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The Stability of Flexibly Supported, Externally Pressurized Gas Journal Bearings*

(Case of a Rigid Rotor)

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The stability of a self excited whirl is an important problem of externally pressurized gas bearings used in high speed turbomachines. Bearings are often flexibly supported in order to suppress this whirl.

In this study, the stability characteristics are shown of a system connected with a symmetrical rigid rotor.

Also, analytical results are presented and compared with an experiment on bearings flexibly supported by 'O' rings.

Main conclusions are:

(1) Whirl on-set speed can be made very high by adequately selecting stiffness, damping and other parameters in relation to rotor and bearings.

(2) Comparison shows theory and experimental results in near-coincidence, and effectiveness of flexible support is exhibited remarkably in contrast with cases of rigidly supported bearings.

1. Introduction

Whirl instability is one of the problems to be solved when applying gas bearings to high speed rotors.

Some papers [1-4] have already indicated that, in case of rigidly supported externally pressurized gas bearings, half-frequency-whirl occurs at a limited speed, and especially, in case of a vertical rotor, it occurs at a speed equal to about twice its lowest critical speed, in spite of bearing supply pressure.

In order to avoid this instability, various means have been applied to gas journal bearings, and the most typical and conventional method is to use bearing bushes flexibly supported by elastic bodies like rubber 'O' rings. This method is simple and practical; its effectiveness was experimentally investigated by Powell [5].

However, these flexibly supported bearings have been contrived based on nothing but experience.

In this study, in order to give a quantitative evaluation of the effectiveness of the flexible support which would affect whirl instability of gas journal bearings, analysis is made of a system connected with a symmetrical rigid rotor, and it is compared

with result from an experiment.

2. Nomenclature

- F_x, F_y : gas-film forces of x, y , respectively kg
 $K_1 \sim K_4 (K_{10} \sim K_{40})$: stiffness and damping coefficients of bearing [non-dimensional]
 L : bearing length cm
 $P_s = p_s/p_a$
 p_s : supply pressure of bearing kg/cm² (abs)
 p_a : ambient pressure kg/cm² (abs)
 R : bearing radius cm
 b_p : damping coefficient of elastic body supporting bearing kg sec/cm
 c : radial clearance of bearing cm
 k_s : hydrostatic stiffness of bearing kg/cm
 k_p : stiffness of elastic body supporting bearing kg/cm
 m_1 : rotor mass sec²/cm
 m_2 : bearing bush mass kg sec²/cm
 s : Laplace's operator
 Γ : restricter coefficient = $\{6\mu n\eta^{1/2}d(\Re/T)^{1/2}/p_a c^2\}(L/D)$

where

- μ : viscosity kg sec/cm²
 η : adiabatic efficiency
 d : feed hole diameter cm
 $\Re T$: the product of gas constant and absolute temperature cm²/sec²

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- n : number of feed holes
 $D = 2R$
 A : compressibility number $= 6\mu\omega R^2/p_a c^2$
 $A_s = A/Q$
 δ : whirl frequency $= \omega_w/\omega_s$ [non-dimensional]
 λ : rotating speed of displaced shaft around the center of bearing bush $\omega_s \lambda = \partial\theta/\partial t$ [non-dimensional]
 θ : attitude angle [rad]
 $Q = \omega/\omega_s$
 $Q_L = \omega_{ON}/\omega_s$
 ω : rotating speed of rotor rad/sec
 ω_s : critical speed calculated with k_s rad/sec
 ω_{ON} : whirl on-set speed rad/sec
 ω_w : whirl frequency rad/sec
 ω_f : frequency rad/sec
 $\tilde{m} = 2m_2/m_1$

