Transient Phenomena Caused by Directional Control Valve in a Hydraulic Pipeline

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This paper deals with the surge pressure and pressure fluctuation in a pipe caused by the instantaneous operation of a valve in a hydraulic system made up of an accumulator, a directional control valve and a pipeline. Theoretical and experimental results indicate that when the valve attached to one end of the pipe is operated rapidly and a step change in pressure is applied to the pipeline, the surge pressure which is caused by the spool-type directional control valve is proportional to the spool velocity. Moreover it is shown that the period and the logarithmic decrement of the pressure fluctuation at the other end are influenced only by a dimensionless quantity defined as the square of the radius of the pipe times the natural frequency of the fluid column divided by the kinematic viscosity; in other words, the pressure fluctuation is similar in cases where the values of this dimensionless quantity are equal to each other.

1. Introduction

In hydraulic systems almost all components are connected by pipe lines. Since the flow in the line is unsteady, and the pressure and the velocity always fluctuate, it is very important to analyze the unsteady flow in the line.

Since Alilevi's work(12), many reports have been published about the unsteady flow in the pipeline based on the one dimensional theory. However the Reynolds number of the flow in the hydraulic transmission line is low, and the flow can be regarded as laminar, so that the velocity distribution in the radial direction can not be neglected. Brown(22) derived the propagation operator of the flow in transmission lines considering the velocity distribution in the radial direction, and D'Souza and Oldenburger(33) derived the transfer matrix. They established a new theory about axisymmetric unsteady flow. Using this new theory, Ichikawa et al.(44~(55) and Sato(55) studied the pressure transfer characteristics of the hydraulic transmission line from a practical point of view, and derived the equivalent viscous resistance of unsteady laminar flow. Urata(99) solved exactly Navier-Stokes' equation for the axisymmetric unsteady flow and indicated that the first approximation coincided with D'Souza's result. Okamura et al.(99) presented in detail the effects of the branch and the bend on the hydraulic transients in the axisymmetrical line. Nakano and Yoshimoto(99) reported the dynamic characteristics of a hydraulic pipeline with a viscoelastic pipe wall and showed that the damping constant and wave length heavily depend on the viscoelasticity of the pipe wall.

Thus the transient phenomena in a hydraulic transmission line have been studied in detail, but the surge pressure and the pressure oscillation which are caused by the sudden operation of a directional control valve in the hydraulic circuits have not been studied so much.

The authors clarify theoretically the hydraulic transients caused by the valve operation in the pipeline shown in Fig. 1, and compare the results with the experimental results. This pipeline system is often used in the impulse pressure test of hydraulic components.

2. Theoretical analysis

2.1 The surge pressure produced by valve operation

In the pipeline system shown in Fig. 1, after rapid operation of the valve the pressure stored in the accumulator, $p_a$, is propagated downstream and is

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amplified to $2p_0$ by reflection at the downstream closed end, if the accumulator is large enough and the fluid is nonviscous. So the maximum pressure rise, $p_{max}$, becomes

$$p_{max} = 2p_0$$

(1)

However, in actual hydraulic systems the surge pressure observed often becomes higher than that expressed by Eq. (1). The reasons may be as follows; (1) The fluid in the valve gets extra energy from the spool in a spool-type direction control valve. (2) The accumulator is not a true constant pressure source because of its finite volume. (3) The measuring instruments do not operate exactly because the signal involves a wide range of frequencies.

In this paper, we considered only the first reason, and for simplicity it is assumed in this section that the viscosity of the fluid can be neglected and that the velocity is small enough.

For the flow in a uniform pipe, Euler’s equation of motion can be written as follows;

$$\frac{\partial u}{\partial t} + \frac{u}{\gamma} \frac{\partial p}{\partial x} = 0$$

(2)

where $u$ is the velocity of the fluid, $p$ is the pressure, $\gamma$ is the specific weight of the fluid, $g$ is the gravitational acceleration and $x$ is the coordinate in the axial direction.

