621-522 : 532.59

Surge Absorption by Accumulator*

(Study in the Case of Air Content in Oil)

By Ken ICHIRYU**

In the hydraulic system with pressure source cylinders are in many cases operated by opening an electro-magnetic valve.

In this case, when switching time of the valve is short, a surge wave is generated and it damages the equipment.

The author investigated the method of surge absorption by means of accumulators.

In a hydraulic system, mixing of air into oil happens easily, and surge pressure is influenced considerably by it.

By theoretical and experimental study, the author clarified that surge increase resulted from the fact that the oil column, stiffened by the mixture of air into oil, took on the characteristic of a hard spring.

Supposing that the inertia of the cylinder is large enough, the hydraulic system is restricted to the fixed end condition.

1. Introduction

In a hydraulic system in which quick cylinder operation is required, in many cases large accumulators are provided as a hydraulic source.

In such a system, by rapid opening of an electro-magnetic valve, a surge is generated in the pipe, as a result of rapid discharge of oil from the accumulator of hydraulic source.

Factors which influence the magnitude of the surge are:

(1) Quantity of air content in oil. (2) open time of electro-magnetic valve (3) fluid resistance of Acc(abrev. of accumulator) for oil feed.

The author discusses these factors.

As a method of absorbing the surge positively, the method of inserting a small volume Acc in the conduit is available and the author studied the influences of the factors on the magnitude of surge by using the method of characteristics for surge calculation. The method itself is not new, but examples of systematic study of factors which influence the surge in oil hydraulic system, especially the effect of air content and the volume of Acc are few. In this research the author restricted his study to the cases of conduit termination with fixed end condition, supposing that the inertia of

cylinder is sufficiently large.

Nomenclature used in the following are shown below.

Acc: accumulator

Accp: accumulator for oil feed

Accd: accumulator for surge absorption

B : bulk modulus of oil (pipe's flexibility is ignored assuming it is sufficiently large) kg/cm^2
 $1.67 \times 10^4 \text{ kg/cm}^2$

B_a : bulk modulus of air kg/cm^2 , N_p

c : sound velocity (case of general use)

$c_{\max}$: sound velocity of $P \rightarrow \infty$ at air content in oil cm/sec

c_L, c_R : sound velocity of left and right medium at mesh point $N \text{ cm/sec}$

c_v, c_{pa}, c_{ad} : discharge coefficient of orifices supposed at directional valve, Accp and Accd respectively.

n : polytrope index

$NA, NB, \dots, NF$: mesh numbers of conduits

P_m : mean line pressure kg/cm^2

$P_{\max}$: maximum pressure of surge generated at conduit end

P : pressure kg/cm^2

R : fluid resistance (laminar flow) $= \text{sec}^{-1}$

R_e : Reynold number

r : pipe radius cm

S_p : pipe sectional area cm^2

S_v : valve opening cm^2

* Received 18th December, 1970.

** Senior Researcher, Hitachi Research Laboratory, Hitachi Ltd, Hitachi.

- S_{a1}, S_{a2} : orifice's sectional area of Accp and Accd
 t : time sec
 t_0 : valve open time sec
 u : flow velocity cm/sec
 u_{max} : maximum flow velocity discharged from Accp (converted value to pipe flow velocity) cm/sec or m/sec.
 v_1, v_2 : volume of Accp and Accd cm^3
 x : position coordinate cm
 β : pressure rise ratio by surge
 Δt : time mesh at calculation sec
 δ : opening ratio of Acc, S_a/S_p
 ϵ_0 : air content ratio at atmospheric pressure
 ρ : oil's density $\text{kg sec}^2/\text{cm}^4$
 ν : oil's kinematic viscosity cm^2/sec

2. Calculation method of surge

Difference between oil hammer and water hammer lies in the two facts, that is, (a) the former has larger viscosity and extinction of surge is more rapid and (b) in the former case, air is more easily mixed and it is difficult for air to escape from the air pocket.

Calculation of oil hammer is reduced to a solution of wave propagation equation covering the influences of viscosity and air content.

As this solution, the author used the well known method of characteristic⁽¹⁾⁽²⁾. Also, in the following, since Reynold number R_e is below 3 000, the laminar flow resistance is considered and the viscosity term is supposed to be proportional to the mean flow velocity.

