L$ as mentioned before (Fig. 3). Besides, Young's modulus of the lower right grain 2 in I layer is smaller than that of the upper grain 4. Therefore, the mean strain of I layer becomes large as compared with that of II layer.

For Model A, we have

$$\frac{\Delta E_L}{\bar{E}_L} = \frac{\psi + \theta\psi}{2 + \psi - \theta\psi}, \quad \frac{\Delta E_R}{\bar{E}_R} = \frac{\psi + \theta\psi}{2 - \psi + \theta\psi} \quad \dots(19)$$

The difference ξ between $\Delta E_R/\bar{E}_R$ and $\Delta E_L/\bar{E}_L$ is given by

$$\xi = \frac{\Delta E_R}{\bar{E}_R} - \frac{\Delta E_L}{\bar{E}_L} = \frac{2(\psi^2 - \theta^2\psi^2)}{4 - (\psi - \theta\psi)^2} \quad \dots\dots\dots(20)$$

For the case of $\theta=0$,

Fig. 17 Relation between $\bar{\eta}^*$ and θ

Fig. 18 Relation between the averaged strain $\bar{\epsilon}_i$ for each grain and θ ($\psi=0.8$)

Fig. 19 Relation between the averaged strain for each layer and θ

$\xi = 2\psi^2/(4-\psi^2) \dots \dots \dots (21)$
 $\xi = 0.38$ for $\psi = 0.8$ and $\xi = 0.08$ for $\psi = 0.4$. It is concluded that ξ increases with an increase of ψ and the difference between $\bar{\epsilon}_I$ and $\bar{\epsilon}_{II}$ becomes large as shown in Fig. 19.

The values of $\bar{\epsilon}_I$ and $\bar{\epsilon}_{II}$ for $\theta = -1.0$ of Model B shown respectively with the marks $\odot$ and $\ominus$ in Fig. 19(b) differ from those values for the deformation under constant stress shown with the marks $\circ$ and $\bullet$. In the case of constant stress, $\bar{\epsilon}_I = 0.2$ and $\bar{\epsilon}_{II} = 0.8$ for $\psi = 0.8$. Meanwhile, the calculated results for $\psi = 0.8$ are $\bar{\epsilon}_I = 0.27$ and $\bar{\epsilon}_{II} = 1.73$ for plane stress and $\bar{\epsilon}_I = 0.30$ and $\bar{\epsilon}_{II} = 1.70$ for plane strain.

In the cases of Model B and Model C shown in Figs. 19(b) and (c) respectively, the mean strain for the layer is approximately inversely proportional to the mean value of Young's modulus $\bar{E}_I$ or $\bar{E}_{II}$ for each layer, namely, $\bar{E}_I \bar{\epsilon}_I \cong \bar{E}_{II} \bar{\epsilon}_{II}$. In the case of Model A shown in Fig. 19(a), however, there are some differences between $\bar{\epsilon}_I$ and $\bar{\epsilon}_{II}$, though E_I is equal to E_{II} . These facts show that the deformation behavior of each layer of the square grain model is

nearly equal to that of the constant stress, though in some cases the deformation of each layer deviates from that under the constant stress.

It is possible to show that the mean value of constraint ratio is related to the mean Young's modulus for polycrystals. This, however, will be discussed more in detail separately.

4. Conclusions

The influence of the values of Young's modulus of the neighbor grains on the elastic deformation behavior of a grain in polycrystals is studied quantitatively. The results are summarized as follows.

(1) If the values of Young's modulus of the neighbor grains adjacent in the direction of stress axis are nearly equal, the deformation behavior is close to that of the constant strain. Meanwhile, if the values of Young's modulus of the neighbor grain adjacent perpendicular to the stress axis are nearly equal, the deformation behavior of the grains comes close to that under the constant stress.

(2) The value of constraint ratio of a grain calculated from the overall mean Young's modulus E_0 for polycrystals is dependent on the values of Young's modulus of the neighbor grains. Especially, when the value of Young's modulus of the grain is nearly equal to that of the whole polycrystals, the value of constraint ratio of the grain is strongly affected by the neighbor grains.

(3) On the other hand, the values of constraint ratio in those grains having the largest or the smallest values of Young's modulus are nearly equal to the mean value of constraint ratio for polycrystals.

(4) When constraint ratio is calculated based on the mean value of Young's modulus for the adjacent grains in the direction of stress axis instead of the overall mean Young's modulus for polycrystals, the values of the constraint ratio in the adjacent grains are nearly equal, with the exception of the region near the grain boundary parallel to the stress axis.

(5) The deformation behavior near the grain boundary parallel to the stress axis is affected by the relative values of Young's modulus of the two neighbor grains lying on both side of the grain boundary.

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