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Delayed Yielding and Hysteresis Phenomenon of Low Carbon Steel under Tensile Fatigue Load*

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Tensile fatigue tests were carried out on plain specimens of annealed S20C steel with two stress conditions of constant minimum and constant mean stresses, and observations were made on the variations of stress-strain hysteresis and permanent fatigue strain with special attention to the occurrence of delayed fatigue yielding. Marked delayed yieldings were observed when the maximum stress $\sigma_{\max}$ was in the range between the upper and lower yield points of the material, with delayed period ranging from 10^8 to about 5×10^6 . This yielding was characterized by a sudden increase of permanent strain ϵ_f and simultaneous initiation of hysteresis loop. After yieldings, ϵ_f and $\Delta\epsilon_p$, the half width of hysteresis loop, showed characteristic variations according to the magnitude of $\sigma_{\max}$. Discussions were made on these points in some detail, and also on the relation between the total hysteresis energy and the fatigue life.

1. Introduction

Many investigations have been carried out on the relationship between stress-strain hysteresis and fatigue strength of metallic materials under various cyclic loads. Especially, in the fields of low cycle fatigue and thermal fatigue, plastic strain in each cycle is regarded as one of the most important parameters which govern the fatigue strength⁽¹⁾. In the field of ordinary fatigue, the variations of hysteresis loop or cyclic inelastic strain have also been observed by many researchers and discussions have been made on their relation with fatigue strength^{(2)~(5)}. But these previous observations were mostly made under fully reversed cyclic stress conditions, and, consequently, no attention was usually paid to gross deformation or permanent fatigue strain. Such a deformation, however, becomes important when a material is subjected simultaneously to mean stress and cyclic stress as in the case of repeated tension with zero minimum stress. Therefore, under such a stress condition, simultaneous observation of the cyclic inelastic strain and the gross deformation of specimen must be an interesting research subject.

The authors have already reported a study on these strain behaviors of S20C steel under tensile fatigue loads⁽⁶⁾. In this previous study, however, since all the stress levels in the fatigue test were set at

their maximum values above the upper yield point of the material, yielding occurred in the first few cycles and specimens showed hysteresis phenomena from the beginning of the tests. Consequently, the lower limit of the fatigue yielding stress was not found, nor the strain behaviors immediately before and after the fatigue yielding were observed.

The present study was planned to clarify these points. And some modifications were made on detecting devices of inelastic cyclic strain and permanent fatigue strain to improve the accuracy of small strain measurement. Tensile fatigue tests were again carried out on the same steel, but this time, the maximum value of cyclic stress was set at several levels above and below the upper yield point.

One of the interesting results was that the delayed yielding under fatigue load, which has already been observed by several researchers on its occurrences^{(7)~(9)}, was successfully observed over a wide range of numbers of cycles from 10^8 to about 5×10^6 , and this yielding occurred quite abruptly, as in the case of static tension tests, with simultaneous initiation of an appreciable hysteresis loop. Discussions were made on this sort of delayed yielding and on the subsequent strain behaviors in some detail, as well as on the relation between hysteresis energy and fatigue strength.

2. Material and experimental procedures

Annealed S20C carbon steel was used in the experiment. The chemical composition of the material is listed in Table 1. The heat treatment and mechanical properties are given in Table 2. Specimens

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were machine finished after heat treatment to the shape shown in Fig. 1. A double-lever type tension fatigue testing machine was used with a stress frequency of about 1100 c/min.