Integrating Eq. (2) from $x_1$ to $x_2$, the energy equation is given as follows;

$$p_1 \frac{\alpha_1}{\gamma} u_1 = p_2 \frac{\alpha_2}{\gamma} u_2 + \int_{x_1}^{x_2} \frac{\alpha_2}{\gamma} du$$

(3)

where $\alpha_1$ is the wave propagation velocity, the subscripts 1 and 2 denote the values corresponding to $x_1$ and $x_2$, the minus sign indicates the wave propagation in the positive direction of $x$ and the plus sign in the negative direction of $x$.

At the section of sudden area change,

$$p_2 \frac{\alpha_2}{\gamma} Q_2 = p_{ae} \frac{\alpha_e}{\gamma} Q_{ae}$$

(4)

$$\frac{\alpha_2}{\gamma} A_2 \frac{\alpha_2}{\gamma} Q_2 = \frac{\alpha_e}{\gamma} A_e \frac{\alpha_e}{\gamma} Q_{ae}$$

(5)

where $Q$ is the flow rate, $A$ is the cross sectional area of the pipe, subscripts $e$ and $a$ denote the points (e) and (a) in Fig. 2 (a), the subscript 0 denotes the value before the wave arrives and subscript $t$ denotes the value of the first approaching wave.

Equation (3) shows the relation for different positions at the same instant, and Eqs. (4) and (5) are for different times at the same position.

In the pipeline system with a spool valve, shown in Fig. 3, at the instant when the pipeline is connected with the accumulator by the operation of the valve spool, the fluid particle at the point (e) is at the same pressure as the pressure in the accumulator, $p_{ae}$ and has the same velocity as the spool velocity $v_0$.

Since the pressure and the velocity at the connecting point (e) of the pipe and the valve have the same values as those before the valve operation, that is $p_{ae}=0$ and $Q_{ae}=0$, from Eqs. (4) and (5) we get

$$p_2 = \frac{A_e}{A_s + A_d} \left( p_0 + \frac{\gamma A_s}{g} v_0 \right)$$

(6)

$$Q_2 = \frac{\gamma A_s}{g} p_2$$

(7)

where $A_s$ is the effective area of spool in Fig. 3.

If the spool area $A_s$ is much greater than the cross-sectional area of the pipe $A_d$, then Eq. (6) reduces to

$$p_2 \approx p_0 + \frac{\gamma A_s}{g} v_0$$

(8)

2.2 The pressure oscillation at the closed end of pipe

In this section we consider the pressure oscillation at the point b in Fig. 1, which is caused by a
sudden valve operation. Here we neglect the extra pressure rise caused by the spool velocity \( v_s \) which was treated in section 2.1.

From D’Souza et al., the equation for unsteady flow in the pipe considering the velocity distribution is as follows in the Laplace-transformed form.

\[
\tilde{U}(L, \zeta) = \tilde{U}(0, \zeta) \cos \left( \frac{\zeta}{i}\beta(\zeta) \right) - \frac{i}{Z} \tilde{P}(0, \zeta) \sin \left( \frac{\zeta}{i}\beta(\zeta) \right)
\]

\[
\tilde{P}(L, \zeta) = \frac{Z}{i} \beta(\zeta) \tilde{U}(0, \zeta) \sin \left( \frac{\zeta}{i}\beta(\zeta) \right) + \tilde{P}(0, \zeta) \cos \left( \frac{\zeta}{i}\beta(\zeta) \right)
\]

where

\[
\tilde{U}(x, \tau) = \int_0^\infty \tilde{u}(x, \tau) e^{-\gamma \tau} d\tau
\]

\[
\tilde{P}(x, \tau) = \int_0^\infty \tilde{p}(x, \tau) e^{-\gamma \tau} d\tau
\]

\[
\tilde{u}(x, \tau) = -\frac{1}{\pi \rho \nu} \int_0^\infty 2\pi \rho \nu (x, r, \tau) dr
\]

\[
\tilde{p}(x, \tau) = -\frac{1}{\pi \rho \nu} \int_0^\infty 2\pi \rho \nu (x, r, \tau) dr = p(x, \tau)
\]