In this case, wave equations are⁽³⁾

$$\frac{\partial P}{\partial x} = -\rho \frac{\partial u}{\partial t} - \rho R u, \quad \frac{\partial u}{\partial x} = -\frac{1}{B} \frac{\partial P}{\partial t} \quad \dots (1)$$

Solutions of these equations P, u by the method of characteristic are as shown below;

$$\left. \begin{aligned} dx - c dt &= 0 \\ dP + \rho c du + \rho R u dx &= 0 \end{aligned} \right\} \text{Progressive wave} \quad \dots (2)$$

$$\left. \begin{aligned} dx + c dt &= 0 \\ dP - \rho c du + \rho R u dx &= 0 \end{aligned} \right\} \text{Backward wave} \quad \dots (3)$$

From the intersection point of characteristic curves of progressive and backward waves given by Eqs. (2) (3), solutions of P, u are attained. The object of calculation is a system as shown in Fig. 1, which has a high pressure conduit at the left side of a directional valve, having Accp for oil supply and has a low pressure conduit (atmospheric pressure) at the right side of the valve with fixed end having surge absorption Accd at

Fig. 1 Object of calculation

the intermediate position.

2.1 Case of no air content in oil

In this case, medium is equal at both ends of point N, and choosing distance of $N-1, N$ and $N, N+1$ as $\Delta x = c \Delta t$, Eqs of progressive and backward waves are respectively,

$$\left. \begin{aligned} P(N, 2) &= P(N-1, 1) - \rho c U(N, 2) \\ &\quad + \rho c (1-\gamma) U(N-1, 1) \\ P(N, 2) &= P(N+1, 1) + \rho c U(N, 2) \\ &\quad - \rho c (1-\gamma) U(N+1, 1) \end{aligned} \right\} \dots (4)$$

where, at $P(N, M), U(N, M), N$ in the bracket means mesh point, and M means time, namely, $M=1, 2$ corresponds present time, and time after Δt respectively. Also, $\gamma = 8\pi\nu\Delta t/S_p$ is a term representing viscous friction. From Eq. (4), the fundamental equation for uniform conduit becomes as follows.

$$\left. \begin{aligned} P(N, 2) &= \frac{1}{2} \{P(N-1, 1) + P(N+1, 1)\} \\ &\quad + \frac{1}{2} \rho c (1-\gamma) \{U(N-1, 1) - U(N+1, 1)\} \\ U(N, 2) &= \frac{1}{2\rho c} \{P(N-1, 1) - P(N+1, 1)\} \\ &\quad + \frac{1}{2} (1-\gamma) \{U(N-1, 1) + U(N+1, 1)\} \end{aligned} \right\} \dots (5)$$

2.2 Case of uniform air content in oil

Sound velocity c of the medium of uniform air content, denoting the sound velocity of oil itself as c_{max} , is represented approximately as below⁽⁴⁾⁽⁵⁾.

$$\left. \begin{aligned} c &= c_{max} \sqrt{\frac{1}{(1-\epsilon) + (B/B_a)\epsilon}} \\ \epsilon &= \frac{\alpha}{1+\alpha}, \quad \alpha = \frac{\epsilon_0}{1-\epsilon_0} \frac{P_0}{P} \end{aligned} \right\} \dots (6)$$

In this case, let's consider the state of wave propagation in Fig. 2(b). By taking the case of maximum sound velocity (sound velocity of oil only) as standard, $\Delta x = c_{max} \Delta t$ is chosen as reference mesh.

In order to determine state variables of mesh N, it is required to obtain sound velocity in the neighborhood of N point.

It is supposed that sound velocities at left and right position of N point are determined by mean pressure P_{ML} of $N-1$ and N stations Δt earlier, and by mean pressure P_{MR} of N and $N+1$ stations Δt earlier, respectively.

(a) Uniform medium without air content

(b) Case of air content

Fig. 2 Mesh of conduit

$$\left. \begin{aligned} P_{MR} &= \frac{1}{2}(P(N, 1) + P(N+1, 1)) \\ P_{ML} &= \frac{1}{2}(P(N-1, 1) + P(N, 1)) \end{aligned} \right\} \dots\dots\dots (7)$$

Supposing sound velocities c_L and c_R of left and right side of point N, are determined from Eq. (6), $c_L = f(P_{ML})$, $c_R = f(P_{MR})$ are obtained. From above, distances of wave propagation $X_R = c_R \Delta t$, $X_L = c_L \Delta t$ are obtained.