Techniques of detecting the applied load and strains of the specimen are illustrated in Fig. 2. The load was picked up by means of the wire strain gauges attached to the loop dynamometer, and cyclic strain of the specimen was picked up by the wire strain gauges attached to the specimen itself. The outputs of these strain gauges were given to the bridge circuits shown in the figure, and then led to the vertical and horizontal axes of a synchroscope after amplification to describe a stress-strain hysteresis loop. Difficulty accompanying this technique of cyclic strain measurement was that, when yielding occurred during the test, with a sudden and large variation of the gauge resistance taking place, the bridge circuit came

Fig. 1 Specimen

Fig. 2 Pickups of load and strains

Table 1 Chemical composition of the material (%)

C	Si	Mn	P	S	Ni	Cr	Cu
0.22	0.29	0.51	0.010	0.021	0.02	0.04	0.10

Table 2 Heat treatment and mechanical properties of the material

Heat treatment	Upper yield point σ_{s0} (kg/mm ²)	Lower yield point σ_{su} (kg/mm ²)	Ultimate strength σ_B (kg/mm ²)	Breaking strength on final area σ_T (kg/mm ²)	Elongation ϕ (%)	Reduction of area ψ (%)	Young's modulus E (kg/mm ²)
880°C, 1hr. F.C.	32.0	24.9	42.9	82.8	31.1	60.3	2.07×10^4

out of balance, and this made the subsequent measurement impossible. In order to overcome this difficulty, a few specimens carrying a pair of strain gauges were prepared, each of which was deformed to a different level of plastic strain to introduce some amount of initial variation of gauge resistance in the attached strain gauges, and a pair of these gauges were used as dummy gauges and exchanged for one of the other pairs according to the strain level of active gauges attached to the specimen under the test. This procedure was used whenever a large plastic strain occurred in the specimen even after yielding.

Figure 3 illustrates a hysteresis loop obtained on the synchroscope. The load was calibrated by plotting the moving distance of the light spot against the load statically applied to the dynamometer, and the strain was determined using the linear relation between stress and strain with the value of E given in Table 2. The linear relation can easily be realized by applying a stress lower than the yield point to the specimen. The hysteresis loops thus obtained were photographed and analyzed by a magnifying projector. Two important parameters involved in the hysteresis loop are the maximum horizontal width of the loop and the area enclosed by the loop. The former was denoted by $2\Delta\epsilon_p$, and $\Delta\epsilon_p$ was called here plastic strain amplitude. The latter gives the energy dissipated in a cycle of stress repetitions, which was here approximated by the area of a parallelogram ABCD shown in Fig. 3, multiplied by a correcting factor k . k was about 0.7 and this value slightly differed from 0.75 obtained by G. R. Halford⁽⁵⁾. The total energy W dissipated until failure was then given by the following formula.

$$W = k(4\sigma_a \sum \Delta\epsilon_p) = 2.8\sigma_a \sum \Delta\epsilon_p \dots \dots \dots (1)$$

Next, the elongation of specimen was measured using a differential transformer mounted on the

Fig. 3 Illustration of hysteresis loop

shoulders of the specimen as shown in Fig. 2. The permanent fatigue strain ϵ_f was computed by dividing the obtained elongation by the gauge length and then by subtracting the elastic strain within the gauge length.

Fatigue tests were performed under repeated tensions either with a constant minimum stress of 2.0 kg/mm² or with a constant mean stress of 18.0 kg/mm², and the stress amplitudes were so adjusted that the maximum stress $\sigma_{\max}$ covered a certain range including the upper yield point.

3. Experimental results

The maximum stress $\sigma_{\max}$ applied to specimens and the number of cycles to failure N obtained by fatigue tests are listed on the first and third columns of Table 3(a) and (b), respectively, for the conditions of constant minimum stress and constant mean stress. These results are plotted with hollow circles on S - N diagrams of Figs. 4 and 5, respectively. One specimen tested at $\sigma_{\max}=31.5$ kg/mm² with constant mean stress did not fail at 10^7 cycles, but, from the variation of plastic strain amplitude, the failure could be expected to occur at some finite number of cycles when the test was continued. Therefore, the fatigue limit in the case of constant mean stress was decided

as 31.0 kg/mm² in $\sigma_{\max}$ as shown in Fig. 5.

In Figs. 6 and 7, the variations of plastic strain amplitude $\Delta\epsilon_p$ during fatigue tests are presented for respective stress conditions, by plotting $\Delta\epsilon_p$ against the logarithm of number of stress cycles n . Also, in Figs. 8 and 9, the variations of permanent fatigue strain ϵ_f are presented in the same manner.