\( \tau = \gamma \cdot T, \ \gamma = \frac{a_0}{2}, L \): pipe length, \( \zeta = T \tau, \ \zeta, \beta \) : Laplace operator, \( Z(\gamma a_0/\nu) \) : pipe length, \( a_0 = \sqrt{a_0/2}, \) modulus of elasticity, \( \delta \) : wall thickness of the pipe, \( K \) : bulk modulus, \( r, \tau \) : coordinate in the radial direction, \( \nu \) : velocity component in the \( x \) direction,

\[
\beta(\zeta) = \left[ 1 - \frac{2}{\sqrt{\zeta^2 + \nu L}} \right]^{1/2}
\]

\[
\lambda = r^2 \frac{\nu L}{(\nu L)}
\]

At the time \( t = 0 \) when the valve is just operated, the pressure at the upstream end of pipe is \( p_0 \), so

\[
\tilde{P}(0, \zeta) = p_0 / \zeta
\]

The fluid velocity at the closed end of the pipe is always zero, so

\[
\tilde{U}(L, \zeta) = 0
\]

Substituting these boundary conditions into Eqs. (9) and (10), we get

\[
\frac{1}{\zeta} \tilde{P}(L, \zeta) = \frac{1}{\zeta} \tilde{P}(0, \zeta)
\]

By an inverse transformation of Eq. (15), we can get the pressure oscillation at the closed end of the pipe, that is at the point \( b \) in Fig. 1. Let us find the poles \( \zeta_k \) \((k=0, 1, 2, \ldots)\) of the right side of Eq. (15) first. Then the poles are

\[
\zeta_k = 0 \text{ and } \zeta_k \beta(\zeta_k) = \frac{i}{2} \left( \frac{2k-1}{|\nu L|} \right), \ k \neq 0 \quad \text{..(16)}
\]

All of these poles are of the first order, and if \( k > 0 \), then from Eq. (16)

\[
\zeta_k \beta(\zeta_k) = -\frac{i}{2} (2k-1) = \zeta_k \beta(\zeta_k) = \zeta_k \beta(\zeta_k)
\]

so that we can express

\[
\zeta_k = \overline{\zeta_k}
\]

Therefore the residue \( F_k \) of the right side of Eq. (15), which concerns \( \zeta_k(\nu = 0, \pm 1, \pm 2, \ldots) \), is

\[
F_k = \lim_{\zeta \rightarrow \zeta_k} \frac{\zeta - \zeta_k}{\zeta - \zeta_k}
\]

Thus for \( k = 0 \)

\[
F_k = 1
\]

for \( k > 0 \)

\[
F_k = \left( \frac{\pi}{2} (2k-1) \right)^{-k} \left[ 1 + \frac{\zeta_k}{\beta(\zeta_k)} \right]^{-1}
\]

and for \( k < 0 \)

\[
F_k = \left( \frac{\pi}{2} (2k+1) \right)^{-k} \left[ 1 + \frac{\zeta_k}{\beta(\zeta_k)} \right]^{-1}
\]

where \( \beta(\zeta_k) = (d/\beta(\zeta_k)) \zeta = \zeta_k \).

By an inverse transformation of Eq. (15) using \( F_k \), we get

\[
\frac{1}{\rho_0} \tilde{p}(L, \tau) = 1 + \sum_{n=1}^{\infty} \left[ F_{n+1} e^{\gamma n \tau} + F_n e^{\gamma n \tau} \right]
\]

\[
= 1 + 2 \sum_{n=1}^{\infty} \left( \frac{\omega_n}{\omega_n-\text{arg} F_n} \right) e^{\gamma n \tau}
\]

where \( \omega_n = \sigma_n + i \omega_n, \ |F_n| = |\beta(\zeta_n)| \text{ Re} F_n, \text{ Re} F_n = \tan^{-1} \left( \frac{3(F_n)}{3(F_n)} \right) \), arg \( F_n = \text{ Re} F_n \).