Next, pressure and velocity at A, B position of wave propagation Δt earlier are assumed by 1st order interpolation. That is, taking P_R, U_R as pressure and velocity of point B and P_L, U_L as equal values of point A,

$$\left. \begin{aligned} P_R &= P(N, 1) + \frac{x_R}{\Delta x} \{P(N+1, 1) - P(N, 1)\} \\ U_R &= U(N, 1) + \frac{x_R}{\Delta x} \{U(N+1, 1) - U(N, 1)\} \\ P_L &= P(N, 1) + \frac{x_L}{\Delta x} \{P(N-1, 1) - P(N, 1)\} \\ U_L &= U(N, 1) + \frac{x_L}{\Delta x} \{U(N-1, 1) - U(N, 1)\} \end{aligned} \right\} \dots\dots\dots (8)$$

Then progressive and backward Eqs. for station N are;

$$\left. \begin{aligned} P(N, 2) &= P_L - \rho c_L U(N, 2) \\ &\quad + \rho c_L (1-\gamma) U_L \quad \text{Progressive wave} \\ P(N, 2) &= P_R + \rho c_R U(N, 2) \\ &\quad - \rho c_R (1-\gamma) U_R \quad \text{Backward wave} \end{aligned} \right\} \dots\dots\dots (9)$$

By combining above Eqs.,

$$\left. \begin{aligned} P(N, 2) &= \frac{c_L P_R + c_R P_L}{c_R + c_L} + \frac{\rho c_R c_L}{c_R + c_L} \\ &\quad \times (1-\gamma)(U_L - U_R) \\ U(N, 2) &= \frac{P_L - P_R}{\rho(c_R + c_L)} + \frac{(1-\gamma)}{c_R + c_L} \\ &\quad \times (c_R U_R + c_L U_L) \end{aligned} \right\} \dots\dots\dots (10)$$

Above Eqs. are the solution of uniform air content. In the case of no air content, Eq. (5) is obtained by putting $\epsilon_0 = 0$ in above Eq., and it is sufficient to assume Eq. (10) as general formulae.

2.3 Calculation of directional valve

In the hydraulic system, a spool valve is used as a directional valve. It was found experimentally that its opening increased linearly, and reached maximum opening.

Valve open time t_0 was usually 5/100 sec. Discharge coefficient c_v of spool valve, as shown in literature⁽⁶⁾⁽⁷⁾, is $c_v \propto \sqrt{R_e}$ for low Reynold number, $c_v \doteq c_{\max}$ for middle Reynold number, and c_v decreases after Reynold number surpasses the critical pressure

Consequently, in the strict sense of the word, we must calculate c_v as function of valve opening and R_e , but because of short valve open time, approximate treatment influences little on the surge pressure. In the following, in the case of uniform air content, let's explain calculation method for c_v constant.

In Fig. 3, first, supposing $U(NB, 2) = U(NC, 2) = 0$, comparison of pressure of both sides of valve is made. Denoting A_L, A_R as pressures of left and right side of the valve,

$$\left. \begin{aligned} A_L &= P_L + \rho c_L (1-\gamma) U_L \\ A_R &= P_R + \rho c_R (1-\gamma) U_R \end{aligned} \right\} \dots\dots\dots (11)$$

Also, putting $A_d = A_L - A_R$, if $A_d \geq 0$, flow is left→right, if $A_d < 0$, flow is right→left.

Flow velocity $U(NB, 2)$ passing through valve, is equal to $U(NC, 2)$ from the continuity condition. [$U(NB, 2) = U(NC, 2)$ is flow velocity, converted to pipe].

That is,

$$\left. \begin{aligned} U(NB, 2) &= \pm \sqrt{\xi_v [P(NB, 2) - P(NC, 2)]} \\ &\quad \begin{aligned} &P(NB, 2) \geq P(NC, 2) + \text{sign} \\ &P(NB, 2) < P(NC, 2) - \text{sign} \end{aligned} \\ P(NB, 2) &= A_L - \rho c_L U(NB, 2) \\ P(NC, 2) &= A_R - \rho c_R U(NC, 2) \end{aligned} \right\} \dots\dots\dots (12)$$

where, $\xi_v = \left(c_v \frac{S_v}{S_p} \right)^2 \frac{2}{\rho}$

By combining above Eqs, the solution is obtained as follows.

$$\begin{aligned} U(NB, 2) &= \frac{1}{2} \xi_v \rho (c_L + c_R) \\ &\quad \times \left\{ \mp 1 \pm \sqrt{1 \pm \frac{4A_d}{\xi_v [\rho(c_L + c_R)]^2}} \right\} \dots\dots\dots (13) \end{aligned}$$

where the signs are in the same order, $A_d > 0, -++$, $A_d < 0, ---$ are taken as signs.

By substituting $U(NB, 2)$ of Eq. (13) into 2nd and 3rd Eqs. of Eq. (12), $P(NB, 2)$ and $P(NC, 2)$ are attained. Valve opening is assumed as linear.

2.4 Calculation of accumulator

Acc is a lumped element system and by taking time mesh Δt large, error is introduced unlike in the wave equation. As calculation methods, there are 1st approximation, 2nd approximation, RKG method, integration progress method etc., but it is found that by taking Δt small, sufficient accuracy is attained by 1st approximation method. So this method is adopted.

As fluid resistance of Acc neck, resistance of orifice (a hole is opened at thin plate) is assumed.