Now, as indicated in Figs. 3 and 4, several specimens were tested at stress levels below the fatigue limit and careful observations were made on their strain behaviors. An interesting behavior observed at these stress levels was that the specimen remained

Fig. 4 S - N and S - N' diagrams ($\sigma_{\min}$ is constant)

Fig. 5 S - N and S - N' diagrams (σ_m is constant)

Table 3 Experimental results

Maximum stress $\sigma_{\max}$ (kg/mm ²)	Number of stress cycles to yielding N'	Number of stress cycles to failure N	Total hysteresis energy W (kgmm/mm ³)	Permanent fatigue strain ϵ_{fo} (%)
(a) $\sigma_{\min}=2.0$ kg/mm ²				
32.5	$<1.00 \times 10^3$	2.08×10^5	—	5.49
32.0	$<1.00 \times 10^3$	2.76×10^5	5.85×10^2	4.37
31.0	$<1.00 \times 10^3$	3.95×10^5	8.30×10^2	4.27
30.0	$<1.00 \times 10^3$	8.00×10^6	9.72×10^3	2.45
30.0	1.01×10^3	1.25×10^6	2.37×10^3	2.95
29.5	7.50×10^3	7.90×10^5	1.32×10^3	2.85
29.5	5.15×10^6	7.16×10^6	3.37×10^3	3.15
29.0	5.50×10^3	4.80×10^6	3.92×10^3	1.83
29.0	4.16×10^4	1.62×10^6	2.63×10^3	2.65
28.5	$>1.03 \times 10^7$	$>1.03 \times 10^7$	0	0
28.0	2.87×10^5	$>1.21 \times 10^7$	$>8.18 \times 10^3$	1.90
28.0	7.11×10^5	$>1.00 \times 10^7$	$>9.09 \times 10^3$	1.73
27.5	3.20×10^6	$>1.00 \times 10^7$	$>6.92 \times 10^3$	1.81
27.0	$>1.13 \times 10^7$	$>1.13 \times 10^7$	0	0
(b) $\sigma_m=18.0$ kg/mm ²				
32.5	$<1.00 \times 10^3$	1.17×10^6	2.11×10^3	3.64
32.0	$<1.00 \times 10^3$	4.70×10^5	8.69×10^2	4.38
31.5	$<1.00 \times 10^3$	$>1.00 \times 10^7$	1.05×10^4	5.07
31.0	$<1.00 \times 10^3$	$>1.00 \times 10^7$	—	3.15
30.5	6.05×10^5	$>1.01 \times 10^7$	$>6.12 \times 10^3$	3.03
30.0	$<1.00 \times 10^3$	$>1.09 \times 10^7$	$>8.13 \times 10^3$	2.82
29.5	6.01×10^4	$>1.35 \times 10^7$	$>5.04 \times 10^3$	2.32
29.0	$>1.20 \times 10^7$	$>1.20 \times 10^7$	0	0
28.5	$>1.00 \times 10^7$	$>1.00 \times 10^7$	0	0
28.0	$>1.00 \times 10^7$	$>1.00 \times 10^7$	0	0

Fig. 6 $\Delta\epsilon_p$ - n diagram ($\sigma_{\min}$ is constant)

elastic as a whole in the early stage of the test, but, after a certain number of cycles, yielding suddenly occurred accompanied by an abrupt increase of permanent strain and simultaneous initiation of a hysteresis loop. In other words, typical delayed yieldings under cyclic stress were clearly observed. In Figs. 6~9, such discontinuous strain behaviors are indicated by vertical broken lines. In Fig. 10, three records of hysteresis loops are shown, which were taken before and after delayed yielding: photograph (a) shows a linear relation between stress and strain, which means the specimen was elastic, photograph (b) shows the hysteresis loop immediately after yielding which occurred at 7.11×10^5 cycles, and this hysteresis loop then gradually decayed under subsequent stress cycles as shown in photograph (c).