Assuming \( 1/\sqrt{\lambda_2} < 1 \), using the first three terms of the right side of Eq. (12) for \( \beta(\zeta_n) \) and substituting into Eq. (16), we get

\[
\sigma_n = \frac{i}{2} \left( 2n-1 \right) \left[ 1 - \frac{1}{2} \sqrt{\lambda_2(2n-1)} \right] \quad \text{..(21)}
\]

\[
\omega_n = \frac{i}{2} \left( 2n-1 \right) \left[ 1 - \frac{1}{2} \sqrt{\lambda_2(2n-1)} \right]
\]

Noting that \( \zeta_n = \sigma_n + i \omega_n \) from Eq. (18) we obtain \( F_n \) in the following form

\[
F_n = \left[ \frac{\pi}{2} (2n-1) \right] \left[ 1 - \frac{1}{4 \lambda_2(2n-1)} \right]
\]

Substituting Eqs. (21) into Eq. (20), we get
\[
\frac{\dot{\theta}(L, \tau)}{\rho_0} = \frac{4}{\pi^2 \lambda_1(2n-1)} \sum_{n=1}^{\infty} (-1)^n \frac{1}{2} \sqrt{\pi(2n-1)} \left[ 1 + \frac{1}{\sqrt{\lambda} \pi(2n-1)} \right] \tau \\
\cos \left( \frac{1}{\sqrt{\lambda} \pi(2n-1)} \frac{\pi}{2} (2n-1) \tau \right) \quad \cdots (24)
\]

The result of calculation by Eq. (24) is shown in Fig. 4. For comparison, the result of calculation by the equation for one dimensional unsteady flow, which can be written as

\[
\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \frac{8\nu}{\tau} \frac{\partial u}{\partial x} \quad \cdots \cdots \cdots \cdots (25)
\]

and the result by Eq. (2), which is derived by assuming that the viscosity can be neglected, are shown. In Eq. (25), \( \nu \) is the kinematic viscosity, \( r_0 \) is the radius of the pipe and \( \dot{u} \) is the velocity before the wavefront.

3. Experimental apparatus and method

Figures 5 and 6 show the experimental apparatus and the directional control valves used. Two kinds of valves were used, one a solenoid operated valve and the other a hydraulically operated valve with a solenoid controlled pilot. When the directional control valve is operated rapidly, a stepwise pressure change in the pipeline occurs. The surge pressures are measured at the upstream end of the pipe by a semi-conductor type pressure transducer and recorded by a Brown tube oscillograph. The pressure oscillation at the downstream end is measured by a strain gauge type pressure transducer and recorded by an electromagnetic oscillograph. The valve stroke is measured by an induction type displacement transducer and recorded by an electromagnetic oscillograph. The pipeline consists of a copper tube of 10 mm ID and 1 mm wall thickness. The distance from the directional control valve to the pipe end is 10.68 m, and distance from the directional control valve to the accumulator is 0.65 m. The closing speed of the hydraulically operated valve with the solenoid cont-

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![Graph showing comparison of calculated results](image)

**Fig. 4 Comparison of the calculated results**

| Solenoid controlled pilot operated valve | 26 | 24 | 29.5 |
| Solenoid operated valve | 15.9 | 11.1 | 12.4 |

**Fig. 6 Main parts of directional control valve**
rolled pilot can be varied within the range from 0.488 m/sec to 0.0408 m/sec, and the speed of the solenoid operated valve is 1.625 m/sec.

Daphne Hydraulic Fluid 444 was used at the temperatures of 20°C and 50°C, and its physical properties are shown in Table 1.