In this case, by a few experiments, it is found that discharge coefficient is approximately

Fig. 3 Calculation of directional valve

$$c_a = c_{am} \sqrt{R_e / R_{em}} (R_e \leq R_{em}), \quad c_a = c_{am} \\ (R_e \geq R_{em}, R_{em} \doteq 500) \dots \dots \dots (14)$$

But actually, when surge pressure is large, and the pipe velocity is fast, $R_e > R_{em}$ is the condition, so it is allowable to assume $c_a = \text{constant}$, approximately.

2.4.1 Calculation of Accp

In Fig. 4(a), by 1st approximation,

$$v_1(2) = v_1(1) + S_p U(NA, 1) \Delta t \dots \dots \dots (15)$$

$$P(LA, 2) = P_m v_{10}^n / v_1(2)^n \dots \dots \dots (16)$$

where, P_m : initial pressure at high pressure circuit, V_{10} : initial volume.

Supposing $U(NA, 2) = 0$, in the same way as with valve, comparison of pressure before and behind the orifice is made. $A_R = P_R - \rho c_R(1 - \gamma)U_R$, $A_d = P(LA, 2) - A_R$, then,

$$P(NA, 2) = A_R + \rho c_R U(NA, 2) \dots \dots \dots (17)$$

$$\left. \begin{aligned} U(NA, 2) &= \pm \sqrt{\xi_1 [P(LA, 2) - P(NA, 2)]} \\ P(LA, 2) &> P(NA, 2) \quad U(NA, 2) > 0 \\ P(LA, 2) &< P(NA, 2) \quad U(NA, 2) < 0 \end{aligned} \right\} \dots \dots \dots (18)$$

$$\text{where, } \xi_1 = \left(c_{ap} \frac{S_{a1}}{S_p} \right)^2 \frac{2}{\rho}$$

By combining Eqs. (17)(18),

$$U(NA, 2) = \frac{1}{2} \rho c_R \xi_1 \\ \times \left(\mp 1 \pm \sqrt{1 \pm \frac{4A_d}{\xi_1 (\rho c_R)^2}} \right) \dots \dots \dots (19)$$

Where the signs are in the same order. Signs are taken as $A_d > 0, -++$, $A_d < 0, --$.

By substituting Eq. (19) into Eq. (17), $P(NA, 2)$ is obtained

2.4.2 Calculation of Accd of pipe part

$$\left. \begin{aligned} P(ND, 2) &= P_L - \rho c_L U(ND, 2) + \rho c_L (1 - \gamma) U_L \\ P(NE, 2) &= P_R + \rho c_R U(NE, 2) - \rho c_R (1 - \gamma) U_R \end{aligned} \right\} \dots \dots \dots (20)$$

Assuming that radius of Acc is sufficiently short compared with wave length of surge, supposing $P(ND, 2) = (NE, 2)$ and putting $UD = U(ND, 2) - U(NE, 2)$,

$$U(ND, 2) = \frac{P_L - P_R}{\rho(c_L + c_R)} + \frac{c_R}{c_L + c_R} UD \\ + \frac{1 - \gamma}{c_L + c_R} (c_R U_R + c_L U_L) \dots \dots \dots (21)$$

Further, defining PM, β as follows,

(a) Case of Accp (b) Case of Accd
Fig. 4 Calculation of Acc

$$\left. \begin{aligned} PM &= \frac{(c_L P_R + c_R P_L)}{c_R + c_L} + \frac{\rho c_R c_L}{c_R + c_L} \\ &\times (1 - \gamma)(U_L - U_R) \\ \beta &= \frac{\rho c_L c_R}{c_L + c_R} \end{aligned} \right\} \dots \dots \dots (22)$$

$$P(ND, 2) = PM - \beta UD \dots \dots \dots (23)$$

$$\left. \begin{aligned} UD &= \pm \sqrt{\xi_2 [P(ND, 2) - P(LB, 2)]} \\ P(ND, 2) &\geq P(LB, 2) \quad UD \geq 0 \\ P(ND, 2) &< P(LB, 2) \quad UD < 0 \end{aligned} \right\} \dots \dots \dots (24)$$

where, $\xi_2 = \left(c_{ad} \frac{S_{a2}}{S_p} \right)^2 \frac{2}{\rho}$, putting $A_r = PM - P(LB, 2)$

$$UD = \frac{\xi_2 \beta}{2} \{ \mp 1 \pm \sqrt{1 \pm 4A_r \xi_2 / (\xi_2 \beta)^2} \} \dots \dots \dots (25)$$

Where the signs are in the same order. Signs are taken as $A_r > 0, -++$, $A_r < 0, --$

If UD is obtained, $U(ND, 2)$ from Eq. (21), $P(ND, 2)$ from Eq. (20) are obtained.