Values of number of cycles N' at which a delayed yielding occurred are listed in the second column of Table 3, and are plotted on the previous S - N diagrams with solid circles (Figs. 4 and 5). S - N' curves were drawn through these points up to 10^7 cycles, at which all the tests were discontinued under the assumption that the specimen showing no yielding until 10^7 would never yield in further repetitions of stress. On the other hand, at higher stress levels than those which caused a delayed yielding, some amounts

of $\Delta \epsilon_p$ and ϵ_f were observed in the first strain measurements. In these cases, yielding is thought to have completed immediately after $\sigma_{\max}$ reached the prescribed value. This fact is clear from the comparison of levels of $\sigma_{\max}$ with the upper yield point given in Table 2. The solid circles plotted on the left side ends in Figs. 4 and 5 represent these cases, and, in the following discussion, they are all treated as having yielded before 10^8 cycles.

Now, regarding the variation of plastic strain

Fig. 8 e_f - n diagram ($\sigma_{\min}$ is constant)

Fig. 7 $\Delta \epsilon_p$ - n diagram (σ_m is constant)

Fig. 9 e_f - n diagram (σ_m is constant)

(a) $n = 6.0 \times 10^5$ (b) $n = 8.0 \times 10^5$ (c) $n = 1.0 \times 10^7$
 $\sigma_{\min} = 2.0 \text{ kg/mm}^2$, $\sigma_{\max} = 28.0 \text{ kg/mm}^2$, $N' = 7.11 \times 10^5$, $N > 1.00 \times 10^7$
 Fig. 10 Examples of recorded hysteresis loop

amplitude $\Delta\epsilon_p$ after yielding, a marked decrease of $\Delta\epsilon_p$ is seen from Figs. 6 and 7 in such cases where the specimens showed a delayed yielding after 10^8 cycles but did not fail. But, in those cases where specimens failed before 10^7 cycles, decay of $\Delta\epsilon_p$ was not so rapid but rather gradual even after delayed yielding, and, especially, with the specimens that had yielded before 10^8 cycles, $\Delta\epsilon_p$ showed no marked decrease despite the fact that $\Delta\epsilon_p$ itself had a large value. From these common trends of $\Delta\epsilon_p$, it is possible to predict that the specimen tested with $\sigma_{max}=31.5 \text{ kg/mm}^2$ and $\sigma_m=18.0 \text{ kg/mm}^2$, which

had not failed at 10^7 cycles, would fail at some finite number of cycles after 10^7 if the test was continued further, since $\Delta\epsilon_p$ showed no marked decrease as was seen in Fig. 7. Therefore, the fatigue limit in the case of $\sigma_m=18.0 \text{ kg/mm}^2$ was decided to be 31.0 kg/mm^2 , as mentioned before. In Figs. 11 and 12, the variations of $\Delta\epsilon_p$ are replotted by taking the cycle ratio n/N on the abscissa with uniform scale for further discussion in the later section, where N is the number of cycles to failure for the failed specimens and is 10^7 for run out specimens.

Next, concerning the permanent fatigue strain ϵ_f , it is observed from Figs. 8 and 9 that ϵ_f increased continuously when σ_{max} was large and a failure occurred in a relatively short period, whereas for runout specimens the increase of ϵ_f was rather suppressed and ϵ_f became almost constant at about 10^7 cycles. The final value of ϵ_f was denoted by ϵ_{f0} , and their values are listed in the fifth column of Table 3. Here, ϵ_{f0} is either the value of ϵ_f just before failure or that at 10^7 cycles.