3. Theoretical analysis

Let us consider an externally pressurized journal bearing supported by an elastic body such as rubber 'O' ring, as shown in Fig. 1. In the co-ordinate of Fig. 2, film-forces of bearing, F_x and F_y are as follows [2, 4]:

$$\left. \begin{aligned} F_x &= -\pi L R p_a [K_1 x_1 + K_2 y_1 + K_3 \dot{x}_1 - K_4 \dot{y}_1] \\ F_y &= -\pi L R p_a [-K_2 x_1 + K_1 y_1 + K_4 \dot{x}_1 + K_3 \dot{y}_1] \end{aligned} \right\} \dots\dots\dots (1)$$

in which $J(x_j, y_j)$ is journal center and $B(x_2, y_2)$ is bearing bush center, and

$$x_1 = x_j - x_2, \quad y_1 = y_j - y_2 \dots\dots\dots (2)$$

When λ is rotational speed of displaced shaft (BJ) around bearing bush center B, $K_1 \sim K_4$ can be expressed as

$$\left. \begin{aligned} K_1 &= K_{10} + K_{40} A_s^2 Q^2 (1-2\lambda) \lambda \\ K_2 &= K_{20} A_s Q (1-2\lambda) + K_{30} A_s Q \lambda \\ K_3 &= K_{30} A_s Q \\ K_4 &= K_{40} A_s^2 Q^2 (1-2\lambda) \end{aligned} \right\} \dots\dots\dots (3)$$

where $K_{10} \sim K_{40}$ can be obtained by solving Reynolds equation, and when A is small, $K_{10} \sim K_{40}$ can be approximated by the value as $A=0$.

In Eq. (3),

$$Q = \frac{\omega}{\omega_s}, \quad A_s = \frac{A}{Q} = \frac{6\mu\omega_s R^2}{p_a c^2} \dots\dots\dots (4)$$

in which ω is rotational speed of the rotor and ω_s is the lowest critical speed of the system calculated with hydrostatic stiffness k_s .

Now, let us consider a symmetrical rigid rotor flexibly supported by two bearings. A set of equations of motion of the rotor-bearing system is as follows:

$$\left. \begin{aligned} m_1 c \omega^2 \ddot{x}_j &= 2F_x \\ m_1 c \omega^2 \ddot{y}_j &= 2F_y - m_1 g \\ m_2 c \omega^2 \ddot{x}_2 + k_p c x_2 + b_p c \omega \dot{x}_2 &= -F_x \\ m_2 c \omega^2 \ddot{y}_2 + k_p c y_2 + b_p c \omega \dot{y}_2 &= -F_y - m_2 g \end{aligned} \right\} \dots\dots\dots (5)$$

where k_p and b_p are stiffness and damping coefficients of the flexible support, respectively, m_1 is the mass of rotor and m_2 is the mass of bearing bush.

Substituting

$$\left. \begin{aligned} K_s &= \frac{k_p c}{\pi L R p_a} = \frac{m_1 c \omega_s^2}{2\pi L R p_a}, \quad \tilde{m} = \frac{2m_2}{m_1} \\ K_p &= \frac{k_p c}{\pi L R p_a}, \quad B_p = \frac{b_p c \omega_s}{\pi L R p_a} \end{aligned} \right\} \dots\dots\dots (6)$$

into Eq. (5) gives

$$\left. \begin{aligned} K_s Q^2 (\ddot{x}_1 + \ddot{x}_2) + K_1 x_1 + K_2 y_1 + K_3 \dot{x}_1 - K_4 \dot{y}_1 &= 0 \\ K_s Q^2 (\ddot{y}_1 + \ddot{y}_2) - K_2 x_1 + K_1 y_1 + K_4 \dot{x}_1 + K_3 \dot{y}_1 &= 0 \\ K_s Q^2 [\ddot{x}_1 + (1+\tilde{m})\ddot{x}_2] + K_p x_2 + B_p Q \dot{x}_2 &= 0 \\ K_s Q^2 [\ddot{y}_1 + (1+\tilde{m})\ddot{y}_2] + K_p y_2 + B_p Q \dot{y}_2 &= 0 \end{aligned} \right\} \dots\dots\dots (7)$$

where constants are omitted.