3. Result of calculation

3.1 Relationship between air content and surge pressure

In Fig. 5, as the object of model system of total pipe length 17m's line, (details of piping are shown in the figure) relationship between pressure rise ratio, U_{max} and air content ratio, is obtained. Maximum sound velocity of oil in case of $\varepsilon_0 = 0$ is 1295 m/sec, full open time of valve t_0 is 5/100 sec, kinematic viscosity of oil ν is 1 St, and pipe's sectional area S_p is 8.56 cm².

Air content ratio ε_0 is varied from 0~5%. Discharge coefficient of directional valve is assumed con-

(Note; ● mark shows experimental value obtained by forced air mixture)

Fig. 5 Relationship between surge pressure and air content ratio

stant value of 0.6.

Case of $\varepsilon_0=0$ is linear system, and the pressure rise ratio β doesn't become larger than 1, as well known. In this case β becomes 0.95.

In proportion to the increase of air content ratio, the pressure rise ratio and maximum discharge flow velocity from Accp increase, and when $\varepsilon_0=2.5\%$, $\beta=2$ and $U_{\max}=11\text{m/sec}$, when $\varepsilon_0=5\%$, $\beta=2.4$ and $U_{\max}=13.5\text{m/sec}$.

In these cases, waves of surge are different from case of $\varepsilon_0=0$, as shown in Fig. 6, and become as sharp as a saw tooth. At the same time, time t_s which is required for the surge 1st wave to reach the fixed end, becomes long nearly in proportion to ε_0 .

3.2 Relationship between surge pressure and valve open time

In the relationship between valve open time and surge pressure, the problem is how to choose the discharge coefficient of valve.

Actually, discharge of valve is an instantaneous phenomenon ending in about 50msec, so, it is questionable whether c_v characteristic for stationary flow as shown in the literatures⁽⁶⁾⁽⁷⁾, holds true or not in the transient phenomenon.

Especially, value of c_v for high R_e , depends on generation of cavitation, so, it is controversial to use c_v of stationary flow for the instantaneous phenom-

non.

Consequently, comparison of the calculation which assumed $c_v=c_{vm}\sqrt{R_e/R_{em}}$, $R_e \leq R_{em}$, $c_v=c_{vm}$, $R_e > R_{em}$ ($R_{em}=200$, $c_{vm}=0.7$), approximately modelling these experimental results, and the calculation of c_v constant ($=c_{vm}$) independent of R_e , was made.

As the result, it became clear that calculation of c_v constant was sufficient, because in case of $t_0 \leq 0.25$ sec R_e exceeded 200 in about $t=10^{-4}$ sec.

In Fig. 7, relationship between valve open time t_0 and β is shown, ε_0 as parameter for the piping system of total pipe length 6.4m line, $L_1=2.2\text{m}$, $L_2=4.2\text{m}$, $P_m=50\text{kg/cm}^2$, $\nu=1\text{St}$, Accp=6 L .

By actual measurement, under the standard condition, t_0 is 5/100sec, and in order to reduce pressure rise ratio to about 1/2 from this state, it is necessary to take t_0 about 5 times longer. β is also large in proportion to ε_0 .

Allowable value of pressure rise ratio differs with the characteristics of given conduit. In this study, allowable value is assumed 0.4 and further discussions are based on this value.

3.3 Relation between Accp's opening ratio and surge pressure

As a surge is superposed free vibrations whose modes are maximum velocity at Accp and maximum pressure at fixed end, the resistance at Accp's neck is assumed to be most effective.

Calculated result of surge pressure rise ratio to opening ratio S_{a1}/S_p , taking ε_0 as parameter, is shown in Fig. 8.

If opening ratio δ is kept below about 0.1, the pressure rise ratio can be kept below allowable value 0.4, independently of ε_0 .

3.4 Surge absorbing effect by Accd

Accd plays a most effective role when it is inserted at the large pressure position of vibrational mode, and in this case, the fixed end provides a best insertion position.

In the following, discussion about the case that Accd is inserted close to the fixed end, is given.

Fig. 6 Surge wave form vs. air content ratio (17 m line)

Fig. 7 Relationship between surge pressure and valve open time
(Calculation of total line length 6.4 m)

Largest parameter in surge absorbing effect by Accd is fluid inertia, that is, conduit length. Large volume of Accd in proportion to conduit length is necessary.

3.4.1 Case of conduit length 6.4m

In Fig. 9, in the case of air content ratio $\epsilon_0 = 0$, with opening ratio $\delta = S_{a2}/S_p$ as abscissa, pressure rise ratio's reduced effect is plotted, taking Accd's volume v_{02} as parameter.

