Study of Critical Flow*

(Completely Separated Gas-Liquid Two-Phase Flow)

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Critical flow is studied for a completely separated two-phase flow which is a compressible gas flow coexisting with an incompressible liquid flow without mutual mixing within a constant-area duct. It is predicted from a theoretical analysis that the critical state appears at the exit of the duct with the condition :

$$M_g = 1 + KM_g^3; \text{ where } K = \frac{1-x}{x} \left(\frac{\rho_g}{\rho_l} \right)^2 \left(\frac{a}{u_l} \right)^3,$$

M_g : mach number of gas flow, x : quality, ρ_g : density of gas, ρ_l : density of liquid, a : sonic velocity of gas, and u_l : velocity of liquid flow. The two quantities M_g and K included in the afore-mentioned equation have a possible range of $1 \leq M_g < \sqrt{3}$ and $0 \leq K < 2/\sqrt{27}$ respectively.

For the experimental verification of the analytical prediction, the condition of a completely separated two-phase flow is successfully realized in a specially designed apparatus with use of air and water. The state of flow at the exit of the duct is determined quantitatively by making use of the characteristics of the present two-phase flow, and the data thus obtained reveal that the analytical prediction afore-mentioned is satisfied in the experiment.

1. Introduction

One of the authors has given a description of the general mechanism with which critical flows are generated in compressible two-phase flows^{(1) (2) (3)}. The cause which brings about a critical flow in a compressible flow should be included in natural laws which control a given compressible flow, and any other special supplementary conditions are unnecessary for the generation of a critical flow. In other words, the generation of a critical flow should be predicted only by a group of the equations which are necessary and sufficient to give a physically complete expression to a given compressible flow. The critical flow is connected generally with a flow in a duct having an axial variation of the state of flow under the influence of forced actions such as the friction and others, and consequently the critical flow appears as the possible limit of the variation of the state of flow corresponding to the condition of zero forced actions.

In order to examine the principle of the

generation of critical flows afore-mentioned, an extremely simple system of a compressible two-phase flow is specially employed in the present study and a comparison is attempted between the analysis and the experiment.

2. Analysis

2.1 Conditions of two-phase flow

As shown schematically in Fig. 1, a simple system of two-phase flow is intentionally employed with the conditions that (1) a gas and a liquid, being separated completely from each other, flow

Fig. 1 Completely separated two-phase flow (G : total flow rate)

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through an insulated constant-area duct, (2) the gas behaves itself as a compressible fluid and the liquid as an incompressible fluid, and (3) there is no heat and mass transfer between the two phases

2.2 Fundamental equations for critical flow

For the sake of simplicity, it is assumed over a cross-section of the duct that the pressure p is uniform, and each phase has a uniform velocity (u_g for the gas and u_l for the liquid). For the variation of the independent variables (dp , du_g and du_l) across the two control surfaces shown in Fig. 1, the principle of the generation of critical flow (Chapter 1) as well as the conditions of flow (section 2.1) yield the fundamental equations of critical flow as follows:

Equation of continuity,

$$x \frac{\rho_l u_l}{\rho_g} \frac{d\rho_g}{dp} dp + (1-x) \frac{\rho_g u_g}{u_l} du_l + x \frac{\rho_l u_l}{u_g} du_g = 0 \quad \dots\dots\dots(1)$$

Equations of momentum

$$\frac{1}{\rho_l u_l} dp + \frac{1}{g_c} du_l = 0 \quad \text{for the phase of liquid} \quad \dots\dots\dots(2)$$

$$\frac{1}{\rho_g u_g} dp + \frac{1}{g_c} du_g = 0 \quad \text{for the phase of gas} \quad \dots\dots\dots(3)$$

Equation of energy,

$$\frac{dh_g}{dp} dp + \frac{u_g}{g_c J} du_g = 0 \quad \text{for the phase of gas} \quad \dots\dots\dots(4)$$

where x is the quality (ratio of mass flow rate of gas to total mass flow rate, $0 < x \leq 1$), ρ_l the density of liquid, ρ_g the density of gas, g_c the conversion factor of force, h_g the enthalpy of gas, and J the mechanical equivalent of heat. The change of the state of gas $d\rho_g/dp$ and dh_g/dp will be determined in the next section. It is assumed in the present study that the liquid is incompressible and no heat transfer exists between the liquid and gas, so that it is unnecessary to take the internal energy of liquid into account. Since there is no mass transfer between the two phases, it is also unnecessary to consider the equations of concentration.