Fig. 1 Bearing flexibly supported

Fig. 2 Co-ordinate of bearing system

Using the value as $A=0$ for $K_{10} \sim K_{40}$, by supposing that A is small, substitution of

$$\left. \begin{aligned} \frac{K_{10}}{K_1} A_s &= c_1, & \frac{K_{20}}{K_2} A_s &= c_2, & \frac{K_{30}}{K_3} A_s &= c_3 \\ \frac{K_{30}}{K_1} A_s^2 &= c_4, & \frac{K_2}{K_1} = \frac{k_p}{k_s} &= c_5, & \frac{B_2}{K_1} = \frac{b_p \omega_s}{k_s} &= c_6 \end{aligned} \right\} \quad (8)$$

and Eq. (3) into Eq. (7) gives the characteristic equation:

$$D = [(1+\bar{m})Q^2 s^2 + c_6 Q s + c_5] \{ Q^2 s^2 + c_5 Q s + c_1 + c_4 Q^2 (1-2\lambda) - Q^4 s^4 \} + Q^2 [c_4 (1-2\lambda) Q s - c_2 (1-2\lambda) - c_3 \lambda]^2 [(1+\bar{m})Q^2 s^2 + c_6 Q s + c_5]^2 = 0 \quad (9)$$

In Eq. (8), when $A=0$, $c_1=1$, but when $A \neq 0$, $c_1 \neq 1$.

Now, because the real part of s is zero in stability threshold, we can write

$$Q_L s = \delta i \quad (10)$$

where $i = \sqrt{-1}$.

Substituting Eq. (10) into Eq. (9), characteristic equation becomes as follows:

$$D = [c_5 - (1+\bar{m})\delta^2] \{ c_1 + c_4 Q^2 (1-2\lambda) - \delta^2 \} - c_3 c_6 \delta^2 - \delta^4 + c_6 [c_1 + c_4 Q^2 (1-2\lambda) - \delta^2] \delta i + c_3 [c_5 - (1+\bar{m})\delta^2] \delta i^2 + Q_L^2 [c_4 (1-2\lambda) + c_3 \lambda] \{ c_5 - (1+\bar{m})\delta^2 \} + c_4 c_6 (1-2\lambda) \delta^2 + c_6 [c_2 (1-2\lambda) + c_3 \lambda] \delta i + c_4 (1-2\lambda) \{ c_5 - (1+\bar{m})\delta^2 \} \delta i^2 = 0 \quad (11)$$

As Eq. (11) is transformed into

$$D = (D_1 + i D_2)(D_1 - i D_2) = 0$$

next relations are established:

$$\left. \begin{aligned} \text{Real}[D_1 + i D_2] &= \alpha_2 \mp \alpha_1 Q_L = 0 \\ \text{Imag.}[D_1 + i D_2] &= \alpha_4 \pm \alpha_3 Q_L = 0 \end{aligned} \right\} \quad (12)$$

in which $\alpha_1 \sim \alpha_4$ are function of only δ .

Then if $n=0$ in case of horizontal rotor (therefore $\lambda=0$) and $n=\pm 1$ in case of vertical rotor (therefore $Q_L=\delta$),

$$\left. \begin{aligned} \alpha_1 &= [c_2 c_6 - (n+1)c_4 c_5] \delta + (n+1)(1+\bar{m})c_4 \delta^3 \\ \alpha_2 &= c_1 c_5 - [c_5 + (1+\bar{m})c_1 + (n+1)c_3 c_6 - 2n(c_2 c_6 - (n+1)c_4 c_5)] \delta^2 + [\bar{m} + 2n(n+1)(1+\bar{m})c_4] \delta^4 \\ \alpha_3 &= c_2 c_5 - [(1+\bar{m})c_2 - (n+1)c_4 c_6] \delta^2 \\ \alpha_4 &= [c_1 c_6 + (n+1)c_3 c_5 - 2nc_2 c_5] \delta - [c_6 + (n+1)(1+\bar{m})c_3 - 2n((1+\bar{m})c_2 - (n+1)c_4 c_6)] \delta^3 \end{aligned} \right\} \quad (13)$$