Where, discharge coefficient of Accd's orifice is assumed to be constant value of 0.6.

To increase or decrease the discharge coefficient in relation to this value, is to change the frictional coefficient, for example, to change position of minimum of pressure rising ratio β , to left or right.

In this case, minimum value of β does not change. As the internal damping of rubber bladder is small, the effect appears only when the opening ratio is large, and the effect gives no influence on β substantially when opening ratio is small.

Choosing the pressure rise ratio $\beta = 0.4$ as allowable value, it is possible to keep it below the allowable value, by choosing $\delta \approx 0.075$ as its centre, if Accd's volume is more than 0.05 l.

Fig. 8 Relationship between pressure rise ratio β and opening ratio of Accp (Total length of line 6.4 m)

o: Calculated value of Eq. (30) of this paper
Fig. 9 Surge absorption effect by Accd (Case of $\epsilon_0 = 0$, total line length 6.4 m)

It is necessary to take the domain of opening ratio is wide to some extent; there appears an increased room in proportion to Accd's volume.

When the opening ratio is large, it sometimes occurs that, by the spring action of Accd's a large flow velocity and larger surge pressure are caused.

On the condition that U_{max} is close to the flow velocity without Accd, the pressure rise ratio becomes smallest.

Time history of surge absorption states by these Accd is shown in Fig. 10.

3.4.2 Case of conduit length 64m

The case of 65 times the oil's inertia, that is, pipe's length 64m, pipe's sectional area 19.6cm² which is 10 times the original pipe length, 6.5 times the original sectional area, was calculated.

In Fig. 11, in the case of $\epsilon_0 = 0$, surge absorption

Fig. 10 Surge absorption effect by Accd (6.4 m line)

o: Calculated value of Eq. (30) of this paper
Fig. 11 Surge absorption effect by Accd ($\epsilon_0 = 0\%$, total line length 64 m)

effect on opening ratio δ is shown, with parameter of Accd's volume.

Where, $t_0=5/100$ sec, $\nu=1$ st, Accp=60l, line pressure 70kg/cm², Accd's feed pressure=42kg/cm², Accp's opening ratio=0.29.

It is known that in this case, in order to be $\beta \leq 0.4$, Accd's volume of 3l or above is necessary and Accd's volume of 6l is sufficient.

These volumes are about 60 times that of the case of total pipe length 6.4m, and this fact shows that Accd's volume increases in proportion to oil's inertia.

4. Discussion

4.1 Relationship between air content ratio and surge pressure

State of air mixture is assumed to show versatile states according to pipe condition.

There may be the case of discontinuous minute air pocket's distribution. Here, the author treated the case of uniform air content. By preliminary experiment, it was observed that $\beta < 1$, when sound velocity in low pressure conduit didn't become low' $3 > \beta > 1$, when it became as low as 100m/sec.

The cause of surge pressure increase in proportion to air content ratio as shown in Fig. 5, is that the oil column acts like a strong hard spring due to the existence of air content.

That is, relationship between P and volume ratio of compression $\Delta v/v$ (corresponding to relationship between spring force versus displacement) is obtained in Fig. 12, with ϵ_0 as parameter, and its non-linearity becomes stronger in proportion to ϵ_0 .

Tangent of $P-\Delta v/v$ curve is in proportion to spring constant of oil column, and it is the spring constant of static iso-thermal compression.

In the hydraulic system in Fig. 1, equivalent vibrational system may be considered approximately in which fluid mass between Accd and fixed end moves uniformly supported by oil column's stiffness.

This is a case of treating the so called solid body theory of fluid.

In this case, an equivalent mechanical system is

Fig. 12 Stiffness characteristic of oil in case of air content (Isothermal compression)

shown in Fig. 13, where stepwise external pressure acts on oil mass m .

Stiffness shows a hard spring characteristic of oil column.

As damping isn't considered, potential energy E_a stored in the spring is equal to E_E which is the work done by external pressure.

Potential energy E_a stored in the spring,

$$E_a = \int_{v_0}^v P dv = v_0 \left[\frac{P_0 \epsilon_0}{n-1} \left\{ \left(\frac{P}{P_0} \right)^{1-1/n} - 1 \right\} + \frac{1-\epsilon_0}{2\beta} (P^2 - P_0^2) \right] \quad (26)$$

Where, at pressure P_0 , volume is v_0 , and adiabatic expansion is considered because of vibrational phenomenon. Work done by external pressure E_E is from displacement volume $v_0 - v$,

$$v_0 - v = v_0 \left[\frac{(1-\epsilon_0)(P-P_0)}{B} + \epsilon_0 \left\{ 1 - \left(\frac{P_0}{P} \right)^{1/n} \right\} \right] \quad (27)$$

$$E_E = (P_E + P_0)(v_0 - v)$$

Where, putting $\beta = (P+P_0)/(P_E+P_0)-1$, β shows the ratio of external pressure and increment of internal pressure caused by it and corresponds to pressure rise ratio of surge.