2.3 State of critical flow

As described in Chapter 1, the critical flow appears generally in a flow accompanied by the variation of state. Accordingly, the critical flow can be determined from the condition that the variations dp , du_l and du_g do not become zero simultaneously in Eqs. (1) to (4).

First, applying this condition to Eqs. (3) and (4) yields

$$\left| \begin{array}{cc} \frac{1}{\rho_g u_g} & \frac{1}{g_c} \\ \frac{dh_g}{dp} & \frac{u_g}{g_c J} \end{array} \right| = 0, \quad \therefore \frac{dh_g}{dp} = \frac{1}{\rho_g J}$$

Remembering one of the well-known relations in thermodynamics, $T_g ds_g = dh_g - \frac{dp}{\rho_g J}$ (T_g : temperature of gas, and s_g : entropy of gas), the result obtained above shows nothing but the change of state $ds_g = 0$. That is to say, the phase of gas is subjected to an isentropic change at the critical flow, and consequently $d\rho_g/dp$ on the left-hand side of Eq. (1) can be written as $(d\rho_g/dp)_s$.

Next, applying again the same condition as before to Eqs. (1), (2) and (3) yields

$$\left| \begin{array}{ccc} x \frac{\rho_l u_l}{\rho_g} \left(\frac{d\rho_g}{dp} \right)_s & (1-x) \frac{\rho_g u_g}{u_l} & x \frac{\rho_l u_l}{u_g} \\ \frac{1}{\rho_l u_l} & \frac{1}{g_c} & 0 \\ \frac{1}{\rho_g u_g} & 0 & \frac{1}{g_c} \end{array} \right| = 0 \quad \dots\dots\dots(5)$$

Making use of a well-known relation of sonic velocity a , that is $a = \sqrt{g_c / (d\rho_g/dp)_s}$, Eq. (5) can be rewritten as follows:

$$M_g^2 = 1 + K M_g^3, \quad \text{where } K \equiv \left(\frac{1-x}{x} \right) \left(\frac{\rho_g}{\rho_l} \right)^2 \left(\frac{a}{u_l} \right)^3 \quad \dots\dots\dots(6)$$

In Eq. (6), M_g denotes the Mach number of a flow of gas in the critical flow ($M_g \equiv u_g/a$) and it is significant physically only for $M_g > 0$. K , a non-dimensional group independent of the velocity of gas flow, is concerned with the velocity of liquid flow, and $K \geq 0$ due to the restriction of quality $0 < x \leq 1$. If $x=1$ in Eq. (6), $M_g=1$, and $K=0$ is obtained corresponding to the well-known condition of the critical flow in the case of a compressible single-phase flow. In the limit as K approaches zero, Eq. (6) can also have a root such as $M_g \rightarrow 1/K$ (that is $M_g \rightarrow \infty$), but this root, not agreeing with the afore-mentioned condition of $K=0$ and $M_g=1$, is meaningless physically.

Equation (6) is a cubic equation for M_g and the characters of the roots of Eq. (6) are shown in Fig. 2, where a heavy curve $y = M_g^2$ represents the value of left-hand side of Eq. (6) while a group of light curves $y = 1 + K M_g^3$ represents the value of right-hand side of Eq. (6) with K as a parameter. It may be noticed in Fig. 2 that in the region of $0 < K < 2/\sqrt{27}$, Eq. (6) has one negative root α_1 as well as two positive roots α_2 and α_3 ($\alpha_2 = \alpha_3$ at $K = 2/\sqrt{27}$) and in the region of $2/\sqrt{27} < K$, Eq. (6) has only one negative root α_1 . However, the negative root α_1 and the positive

root α_3 (as K approaches zero, $\alpha_3 \rightarrow \infty$) can be left out of consideration, because they have no physical meaning. Consequently, it is concluded that the Mach number of gas flow M_g and the quantity of state of flow K for the critical flow in the present study can exist only within the following ranges respectively,

$$1 \leq M_g < \sqrt{3}, \text{ and } 0 \leq K < 2/\sqrt{27} \dots\dots\dots (7)$$

According to Eq. (7), the critical flow in the present study appears being accompanied with a super-sonic state of gas flow, for which the Mach number ranges from 1 to $\sqrt{3}$.