Eliminating Q_L from Eq. (12), a cubic expression for δ^2 :

$$A(n) = \alpha_1 \alpha_4 + \alpha_2 \alpha_3 = a_0(n) + a_1(n) \delta^2 + a_2(n) \delta^4 + a_3(n) \delta^6 = 0 \quad (14)$$

is obtained, in which

$$\left. \begin{aligned} a_0(n) &= c_1 c_2 c_5^2 \\ a_1(n) &= c_2 [c_1 c_6^2 - c_5^2 - 2(1+\bar{m})c_1 c_5] - (n+1)^2 c_3 c_4 c_5^2 \\ a_2(n) &= c_2 [(1+\bar{m})^2 c_1 + (1+2\bar{m})c_5 - c_6^2] - (n+1)^2 c_3 c_4 [2(1+\bar{m})c_5 - c_6^2] \\ a_3(n) &= -\bar{m}(1+\bar{m})c_2 - (n+1)c_4 c_6 - (n+1)^2 (1+\bar{m})^2 c_3 c_4 \end{aligned} \right\} \quad (15)$$

Solving Eq. (14), whirl on-set speed ratio is given from Eq. (12),

$$Q_L = |\alpha_2/\alpha_1| = |\alpha_4/\alpha_3| \quad (16)$$

when $n=0$ in horizontal rotor,

$$Q_L = n\alpha_2/\alpha_1 = -n\alpha_4/\alpha_3 \quad (17)$$

when $n=\pm 1$ in vertical rotor. Solution $Q_L < 0$ is trivial.

And then, whirl on-set speed and whirl frequency are respectively

$$\omega_{ON} = Q_L \omega_s, \quad \omega_W = \delta \omega_s \quad (18)$$

and whirl ratio is

$$\bar{R} = \omega_{ON}/\omega_W = Q_L/\delta \quad (19)$$

4. Calculation results

Assumptions for calculation are as follows:

(1) Externally pressurized bearing is of $L/D=1$ and has a single plane admission. It is designed under the optimum condition for maximum stiffness.

(2) Effectiveness of flexible support is expressed by using next parameters: $\bar{m}$, A_s , k_p/k_s and ζ_p , in which ζ_p is damping ratio which is rewritten as

$$\left. \begin{aligned} c_6 &= b_p \omega_s/k_s = 2(1+\bar{m})\zeta_p \omega_p/\omega_s \\ &= 2(1+\bar{m})^{1/2} c_5^{1/2} \zeta_p \\ \omega_p &= \sqrt{\frac{2k_p}{m_1 + 2m_2}}, \quad \zeta_p = \frac{b_p}{(m_1 + 2m_2)\omega_p} \end{aligned} \right\} \quad (20)$$

Parameters are set as follows:

$$P_s = 2 \sim 7, \quad \bar{m} = 0.05 \sim 1.0, \quad A_s = 0.05 \sim 2.0$$

$$k_p/k_s = 0.3 \sim 5.0, \quad \zeta_p = 0.01 \sim 0.5$$

Figure 3 is a representative chart of stability threshold. The roots of Eq. (14) vary in accordance with k_p/k_s . Two curves of Fig. 3 are made by plotting their roots, in which the stable regions are both lower-side of the real line and left-side of the broken line. The stable region is hatched by oblique line in Fig. 3. In the later figures, only the boundary curves will be shown.

Fig. 3 Stability region

Figures 4~7 show some results with a vertical rotor.

In Fig. 4, the smaller the hydrodynamic effect Λ_s is, the higher the whirl on-set speed ratio Q_L becomes.

Figure 5 for mass ratio $\bar{m}$ has a special feature: the left-side of curve which corresponds to the real line in Fig. 3 is influenced by m , while the right-side is hardly influenced.

The effect of damping ratio is shown in Fig. 6 and ζ_p had better be large for stable region of whirl to spread within the limit of this parameter range.

This rotor operates vertically, therefore, as k_p/k_s approaches infinity, Q_L approaches 2, that is the

whirl onset speed ratio of rigidly supported bearings.

In each figure, curves corresponding to the real line in Fig. 3 are hardly affected by k_p/k_s , but curves corresponding to broken line in Fig. 3 are changed remarkably by k_p/k_s . After all, it is good design to select rather small k_p/k_s , and if k_p/k_s at peak point between two curves can be obtained, flexible support produces the greatest effectiveness.