By equalizing Eq. (26), (27),

$$\beta = \frac{(P+P_0) \left[\frac{(1-\epsilon_0)(P-P_0)}{B} + \epsilon_0 \left\{ 1 - \left(\frac{P_0}{P} \right)^{1/n} \right\} \right]}{\left[\frac{P_0 \epsilon_0}{n-1} \left\{ \left(\frac{P}{P_0} \right)^{1-1/n} - 1 \right\} + \frac{1-\epsilon_0}{2B} (P^2 - P_0^2) \right]} - 1 \quad (28)$$

From this Eq., relationship between P_E and β is obtained in Fig. 14, putting $P_0=1$ kg/cm², with ϵ_0 as parameter.

In the case of $\epsilon_0=0$, that is, case of oil only, $\beta=1$ and it corresponds to a linear system. In proportion to ϵ_0 , non-linearity becomes stronger and the pressure rise ratio becomes larger. β becomes smaller gradu-

(a) Rigid body approximation model of Fig. 1

(b) Rigid body model which shows effect of Accd
Fig. 13

ally in proportion to the external pressure P_E .

In the calculation of wave theory, as described previously, external pressure does not act stepwise (valve open time $t_0 \neq 0$) and by damping of oil, does not become extremely large, as with the case of 1st degree of freedom model.

In order to ascertain validity of calculation of air content, air is forcibly mixed into oil conduit which has equal condition of calculation. Data obtained from these experiments are shown in $\bullet$ mark in Fig. 5.

Both are acknowledged to agree in tendency.

In Fig. 6, actual measured examples of surge pressure in case of $\varepsilon_0 = 0$ and 5% are shown in dotted line.

In the latter case, the wave form of 1 cycle agrees well with calculated curve, but successive wave form differs somewhat.

This discrepancy is assumed to be caused by a complex phenomena, such as air's re-dissolution into oil.

4.2 Reducing method of surge pressure by Accd

Reducing effect of surge pressure by Accd, generally, becomes large in proportion to Accd's volume (gas volume). Accd's volume in proportion to fluid inertia in pipe, is necessary and in order to give effective damping to the system, it is necessary to set its opening ratio appropriate range too.

A simplified solid body model of surge phenomenon in case of valve opening is such that, as shown in Fig. 14, the external pressure P_E acts on the system which has oil mass m and oil column stiffness K_0 with air spring K_a and oil damper c_a in series connection.

As the external pressure is input, velocity of m becomes function of external pressure and usual equation of valve closure of velocity input cannot be used generally.

In the case of no air content, by $\delta - U_{\max}$ curve in Fig. 9, at the region of $\delta \leq 0.075$, $U_{\max}$ converges to constant value independent of Accd's volume v_{02} .

Fig. 14 Relationship between P_E and pressure rise ratio β by non-linear spring characteristic of oil-air mixture when external pressure acts

Consequently, in these domains, it is assumed that Accd's volume can be determined simply under the condition that velocity input is constant.

In this case, the relation between maximum velocity $U_{\max}$ of oil mass and ratio of line's max pressure $P_{\max}$ and line's mean pressure P_m is, when $K_0 \gg K_a$, kinetic energy of the system is equal to Accd's potential energy, that is;

$$\frac{1}{2} m U_{\max}^2 = \int_{v_m}^{v_{\min}} P dv = \frac{P_m v_m}{n-1} \times \left\{ \left(\frac{P_{\max}}{P_m} \right)^{1-1/n} - 1 \right\} \quad (29)$$

$v_{\min}$: Accd's smallest volume, v_m : Accd's volume at pressure P_m .

$$\frac{P_{\max}}{P_m} = \left\{ 1 + \frac{(n-1) m U_{\max}^2}{2 P_m v_m} \right\}^{n/(n-1)} \quad (30)$$

From the above equation, relationship between pressure rise ratio $\beta = P_{\max}/P_m - 1$ and Accd's volume is obtained. Calculated result in the case of taking whole mass as m , is shown in $\circ$ mark in Fig. 9.

These results agree comparatively well with minimum value of $P_{\max}$ at $\delta \approx 0.075$.

Consequently, calculation which assumes velocity without Accd as input, in reality, assumes appropriate resistance at the Acc's neck, and generally gives maximum of surge absorbing capacity of Accd.