4. Discussions of the experimental results

4.1 Phenomenon of delayed fatigue yielding

As far as the occurrence of delayed yieldings under cyclic stresses is concerned, there have been several reports on its observations with carbon steels having distinct upper and lower yield points under static load, when they are subjected to repeated tensile stresses whose maximums are in the vicinity of static yield points. However, since these observations were made within a narrow range of numbers of cycles less than about 10^4 , the following problems have not yet been studied: whether delayed yielding may occur after a large number of cycles such as several million cycles, and what the accompanied strain behaviors and their mutual relations are⁽⁷⁾⁻⁽⁹⁾.

It is now clear from the present study that delayed yielding does occur even in the range of numbers of cycles from 10^4 to about 5×10^6 , and it is accompanied by a sudden increase of permanent strain and a simultaneous initiation of hysteresis loop. Furthermore, the number of cycles until yielding can be related to σ_{max} by $S-N'$ curves as shown in Figs. 4 and 5.

These $S-N'$ curves can be regarded as a sort of damage lines that indicate some degrees of accumulation of damage in materials which occurs in the course of fatigue. Many attempts have been made to construct such damage lines based on the observations of some physical and metallurgical changes occurring in materials during fatigue process⁽¹⁰⁾⁻⁽¹⁵⁾. In Fig. 13, one of such investigations, carried out by B. C. Mahoba, is shown, in which the fatigue process of low

Fig. 11 $\Delta\epsilon_p$ - n/N diagram (σ_m is constant)

Fig. 12 $\Delta\epsilon_p$ - n/N diagram (σ_{min} is constant)

carbon steel was divided into four periods based on microscopic observations. These are (I) the latent period for accumulation of lattice distortions, (II) the period for initiation and growth of sub-micro-cracks to micro-cracks, (III) the period for growth of micro-cracks to macro-cracks of critical size, and (IV) the period for unstable propagation of macro-cracks (this period is often negligible for small specimens). The lower two curves in Fig. 13 correspond to the initiations of the periods (II) and (III), respectively⁽¹⁵⁾.

Now, referring again to the S - N' curves in Figs. 4 and 5, it can be supposed that lattice distortions and local preparatory slips⁽¹⁶⁾ are developing continuously during the first period until the number of cycles reaches N' , although the specimen remains elastic as a whole during this period; and when their accumulations reach some critical values, a gross yielding occurs instantaneously over the gauge length of the specimen. In this sense, the S - N' curves in Figs. 4 and 5 are thought to correspond to the lowest curve in Fig. 13. Further discussion is, however, difficult at present, since most of those observations including that of the authors have been made by focusing on only one or two parameters among many influencing factors, and have traced their variations during fatigue

Fig. 13 Damage lines of steel (plane bending)

Fig. 14 σ_a - σ_m diagram

process, and therefore mutual relations among many factors cannot be established nor results of different investigators are well incorporated together.

In Fig. 14, boundary lines for the occurrences of yielding and failure are drawn by taking stress amplitude σ_a and mean stress σ_m on respective axes. The heavy line indicates the fatigue limit below which no failure occurs. The highest and lowest lines inclined at 45° to both axes represent the upper and lower yield points and the middle line between them indicates the lower limit of stresses above which yielding occurs before 10^8 cycles in fatigue test. Therefore, in the domain below this line, a delayed fatigue yielding can be observed, and thus the maximum stresses causing a delayed yielding are located between the upper and lower yield points. Their lower limit is, however, somewhat higher than the lower yield point, and this fact differs from the conclusions in the previous reports that the fatigue yielding stress is approximately equal to the lower yield point for many carbon steels which show distinct yielding under static load⁽⁸⁾, although some exceptional cases have also been reported⁽⁷⁾.

Contrary to the delayed yielding under cyclic stress, there have been many investigations on the delayed yielding under statically sustained stress. Among others, D. C. Clark & D. S. Wood⁽¹⁷⁾, I. R. Kramer & R. Maddin⁽¹⁸⁾ and J. M. Krafft⁽¹⁹⁾ have reported that the relations between sustained stress σ and delayed time t until yielding could be represented by σ - t curves just similar to S - N' curves in Figs. 4 and 5, and explanations were given based on theories of dislocation⁽²⁰⁾. However, delayed time t in these observations was at most 10 sec and the lowest stress causing a delayed yielding was approximately equal to the upper yield point of material. Therefore, it is difficult to find any explicit relation between the delayed yielding under static load and that under cyclic load, for, in the latter, the yielding can occur even after millions of cycles and at the stress between upper and lower yield points. But there was an interesting report that a delayed yielding could occur under static stress below the upper yield point with a certain probability⁽²¹⁾, and this sort of phenomenon is thought to be correlated rather closely to the delayed yielding under cyclic load.