2.4 Another expression of Eq. (6)

For the cross-section of the duct through which a gas and a liquid flow in such a manner as in the present study, the following relation is established from the definition of quality x ,

$$\frac{\rho_g u_g A_g}{\rho_l u_l (A - A_g)} = \frac{x}{1-x}, \quad \therefore \quad \frac{u_g}{u_l} = \frac{\rho_l}{\rho_g} \frac{x}{1-x} \times \frac{A - A_g}{A_g} \dots\dots\dots (8)$$

where A_g is the area of the cross-section occupied by gas flow and A the total area of the cross-section. For a given state of u_g , u_l , ρ_l and ρ_g , arbitrary values can be given independently of A_g so that the corresponding values of x are determined; that is to say, x is an independent variable in this sense.

Eliminating u_l from Eq. (6) to Eq. (8) yields the following equation,

$$M_g^2 = 1 + B, \quad \text{where} \quad B = \left(\frac{\rho_l}{\rho_g} \right) \left(\frac{x}{1-x} \right)^2 \left(\frac{1 - A_g/A}{A_g/A} \right)^3 \dots\dots\dots (9)$$

Since the physical quantities included in Eq. (9) except x and ρ_l are concerned with the state of

gas flow alone, Eq. (9) will be more convenient than Eq. (6) when a comparison is made afterwards between the analysis and the experiment.

The relation of Eq. (9) is represented in Fig. 3, where the critical flow appears within the region of $1 \leq M_g^2 < 3$ and $0 \leq B < 2$ corresponding to the restrictions which have been shown by Eq. (7). x and A_g/A in Eq. (9), being related through Eq. (8), are not independent of each other, but in the experiment described afterwards, x and A_g/A can be determined respectively.

3. Experimental apparatus and its characteristics

3.1 Experimental apparatus

Air and water are used as the gas and liquid in the present experiment. Experimental apparatus is shown schematically in Fig. 4, where both an air-supply system (which includes two air compressors) and a water-supply system (which includes a water-pump and a water reservoir) are connected with the testing part of a two-phase

Fig. 3 Critical-state line [Eq. (9)]

Fig. 2 Root of Eq. (6)

Fig. 4 Outline of experimental apparatus

flow. Each air compressor of reciprocating type (pressure 10 kg/cm^2 and capacity 730 l/min) is equipped with an after-cooler, thereby air of nearly atmospheric temperature is supplied to the testing part. Water of atmospheric temperature which comes from the water reservoir is pressurized by the water-pump of centrifugal type (delivery head 200 m and discharge $0.3 \text{ m}^3/\text{min}$) and then supplied to the testing part.

Details of the testing part of a two-phase flow are shown in Fig. 5, where a horizontal, straight and constant-area duct between the sections B and C serves as the testing duct of the two-phase flow. At the inlet section B of the testing duct, air coming out of an air nozzle flows through the central part, while water entering from the circumference of the air nozzle flows along the duct wall. In this way, a separated two-phase flow of annular flow type is formed in the testing duct, and the reduction of pressure through the duct is measured with pressure taps (0.3 mm I.D.) which are set along the duct wall. The outlet section C of the testing duct opens to the atmosphere, and the discharged water is returned to the water reservoir.

As shown in Fig. 5, an air nozzle of $d=6 \text{ mm}$ I.D. at the exit is fixed, but the outer wall of passage of water together with the testing duct is easily replaced with another so that the internal diameter D and the axial length L of the testing duct are changeable without yielding a discontinuous step on the outer wall of passage of water. Experiments are made in the range of $D=7.6\sim 10.0 \text{ mm}$ and $L/D=1\sim 5$, for which the ratio of the sectional area occupied by air flow to

the total sectional area at the inlet section of the testing duct is in the range of $(d/D)^2=0.62\sim 0.36$.

3.2 State of air flow at the exit of air nozzle

As shown in Fig. 5, air is supplied to the air nozzle through two air pipes, but the two air pipes are perfectly symmetrical and a source pressure of air is always the same. Since experiments are made with source pressures of air sufficiently lower than the critical pressure with respect to the exit pressure of the air nozzle, the discharge velocity of air at the exit of the air nozzle is always considerably lower than the sonic velocity. According to the experimental calibration which has been made for the relation between the source pressure of air, the exit pressure of air nozzle and the flow rate of air, the discharge coefficient⁽⁴⁾ is kept constant within the range of the present study. Besides, through the air pipe and air nozzle, total temperature of air is kept constant. Accordingly, making use of these relations, the pressure P_2 , the temperature T_2 and the Mach number M_2 at the exit of the air nozzle can be determined with sufficient accuracy from the measurements of the air flow rate, the source pressure of air P_1 and the air temperature T_1 of air pipes.