4. Experimental results and discussion

4.1 The surge pressure produced by the valve operation

Figure 7 shows an example of the surge pressure at the upstream end of the pipeline. Figure 8 shows the relations between the surge pressure and the valve operating speed. In this figure, the solid line indicates the result of calculation by Eq. (8), and the broken lines the results of calculation by Eq. (6) for cases $A_d/(A_s + A_d) = 0.702$ (the hydraulically operated valve: $A_s = 1.85 \text{ cm}^2$, $A_d = 0.785 \text{ cm}^2$) and $A_d/(A_s + A_d) = 0.564$ (the solenoid operated valve: $A_s = 1.01 \text{ cm}^2$, $A_d = 0.785 \text{ cm}^2$). The value of $A_d/A_s$ is not zero in practice, so the experimental value of the surge pressure is smaller than that calculated by Eq. (8), and the value calculated by Eq. (6) agrees well with

<table>
<thead>
<tr>
<th>Specific weight</th>
<th>Kinematic viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T \text{ kg/cm}^2$</td>
<td>$\nu \text{ cm}^2/\text{sec}$</td>
</tr>
<tr>
<td>20°C</td>
<td>$0.864 \times 10^{-3}$</td>
</tr>
<tr>
<td>50°C</td>
<td>$0.843 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

**Table 1** Physical properties of the hydraulic fluid

![Surge pressure vs. valve operating speed](image)

![Pressure-time diagram](image)

(a) Oil temperature: 20°C

(b) Oil temperature: 50°C

*Fig. 7 Example of surge pressure*

*Fig. 7 Example of surge pressure*

*Fig. 8 Surge pressure vs. valve operating speed*

*Fig. 9 Pressure-time diagram*
the experimental results.

From these results, it is clear that when the valve closing speed is high and the effective piston area $A_v$ of the valve spool is large, the surge pressure becomes unexpectedly high and often dangerously so. In other words, as seen from Eq. (8) the pressure at the pipe inlet which occurs when the valve is closed becomes higher by $\gamma a_{0}a_{0}/g$ than the pressure stored in the accumulator. After the reflection at the closed end of the pipe, the maximum pressure rise reaches the value of $2(p_0 + \gamma a_{0}a_{0}/g)$. When a valve which does not have a closed valve chamber, for example, a gate valve is used, the oil is not influenced by the valve movement, so that the pressure does not rise by more than $2p_0$. On the other hand, when the spool valve, which has a closed chamber, is used, the extra pressure rise occurs which corresponds to the kinetic energy of the oil given by the spool valve.

4.2 The pressure oscillation at the closed end of pipe

Figure 9 shows the pressure oscillation at the closed end of the pipe. In the figure, the solid lines indicate the experimental results, and the broken lines indicate the results of the calculation by Eq. (24). As seen from the figures, the results of the experiment and those of the calculation agree well.

Equations (9) and (10) indicate that the unsteady characteristics of the transmission line are determined by the pipe line impedance $Z$ and the constant $\lambda$. And Eqs. (11) and (15) indicate that the pressure oscillation at the downstream end depends on the value of $\lambda$. So we can say that the unsteady flow in a pipe is similar in cases when the values of $\lambda$ are the same. The constant $\lambda$ can be considered as a kind of Stokes number\(^{12}\).

When we denote the fundamental frequency by $f_1$, from Eq. (24), we get

$$f_1 = \frac{a_0}{4L} \left(1 - \frac{1}{\sqrt{\lambda^2}}\right)$$

The propagation velocity of the fundamental pressure wave, $a_{01}$, is expressed as follows,

$$a_{01} = a_{0} \left(1 - \frac{1}{\sqrt{\lambda^2}}\right)$$

As seen from this equation, it is a little slower than the wave propagation velocity in the case of nonviscous fluid, $a_{00}$\(^{10}\).

Moreover, when we express the fundamental pressure oscillation by subscript 1, the logarithmic decrement of the pressure fluctuation, $\delta_1$, can be expressed as follows:

$$\delta_1 = -\frac{\pi}{a_1} \frac{2}{a_1} = -\frac{2}{\lambda^2} \frac{\pi}{\sqrt{\lambda^2} + 1}$$

$$\delta_2 = \frac{2}{\lambda} \left(1 + \frac{2}{\sqrt{\lambda^2}}\right)$$

From this equation, we can see that the effect of viscosity on the decrement is more than was expected.