For the further discussion, in Fig. 15, the final values of permanent fatigue strain, ϵ_{f0} , are presented in relation to σ_{max} and are compared with the stress-strain curve under static load. It is seen that the results for both conditions of constant minimum stress and constant mean stress are located together in the indicated scatter band, and in the range $0 < \epsilon_{f0} < 1.7\%$, there exists no experimental point. This means that this range is equivalent to the flat part at lower

yield stress in a static stress-strain curve. Moreover, as indicated by the experimental points located on the ordinate, the occurrence of yielding seems to have some probabilistic nature also under cyclic stress, when the applied stress is comparatively low.

4.2 Classification of fatigue processes according to strain behaviors

As were explained in 3, the plastic strain amplitude $\Delta\epsilon_p$ and the permanent fatigue strain ϵ_f showed characteristic behaviors according to the value of $\sigma_{\max}$

Fig. 15 $\sigma_{\max}$ - ϵ_{f0} diagram

Fig. 16 Classification of strain behaviors according to $\sigma_{\max}$

in tensile fatigue tests. In order to examine this point more in detail, S - N and S - N' curves and the modes of variations of $\Delta\epsilon_p$ and ϵ_f are represented in Fig. 16 schematically. And three stress levels σ_I , σ_{II} and σ_{III} are defined, where σ_I is the upper limit of stress which causes no yielding until 10^7 cycles, σ_{II} is the fatigue limit and σ_{III} is the lower limit of stress above which yielding occurs before 10^3 cycles. σ_{III} is approximately equal to the upper yield point of material from Fig. 14. Then, in each stress range bounded by σ_I , σ_{II} and σ_{III} , the material shows characteristic strain behaviors as shown in diagrams (b) and (c). These can be specified as follows.

(I) $\sigma_{\max} \leq \sigma_I$: In this stress range, no hysteresis loop nor permanent fatigue strain occurs even at 10^7 cycles, and the material remains elastic from a macroscopic point of view. Stress σ_I , which is always lower than the fatigue limit σ_{II} , is thought to correspond to the "ultimate endurance limit" suggested by M. S. Hunter & W. G. Fricke⁽¹³⁾.

(II) $\sigma_I < \sigma_{\max} < \sigma_{II}$: In this range, the delayed fatigue yielding occurs at a number of cycles less than 10^7 , but no failure occurs. Plastic strain amplitude $\Delta\epsilon_p$ sharply decays in the subsequent stress cycles as shown in diagram (b). In other words, a typical cyclic hardening of material is observed⁽²²⁾. $\Delta\epsilon_p$ attains a certain steady value at about 10^7 cycles but never disappears, and this means that a steady dissipation of energy is taking place even at and after 10^7 cycles. Consequently, this energy dissipation seems to have no contribution to fatigue failure. On the other hand, permanent fatigue strain ϵ_f , which was simultaneously initiated with hysteresis loop, shows no marked increase but remains almost constant during the subsequent stress cycles.

(III) $\sigma_{II} < \sigma_{\max} \leq \sigma_{III}$: In this stress range, a delayed yielding is also observed after 10^3 cycles, but the specimen fails at a finite number of cycles. Sharp decay of $\Delta\epsilon_p$ is not observed but it decreases rather gradually, and ϵ_f increases continuously up to failure.

(IV) $\sigma_{III} < \sigma_{\max}$: In this highest stress range, yielding is completed before 10^3 cycles and the specimen fails at less number of cycles. The variations of $\Delta\epsilon_p$ and ϵ_f are similar to those in (III), but the increase of ϵ_f is more rapid.