As for the state of flow of air and water, the ranges within which the experiment is carried out are as follows:

flow rate of water: $G_t=1.00\sim 0.58 \text{ kg/sec}$

quality: $x=0.0242\sim 0.0028$

$P_1=4.93\sim 1.18 \text{ kg/cm}^2 \text{ abs.}$, $T_1=32^\circ\sim 13^\circ\text{C}$

$P_2=4.52\sim 1.14 \text{ kg/cm}^2 \text{ abs.}$, $M_2=0.58\sim 0.16$

3.3 State of two-phase flow in the testing duct

One of the reasons of employing ducts of comparatively short axial length ($L/D=1\sim 5$) in the present study as described in section 3.1 is to facilitate the maintenance of a completely separated two-phase flow. According to a lot of photographic observations which have been made with transparent testing ducts made of acrylic resin, it may be said that flows of air and water are sufficiently separated from each other and the interface between air and water can be observed through a testing duct.

For the extent of pressures adopted in the present study (below $4.52 \text{ kg/cm}^2 \text{ abs.}$), it may be permissible to regard water as an incompressible fluid. It may be also permissible to assume from the experimental conditions adopted that the heat transfer between two phases and the mass

Fig. 5 Details of testing part of two-phase flow

transfer due to the change of phase do not become serious problems within the testing duct.

3.4 State of air flow at the exit of testing duct

If attention is paid to the part of gas flow in such a completely separated two-phase flow as described in the preceding section, it can be pointed out that a gas flows through a nozzle which is made of a free surface of liquid flow. Being different from a solid wall nozzle, of course, the shape of nozzle (that is, the change of sectional area in the axial direction) is not fixed in the present case, but one shape of nozzle is determined corresponding to one given condition of two-phase flow. There is some turbulence on the wall of nozzle (the interface between gas and liquid), but the wall of nozzle itself flows in the same direction as that of the flow of gas. Accordingly, as for the change of state of gas at least, it may be presumed that the condition in the present case does not differ so much from the ordinary case of a single phase flow through a solid wall nozzle; that is to say, the flow of gas submits itself approximately to an isentropic change (this isentropic change has no connection with the isentropic change in the critical flow described in section 2.3). Here is another reason for employing ducts of comparatively short axial length, and this is for reducing the effect of friction on air flow as far as possible between the inlet and outlet of the testing duct.

In relation to the afore-mentioned matters, Fig. 6 shows an example of preliminary experiment, where the local flow rate of water over the cross-section at the exit of the testing duct of 10 mm I.D. has been measured by traversing a pitot-tube of 0.75 mm I.D. (with a sharp edge at the top) at intervals of 0.5 mm along a vertical line passing through the center of the duct. With respect to the experimental result such as Fig. 6, of course, there should be ambiguities of data

Fig. 6 Distribution of local flow rate of water over the outlet section of a testing duct

especially at the positions of abrupt change of local flow rate of water, due to the dimensional and geometrical relations between the pitot-tube, the water droplet and the testing duct. However, as for the central portion of zero local flow rate of water in Figure 6, it may be supported that there is substantially a flow of air alone (though it includes water droplets to some extent). Besides, it is observed that the local flow rate increases abruptly at the outer edge of the afore-mentioned air flow. Fig. 6 may be, therefore, regarded at least to elucidate how the flow of air is concentrated to the central portion of the cross-section of the duct (though the flow of air is slightly transferred upward due to the action of gravity upon the flow of water), and this agrees well with the photographic observations described in section 3.3.

Theoretical value of the sectional area of the air flow at the exit of the testing duct, which is derived under the assumption of an isentropic change of air from the state at the inlet of the testing duct (determined by the method in section 3.2) to the atmospheric pressure, is shown in Fig. 6. The agreement with the experimental result is good.