Parameter of Fig. 7 is supply pressure ratio $P_s = p_s/p_a$; also results of a horizontal rotor are compared with those of a vertical one. It is shown that the larger P_s is, the more clearly k_p/k_s changes Q_L , and k_p/k_s that gives maximum Q_L is hardly affected by P_s . Therefore, unless k_p/k_s is chosen smaller than optimum, it is not expected that Q_L increases with an increasing P_s .

5. Comparison of theoretical results with experimental ones

Approximations in previous chapters as follows:

- (1) $K_{10} \sim K_{40}$ are the values of $\Lambda = 0$.
- (2) Other parameters remain constant.

However, these approximations deviate from a real system. Especially, stiffness and damping ratio may change with frequency.

Fig. 4 Stability threshold (Parameter: Λ_s)

Fig. 5 Stability threshold (Parameter: $\bar{m}$)

Fig. 6 Stability threshold (Parameter: ζ_p)

Fig. 7 Stability threshold (Parameter: P_s)

Fig. 8 Dynamic properties of 'O' ring

Change in $K_{10} \sim K_{40}$ with rotor speed (i.e. hydrodynamic effect of bearing) has been stated in some previous papers, but the dynamic properties of an elastic body, especially of a rubber 'O' ring are not made clear in any literature.

In this study, in order to make clear the dynamic properties of 'O' rings, vibration tests under conditions identical to the experimental system were carried out. (See appendix)

An example of the test results is shown in Fig. 8 for 'O' ring JIS-B2401-P26 and it indicates a wide variation of stiffness and damping ratio with frequency. This means that coefficients of Eq. (14) for δ are also functions of frequency, i.e. δ . Therefore, in Eq. (14), roots δ must agree with δ giving the coefficients.

With this taken into consideration, calculation to decide the whirl on-set speed must be carried out as follows:

(a) Hydrodynamic effect of bearing is included in the next steps.

Whirl on-set speed ω_{ON} is calculated by using $K_{10} \sim K_{40}$ for rotating speed ω as shown in Fig. 9. ω must agree with ω_{ON} at whirl on-set point. Thus, curve of ω_{ON} is plotted as shown by a real line in Fig. 9, and the straight line of $\omega = \omega_{ON}$ is superposed as done by a broken line. The point of intersection of two curves represents the real whirl on-set speed ω_{ON}^R , where k_p and ζ_p remain constant.

Fig. 9 Consideration of hydrodynamic effect

(b) Moreover, dynamic properties of 'O' ring are taken into consideration as shown in Fig. 10. When ω , i.e. $K_{10} \sim K_{40}$ is fixed, whirl frequency ω_W is calculated by using k_p and ζ_p for ω_f as shown in Fig. 10(a). Intersection ω_f^R of curve of ω_W and that of $\omega_f = \omega_W$ is the point where ω_W or δ as root of Eq. (14) is equal to ω_W or δ , giving coefficients of Eq. (14). As, at the same time, ω_{ON} is calculated, this ω_{ON} is plotted in Fig. 10(b). Finally, intersection of ω_{ON} and $\omega = \omega_{ON}$ gives the real whirl on-set speed ω_{ON}^R as in Fig. 9.

Fig. 11 Air Turbine with flexibly supported gas journal bearing

Fig. 10 Consideration of both hydrodynamic effect and dynamic properties of 'O' ring

Test apparatus consists of an air turbine with a vertical rotor shown in Fig. 11. The rotor has 20 mm diameter and radial inflow turbine blades at one end.

Externally pressurized gas journal bearings are flexibly supported by rubber 'O' rings. Data on bearing bush are as follows:

Materials	BsBM(brass)
$\bar{m}$	0.373 6 (rotor is made of mild steel)
c	0.005 cm
n	16 (double plane admission)
d	0.05 cm (inherently compensated)
L/D	2

This 'O' ring is standardized according to JIS-B2401-P26.

During experiment, rotor vibration was measured by capacitance probes and observed on a synchroscope. Also, a test of rigid support was carried out by using metal bushes inserted between bearing bush and pedestal, instead of 'O' ring. Some tests were repeated at several supply pressure.

Effectiveness of flexible support is compared with rigid support in both theoretical results and experimental ones shown in Fig. 12.

Dynamic properties of 'O' ring in Fig. 8 were used in analysis and whirl on-set speed of rigid support is 2ω , because of the vertical rotor.