It is only remarked here that all the practical variations of strain observed in the experiment cannot always be classified according to the above description. Some specimens showed mixed characters in their strain behaviors, and these are thought to belong to a transient or boundary zone in the above classification.

Now it is interesting to examine the relations between strain behaviors and fatigue damage in the stress range $\sigma_{\max} > \sigma_I$ in which failure occurs. First of all, in the range $\sigma_I < \sigma_{\max} \leq \sigma_{II}$, since no failure

occurs, neither $\Delta\epsilon_p$ nor hysteresis energy should contribute to the accumulation of fatigue damage sufficient to cause failure, and after a large number of cycles, a steady energy dissipation takes place independently of fatigue damage as mentioned before. On the other hand, since the permanent fatigue strain ϵ_f shows no increase but takes a constant value near 10^7 cycles, the hysteresis loop should be completely closed, and this means the strain behavior is almost anelastic after a large number of cycles. Consequently, it is concluded that anelastic strain does not contribute to the fatigue failure. This conclusion agrees with that of C. E. Feltner & J. D. Morrow, who asserted that inelastic strain was composed of plastic and anelastic strains and the latter has no effect on fatigue failure⁽²³⁾.

Next, for stress $\sigma_{\max} > \sigma_L$, $\Delta\epsilon_p$ shows no marked decrease and ϵ_f increases steadily up to failure including the case when a delayed yielding occurs. These facts indicate two things: that slips are developing continuously, and that the hysteresis curve is no longer closed. Therefore, some amount of plastic strain due to slips should be involved in $\Delta\epsilon_p$ together with anelastic strain, and accumulation of the former component necessarily constitutes a fatigue damage causing failure. These reasonings are partly justified by observing the variations of $\Delta\epsilon_p$ and ϵ_f in the case of $\sigma_{\max} = 30.0 \text{ kg/mm}^2$ plotted in Figs. 6 and 8, where the decrease of $\Delta\epsilon_p$ was rather gradual when ϵ_f was increasing, but the decrease of $\Delta\epsilon_p$ was accelerated when ϵ_f ceased to increase. Thus, it is understood that the variations of $\Delta\epsilon_p$ and ϵ_f are closely correlated to each other.

For further discussion, it is necessary to separate quantitatively the respective components included in $\Delta\epsilon_p$. But at present there is no effective method for separation, and this is the major reason why $\Delta\epsilon_p$, the half width of hysteresis loop, is handled as a whole and called the plastic strain amplitude in relation to the previous reports. Simultaneous microscopic observations would be helpful to justify the above reasoning, especially on the relationship between development of slips and fatigue damage. But these are the problems left for future study.

4.3 Relation between total hysteresis energy and fatigue life

Although anelastic component involved in $\Delta\epsilon_p$ is thought to have no effect on fatigue damage, as was pointed out in the previous section, $\Delta\epsilon_p$ still remains as one of the important parameters in the discussion of fatigue damage, since quantitative evaluations can be made in terms of $\Delta\epsilon_p$ or in terms of hysteresis energy derived from $\Delta\epsilon_p$ and stress amplitude σ_a .

In this study, the total hysteresis energy W defined

by Eq. (1) is adopted as a parameter of fatigue damage, and W was computed by numerically integrating the area below respective curves in Figs. 11 and 12, and then by multiplying N . The values of W thus obtained are listed in the fourth column in Table 3, and are plotted against $N-N'$ in Fig. 17, where $N-N'$ means the number of cycles from yielding to failure. The data presented in the author's previous report on the same material are also plotted together⁽⁶⁾. Including the previous data, most of the results are located in the indicated scatter band, and no discrepancy is observed between the results of different stress conditions. The linear relation drawn through the middle of the scatter band is given by the following formula.

$$W = 7.19 \times 10^2 (N - N')^{0.73} \quad \dots\dots\dots (2)$$

It should be noted here that, by plotting W against $N-N'$, especially the data obtained in the case when a delayed yielding occurred show better agreement with the others within the scatter band, than by merely plotting W against total life N . The relation given by Eq. (2) coincides in the tendency with those obtained in the previous studies by several authors⁽³⁾⁻⁽⁶⁾.