As a consequence, it may be assumed without serious error that the change of the state of gas takes place isentropically through a testing duct. In this case, as far as the pressure at the outlet section of the testing duct is known, the density of gas ρ_g , Mach number M_g , and the sectional area of gas flow A_g at the outlet section can be determined with pretty high accuracies by the theoretical calculation afore-mentioned.

4. Experimental result

4.1 Pressure distribution in the testing duct

As to the pressure reduction through the testing duct, Fig. 7 shows a typical example of the experimental data which are obtained with pressure-measuring taps set along the duct wall. The data in Fig. 7 are concerned with the experiment made under the conditions that the internal diameter of the exit of the air nozzle $d=6$ mm, the internal diameter of the testing duct $D=8.5$ mm, the length of the testing duct $L=17.0$ mm, the mass flow rate of water G_l is kept constant at 1.00 kg/sec, and the mass flow rate of air is changed (the marks a, b, c, d, and e in Fig. 7 correspond to the values of quality $x=0.0168$, 0.0153, 0.0117, 0.0090, and 0.0068 respectively). The pressure-measuring tap (0.3 mm I.D.) nearest the exit of the testing duct is at the position of

0.25mm from the outlet section of the duct; and all curves in Fig. 7 have been drawn tentatively under the assumption that the outlet section of the duct is held at atmospheric pressure. Values of M_g written alongside the curves in Fig. 7 are Mach numbers of air flow at the outlet section which have been determined under the assumption of atmospheric pressure at the outlet section (cf. section 3.4).

If any conclusion on the fluid pressure at the outlet section of the duct is drawn from the results

Fig. 7 Pressure distribution in the axial direction within a testing duct (Mark ●: Experimental data)

Fig. 8 Experimental result of the variation of M_g and B (cf. Table 1 for the experimental conditions)

Table 1 Conditions for the experiments in Fig. 8

Experiment	A	B	C	D	E	F	G	H	I	J	K
Diameter of testing duct : D mm	8.5	7.6	8.5	8.5	8.5	10.0	10.0	10.0	10.0	10.0	10.0
Axial length of testing duct : L mm	8.5	7.6	17.0	17.0	20.0	20.0	20.0	40.0	40.0	40.0	50.0
Mass flow rate of water : G_t kg/sec	1.00	0.80	1.00	0.80	0.92	0.95	0.58	1.08	1.00	0.75	1.00

of Fig. 7, a question may be raised of course. At least, however, it seems that a serious error may not be produced if the pressure at the outlet section is assumed to be equal to or near atmospheric pressure in Fig. 7. Then, it is noticed that the experimental data in Fig. 7 may be divided into two groups (one group of a and b, and another of c, d and e) in view of the different trends of the pressure distribution; and on the basis of this fact, it seems reasonable to presume that the experiments c, d and e made in the range of low flow rates correspond to the subcritical state of a two-phase flow (the outlet section is held at atmospheric pressure in this case), while the experiments a and b made in the range of high flow rates correspond to the supercritical state (the outlet section should be held at pressures a bit higher than atmospheric pressure). If the aforementioned presumption is admitted, the result of Fig. 7 means that a critical flow appears when the Mach number M_g of gas flow at the outlet section (the values of M_g in Fig. 7 should be correct in the subcritical state) reaches a value that is higher than unity.

The experimental result afore-mentioned may support the analytical result described in Chapter 2. However, the present discussion necessarily includes a presumption on the pressure at the duct exit, and besides, Fig. 7 has a form which is inconvenient for the comparison with Eq. (6) or Eq. (9). Accordingly, the discussion on Fig. 7 will not be extended further.

4.2 Change of the state of flow at the outlet section

For the experiment of subcritical state of a two-phase flow, the fluid pressure at the outlet section of the testing duct is always held at atmospheric pressure until the critical state is reached. For subcritical conditions, therefore, the state of gas flow at the outlet section can be determined with a high accuracy by the method described in section 3.4, thereby the two quantities M_g and B included in Eq. (9) are determined. Then, if the analysis described in Chapter 2 is assumed to be correct, it may be concluded that as far

as the two-phase flow is retained within a state of subcritical condition, the relation of Eq. (9) is not satisfied by M_g and B , but as the two-phase flow tends to the critical state, the relation between M_g and B approaches Eq. (9).