Experimental results in Fig. 12 contain what could not be recognized as whirl (marked $\uparrow$ in Fig. 12), for maximum running speed is smaller than whirl on-set speed.

Figure 12 leads to the conclusions that theoretical and experimental results overlap closely and that both almost agree quantitatively; that, as compared with rigid support, effectiveness of flexible support which would affect the whirl is remarkable, and that, in the case of 'O' ring used, the whirl on-set speed increases as the supply pressure increases.

Fig. 12 Whirl on-set speed

6. Conclusions

The following conclusions can be drawn:

(1) Through optimum selection of stiffness and damping ratio of an elastic body supporting bearing and other parameters, a very high whirl on-set speed can be obtained.

(2) The use of 'O' ring is a simple and very effective method of whirl stabilization of gas bearings, but it is noted that its dynamic properties vary with frequency.

(3) Analysis of this study can provide a considerably exact evaluation of whirl on-set speed, by taking into consideration variation of dynamic properties of elastic body and characteristics of bearings.

[Appendix] Dynamic properties of 'O' ring

Stiffness and damping ratio of 'O' ring can be obtained by comparison of experimental results using a vibration test model in Fig. 13(a), that gives the same condition as test turbine bearing with that of analytical treatment of the equivalent system in Fig. 13(b).

Equation of motion of the system in Fig. 13(b) is $m\ddot{x}_m + k_p(x_m - x_0) + b_p(\dot{x}_m - \dot{x}_0) = 0$ (21)

and the displacement of mass m is

$$x_m = X_m e^{i(\omega t - \varphi)} = \bar{X}_m e^{i\omega t}, \quad x_0 = X_0 e^{i\omega t} \text{(22)}$$

in which φ is phase angle.

Substituting

$$\bar{X}_m = X_m \cos \varphi - i X_m \sin \varphi \text{(23)}$$

into Eq. (21) ~ (23), the following expressions can be obtained

$$X_m \cos \varphi = \frac{k_p(k_p - m\omega^2) + (b_p\omega)^2}{(k_p - m\omega^2)^2 + (b_p\omega)^2} \text{(24)}$$

$$X_m \sin \varphi = \frac{m\omega^2(b_p\omega)}{(k_p - m\omega^2)^2 + (b_p\omega)^2} \text{(25)}$$

From Eq. (25) and Eq. (24), k_p and ζ_p are obtained as follows:

$$\left. \begin{aligned} k_p &= \left(\frac{X_0}{X_m \sin \varphi} \frac{A}{1+A^2} + 1 \right) m\omega^2 \\ \zeta_p &= \frac{X_0 m \omega}{2 X_m \sin \varphi \sqrt{m k_p}} \frac{A^2}{1+A^2} \\ A &= \frac{X_m \sin \varphi \tan \varphi}{X_m \sin \varphi - X_0 \tan \varphi} \end{aligned} \right\} \text{(26)}$$

Fig. 13 Vibration test model and its equivalent system

By measuring x_0 , x_m (or amplitude of $\ddot{x}_0$ and $\ddot{x}_m$) and φ , k_p and ζ_p can be calculated.

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Discussion

A. MORI (Kyoto University):

Authors concluded that the larger the damping ratio is, the wider the stable region becomes. I want to ask whether there is an optimum value of ζ_p , and whether it is advisable to use a larger value of ζ_p within the limit of properties of 'O' rings. The value of ζ_p may be made infinitely large by other supporting methods. May we use an infinitely large value of ζ_p ?

Authors' closure

In calculation, an optimum value of ζ_p exists.

For example, Q_L becomes maximum when ζ_p is nearly 0.5 under the conditions of $A_s=0.1$, $\mu=0.1$ and $k_p/k_s=1$ (in Fig. 6). Usually, if ζ_p becomes infinitely large, bearing bushes are inflexible and they become similar to those of rigidly supported ones and, therefore, Q_L becomes small. Thus, after all, we can easily deduce that there is an optimum value of ζ_p .

However, as you indicate, the larger ζ_p becomes within the limit of practical materials, the more effectively the whirl will be suppressed.