5. Conclusions

Tensile fatigue tests with constant minimum and constant mean stresses were carried out on plain specimens of annealed low carbon steel, and observations were made on the variations of stress-strain hysteresis and permanent fatigue strain with special attention to the occurrence of delayed fatigue yielding.

Major conclusions obtained are as follows.

(1) Marked delayed yieldings were observed when the maximum stress $\sigma_{\max}$ was in the range between the upper and lower yield points of the material, with delayed periods ranging from 10^3 to

Fig. 17 $W-(N-N')$ diagram

about 5×10^6 . During this period, a specimen remained macroscopically elastic, and after this period, yielding occurred with a sudden increase of permanent fatigue strain ϵ_f more than 1.7%, and with simultaneous origination of hysteresis loop. Thus, the delayed yielding could be observed very distinctly.

(2) $S-N'$ curves similar to $S-N$ curves could be established between the applied stress and the number of cycles to yielding, and this relation is thought to be one of damage lines representing a certain extent of damage accumulation in the material. But systematic analyses and interpretations of various damage lines obtained by many investigators are problems left for future study.

(3) Strain behaviors of low carbon steel under cyclic tensile stress can be classified into four groups on the basis of the magnitude of $\sigma_{\max}$ as illustrated in Fig. 16, and the material shows specific strain behaviors such as a delayed yielding and a cyclic hardening in respective stress ranges.

(4) Close relationship does exist between the variations of permanent fatigue strain ϵ_f and plastic strain amplitude $\Delta\epsilon_p$, and it is concluded that anelastic strain involved in $\Delta\epsilon_p$ has no effect on fatigue failure, and failure is caused only by the accumulation of the plastic strain component in $\Delta\epsilon_p$ due to slips.

(5) The relation expressed by Eq. (2) holds between the total hysteresis energy W and the number of cycles, $N-N'$, and this relation coincides in the tendency with those obtained by several authors. By adopting $N-N'$, the number of cycles from yielding to failure, as a parameter instead of N , data are better represented by Eq. (2) including the cases when a delayed yielding occurred.

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Discussion

H. NISITANI (Kyushu University):

(1) It is supposed that the hysteresis energy represents an average fatigue damage, and the strength of each grain or degree of its imperfection has a certain distribution around the average value. Therefore, these facts should be taken into account in the evaluation of the relation between hysteresis energy and fatigue life. I would like to know your opinion on this point.

(2) Calibration of the dynamometer was made only by means of static load in this study. Please let

me know the magnitude of an additional load which may develop under vibration due to the mass located between the dynamometer and the specimen. (Is there any possibility that the upper limit of a fatigue yielding stress approaches the lower yield point of material?)

Authors' closure

(1) It is naturally supposed that the value of measured hysteresis energy is an average value of those of respective grains in a polycrystalline metal, and each

grain shows its own hysteresis phenomenon which may distribute around the average one, just as you pointed out. Consequently, it is very advantageous to know its distribution for the evaluation of fatigue strength, since failure originates from the weakest grain. Authors are now preparing to observe such a localized phenomenon by means of X-ray, and also preparing to make an observation of deformations of grain boundaries as well as inside the grain using specimens of large grain size.

(2) Since the testing machine used in this

experiment was designed to increase the rigidity of the part involving a loop dynamometer and the specimen, the inertia effect of this part can be neglected. Therefore, the additional load due to the inertia force was not taken into account in the experiment. But, since the static yield point is in general affected by the loading speed, it is expected that the upper limit of fatigue yielding stress approaches more or less the lower yield point, if the monotonic tension test is performed at the same loading speed as the fatigue test.