The examination of the afore-mentioned matter will be made in Fig. 8 (cf. Table 1 with respect to the experimental conditions). Each curve from A to K in Fig. 8 represents the experimental result of the variation of M_g and B which takes place when the mass flow rate of air is increased under the condition of keeping the mass flow rate of water G_l constant. It may be clearly noticed here that every curve has a trend to approach the relation of Eq. (9) (within the range of $1 \leq M_g^2 < 3$ and $0 \leq B < 2$ shown in Fig. 3) as the flow rate increases, and this result is regarded as a strong evidence for the propriety of the analysis described in Chapter 2. As for the discrimination between the subcritical and supercritical states, it should be stated here that the present method gives the same conclusion as that in section 4.1 for the experiments from a to e in Fig. 7.

All curves in Fig. 8 have been obtained under the assumption that the outlet section of the duct is held at atmospheric pressure, but this premise of the pressure is not satisfied after the critical state has been reached. That is to say, the part of each curve of A to E which lies above the critical-state line represents the relation between superficial values of M_g and B . In connection with this matter, the variation of the value of B which takes place along the curve of A in Fig. 8 following the increase of the mass flow rate of air (the flow rate of water is kept constant) is shown in Fig. 9. It may be noticed that the characteristics of the variation of B in the region after the critical state is reached is clearly dif-

Fig. 9 Variation of the value of B due to the increase of mass flow rate of air

ferent from those in the subcritical region.

After the two-phase flow reaches the critical state, the actual relation between M_g and B at the outlet section of the testing duct (where fluid pressure is higher than atmospheric pressure) should be represented by the critical-state line in Fig. 8. On this point, it is easy to presume an actual value of pressure at the outlet section by revising the outlet pressure from atmospheric pressure so as to satisfy Eq. (9). For example, the pressures at the outlet section thus presumed for the experiments a and b in Fig. 7 are indicated by the directed lines a and b in Fig. 10 respectively, and the values of M_g written alongside the curves in Fig. 10 are the actual outlet Mach numbers of air flow which correspond to the actual pressures at the outlet section afore-mentioned.

4.3 Additional remarks

It is noticed in Fig. 8 that the experimental curve is transferred to the right as (1) the internal diameter of the testing duct D becomes greater (the exit diameter of the air nozzle d is constant), (2) the mass flow rate of water G_l becomes smaller, and (3) the axial length of the testing duct L becomes longer. In these cases, A_g/A (ratio of the sectional area of air flow to the total section) at the outlet section is reduced generally, and this is the main cause for the afore-mentioned transfer of the experimental curve.

The bend of the experimental curve which appears when $M_g \approx 1$ in Fig. 8 depends mainly on the variation of A_g/A at the outlet section. The passage of air surrounded by water in the testing

Fig. 10 Distribution of pressure in the axial direction within a testing duct

duct has a form of convergent nozzle when $M_g < 1$ at the outlet section, but it changes to a convergent-divergent nozzle when $M_g > 1$. As for the curve K in Fig. 8, it may have involved the effect of the deviation from the assumed state of a completely separated two-phase flow due to too great value of L/D .

5. Conclusions

(1) The critical flow has been studied for a completely separated two-phase flow in a constant-area duct composed of a compressible gas and an incompressible liquid. Theoretical analysis predicts that the critical flow appears at the exit of duct with the following relation between the Mach number of gas flow M_g , the velocity of liquid u_l , quality x , etc.,

$$M_g^2 = 1 + KM_g^3, \text{ where } K \equiv \left(\frac{1-x}{x} \right) \left(\frac{\rho_g}{\rho_l} \right)^2 \left(\frac{a}{u_l} \right)^3$$

and the two quantities M_g and K afore-mentioned are limited to within the ranges:

$$1 \leq M_g < \sqrt{3}, \text{ and } 0 \leq K < 2/\sqrt{27}$$

For the experimental verification of the analytical prediction, a series of experiments have been carried out with use of air and water. The experimental data thus obtained reveal that the analytical prediction is satisfied in the experiment.

(2) The result afore-mentioned is accompani-

ed with a conclusive notice that the principle which is universally applicable to every kind of choking phenomenon cannot be established on the basis of the sonic-velocity-condition, that is, the condition of the interception of the transmission of pressure fluctuation. In the two-phase flow treated in the present paper, at least, the effect of the pressure fluctuation downstream can be always transmitted to upstream though a continuous flow of incompressible fluid, so that the choking phenomenon never appears in view of sonic-velocity-condition.