5. Conclusions

The surge pressure and pressure fluctuation at the pipe end caused by the instantaneous operation of a valve in a hydraulic system made up of an accumulator, a directional control valve and a pipeline are discussed theoretically. Agreement between theoretical and experimental results is good enough to validate the theory.

The following conclusions can be drawn from the results.

(1) When the valve attached to one end of the pipe is operated rapidly and a step change in pressure is applied to the pipeline, the surge pressure which is caused by the spool-type directional control valve is affected by the spool velocity. The higher the spool velocity, the larger the surge pressure is.

(2) The period and the logarithmic decrement of pressure fluctuation at the other end are influenced only by a dimensionless quantity $\lambda = a_{0}a_{0}/(\nu L)$; in other words, the pressure fluctuation is similar in cases where the values of this dimensionless quantity are the same.

References

Discussion

T. Ito (Nagoya University):

(1) According to the expression in Chapter 3, the directional control valve is connected to the accumulator by a transmission line 65 cm long. It is a matter of course that the state at the valve position propagates downstream in the pipe, but at the same time it should propagate upstream. What is your opinion about the effects of the propagation upstream on the surge pressure and the pressure rise at the pipe end?

(2) In Fig. 9(a), the period of the pressure oscillation in the experimental result is longer than the period in the calculated result, but in Fig. 9(b) the situation is exactly opposite to the above. On the other hand, the decrements of both results agree well in Fig. 9(a), but there is a noticeable difference in Fig. 9(b). What do you think is the reason? Is it because of the degree of approximation in calculation, or because of the degree of accuracy in the experiment?

Authors’ closure

(1) Assuming the validity of Eq. (8), the pressure pulse of magnitude \( \gamma a_0 v_0 / \rho \) propagates toward the accumulator. After reflection at the accumulator, its magnitude is \( -\gamma a_0 v_0 / \rho \) in the ideal case, and the reflected pulse propagates downstream. However, this reflected pulse propagates later by the delay time \( 2t_1 (t_1 = 0.65/a_1) \) than the pressure change, which is produced by the valve operation and at once propagated downstream. Therefore, this reflected pulse does not affect the magnitude of the surge pressure produced by the valve operation. The pressure change with time at each point in a transmission line varies only a little corresponding to this pulse. Appendix—Fig. 1 shows the pressure-time diagram at the point \( \theta \) in Fig. 3 for the ideal case. Practically all of the kinetic energy which is given to the fluid by the solenoid is so small that the pulse disappears because of viscosity in about one return trip (see Fig. 7).

(2) In the calculation of Eq. (24), we assumed that the wave velocity, \( a_{th} \), is not affected by the fluid temperature and is constant. In reality, however, with the rise of the fluid temperature, the bulk modulus, \( K \), decreases and

\[
a_{th} = \sqrt{\gamma K / \rho / \sqrt{1 + (K / \rho) (2v_0 / \delta)}}
\]

also decreases. Therefore, from Eq. (26) the fundamental frequency of the pressure oscillation at the closed end of the line becomes less than that for the case when \( a_{th} \) is assumed to be constant. Also, with the decrease of \( a_{th} \), \( \lambda = r_0 a_{th} / (vL) \) decreases, and the logarithmic decrement \( \xi \) becomes greater than in the case when \( a_{th} \) is assumed to be constant. For the above reasons, if we consider the effect of the fluid temperature on \( a_{th} \), the calculated results will approach the experimental results. In any event, we think that the problem lies more with the calculated results than with the experimental results. In the calculation of the value of \( a_{th} \), we used the value \( K = 1.61 \times 10^4 \text{kg/cm}^2 \), which is the value for normal mineral oil, since the actual measurement of \( K \) is very difficult.