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Discussion

N. ISSHIKI (Tokyo Institute of Technology):

(1) It seems likely that the condition related to the variation of sectional area through a duct, such as $dA=0$ for example, is not involved explicitly in fundamental equations (1) to (4). Where should the condition of area be involved in these equations? Is it possible to generalize and extend these equations to the case of a convergent or a divergent nozzle?

(2) Is it possible to regard the assumption and the result in the authors' paper as inherent in a state composed of a gas flow through a convergent-divergent nozzle and a liquid flow through a convergent nozzle?

(3) How do the authors think of the circumstances where if the interpretation described in (2) is admitted, the concept of the critical pressure ratio comes out as a result?

(4) Experimental data may be correlated by plotting a quality which is similar to the non-dimensional flow-rate coefficient ψ [cf. Eq. (359) in Chap. 11, Handbook of Mechanical Engineering]

against the pressure ratio. Is there not any chance for finding a correspondence between the critical state and a limit of ψ ?

Authors' closure

(1) When there is a variation in the sectional area of a duct, a term concerned with dA is added to Eq. (1). In the present case, however, a constant-area duct ($dA=0$) as shown in Fig. 1 is treated so that the term afore-mentioned vanishes. As for the general principle of the generation of critical flow including a convergent and a divergent nozzle, it is mentioned in references^(*) and it should be stated that the analysis made in the present paper accords with the general principle afore-mentioned.

(2) In the authors' experiment, a liquid flows keeping a contact with the solid wall of the duct, so that the sectional area of the liquid flow increases in the direction of flow usually through a reduction of velocity due to a comparatively strong friction of the wall, and thereby a passage of the

type of a convergent nozzle is offered to the gas flow. However, when the pressure gradient in the direction of flow becomes sufficiently great (especially near the exit of the duct as shown in Fig. 7), the liquid flow is accelerated so much there that a form of divergent nozzle is offered to the gas flow. In this way, a passage of gas flow with a shape of varying-area nozzle is made by the liquid flow; and when the passage of gas flow has the form of a convergent-divergent nozzle, the velocity of gas flow is equal to the sonic velocity at the throat of the nozzle.

(3) Circumstances mentioned in (2) seem to represent nothing but the conventional flow of gas through a solid wall nozzle. However, the present case is definitely different from the case of solid wall nozzle at the point where the shape of nozzle is not fixed for the gas flow and the variation takes place in relation to the gas and the liquid flow. Critical flow is generated also in relation to the gas and the liquid flow, and this is the principal subject in the present study. That is to say, it is meaningless to consider the gas flow alone for the study of the two-phase critical flow.

It should be pointed out that the concept of the critical pressure ratio is very special even in the case of a single-phase flow. This ratio appears only in the case of the isentropic flow of an ideal gas through a nozzle, and it depends on a special circumstance that the critical state at the nozzle exit or the nozzle throat (which is determined under the control of the criterion for the onset of critical flow) can be related to the state at the

nozzle inlet due to the frictionless flow. In other words, the critical pressure ratio is not the general criterion for the onset of critical flow itself, and the authors' paper is concerned with the study of the criterion. Besides, the present system of two-phase flow is under the condition that the liquid flow, at least, is affected by a wall friction, so that it is impossible to get a universal relation between the states of flow at the inlet and outlet of the duct.

(4) As described in (3), it is meaningless to consider the gas flow alone. Even if the pressure ratio between the inlet and outlet of the duct is given, there remains a freedom of changing the state of liquid flow. Accordingly, the method proposed by Prof. Isshiki is considered to be unreasonable in its principle. It is the essence of the present problem that the critical state is not indicated by the flow rate but by the velocity (Mach number) as it is seen in Eq. (6).

It may be also of use to mention the following matter. Putting $x=1$ (a pure gas flow) in Eq. (6) yields $M_g=1$ (sonic velocity), which agrees with the well-known condition of critical flow for a single-phase flow through a constant-area duct with friction. It is a common sense that such a quantity as the flow-rate coefficient ψ does not exist for the single-phase flow of this form.

(*1) Katto, Y. and Sudo, Y., *Preprint of Japan Soc. Mech. Engrs.* (in Japanese), No. 710-17 (1971-11), p. 97.

(*2) Sudo, Y., *Preprint of Japan Soc. Mech. Engrs.* (in Japanese), No. 720-13 (1972-8), p. 279.