In the case of $\varepsilon_0 \neq 0$, it is difficult to determine surge absorbing capacity of Accd simply by Eq. (30), as the potential of residual air must be considered. Since the surge pressure is large, $P_{\max}$ becomes larger than the case of $\varepsilon_0 = 0$.

5. Conclusion

The author discussed a surge generated by the discharge of valve in a hydraulic system with pressure source, and introduced general method of characteristic considering the effect of air content.

(1) In proportion to the increase of air content in oil, surge pressure increases. This reason is explained by the phenomenon in which oil column stiffened by the air content acts like a hard spring.

(2) Relationship between valve open time, fluid resistance of Accp and surge pressure was calculated.

(3) Insertion of Accd in the fixed end for the purpose of surge absorption is effective.

The author clarified that proper volume of Accd became large in proportion to inertia of fluid, and proper fluid resistance was necessary.

As a convenient method for obtaining volume of Accd, the author compared results of energy balance equation based on fluid velocity without Accd as standard and of method of characteristic. As the result, it was known that the former nearly gave lower limit of pressure rise ratio of the latter.

The author appreciates Takenoshita of 82 section,

Yamaguchi of 33 section of this research laboratory, Oishi and Suzuki of Hitachi Work for their help.

References

- (1) Kasahara, E. and Tanahasi, T., *Trans. Japan. Soc. Mech. Engrs.* (in Japanese), Vol. 35, No. 272 (1970-4), p. 774.
- (2) Lister, M., *Mathematical Method for Digital Com-*

- puters*, Chap. 15 (1960), p.165, John Wiley & Sons.
- (3) Takenaka, T. and Urata, E., *Hydraulic Control* (in Japanese), (1967), p. 26, Maruzen.
- (4) Takenaka, T., *Oil Hydraulics*, Vol. 7, No. 2 (1964), p. 3.
- (5) Mitsui, N., et al., *Hitachi District JSME Paper* (in Japanese), (1967-11), p. 1791.
- (6) Anno, Y. and Akiyama, N., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol. 36, No.286, p. 966.
- (7) Ibid p. 974.

Discussion

T. ISHIHARA (University of Tokyo):

Generally, a considerable amount of gas is contained in hydraulic oil and air content ratio of about 8% (which is expressed by volume ratio at atmospheric pressure) is most probable.

In your experiment, when you change ϵ_0 , it is controlled by air volume feeded externally. How do you think about the amount of gas which is already contained in oil?

In your experiment, you consider the region of large change of sound velocity for the increase of ϵ_0 . Please show me your opinion about the assumption (or measurement) of air content ratio, considering the above point.

From your paper, concrete value of air content ratio in P and A lines is not clear. Please explain in this point.

Author's closure

As you mentioned, air content ratio ϵ_0 is about 10%. This value is close to the saturated solution amount of air.

This air content ratio is the sum of ϵ_{0s} which corresponds to the fraction of solution, that is, the state of air combining the oil molecule and ϵ_{0m} which corresponds to the fraction of mixture, that is, the state of air separating in oil in the form of minute bubbles.

Really, this fact is verified by our experimental result of the oil. That is, the author conducted a static compression test and a dynamic sound velocity measurement of conduit of about 5 m length, using the oil whose ϵ_0 is measured to be about 9.5% by gas analyzer.

According to these experiments, ϵ_{0m} which was considered to be a component of the mixture was about 1% or below, and the other component of air which combined oil molecule, were not supposed to influence oil's compressibility directly.

Amount of air content is known from experience to be strongly influenced by pipe condition.

That is, when there is a misalignment or a pocket at flange connection part, the air dwells in the form of

a pocket, and it becomes difficult for it to escape from the pocket.

This results in the same effect of increase of air content.

Really, experimental apparatus of this time consists of 1st and 2nd order. First experimental apparatus uses flanges of different sizes, and reduction of sound velocity at low pressure is remarkable.

The reason for this seems to lie in the formation of the air pocket at the flange pitch.

In the secondary experiment, we totally reconstructed the piping system, matching flange and pipe, and minimized the existence of air pocket. As the result, reduction of sound velocity at low pressure was hardly acknowledged. In this state, as the result of calculating the air content ratio from sound velocity, $\epsilon_{0m} \doteq 0.05\%$ was attained. At this value, non-linearity of oil column stiffness can be ignored and the surge corresponds to a linear system.

The experiment, treated here, was conducted by this secondary apparatus and no consideration of air content ratio of hydraulic oil itself was made especially.

But, really an apparent phenomena of increase of air content ratio which appeared in the 1st experimental apparatus, is frequently acknowledged. The author simulates the phenomena, making uniform air layer feeding air forcibly in order to represent this phenomena.

P line is initially at high pressure and the influence on sound velocity isn't large, if some air is mixed with it.

On the contrary, the influence of air content has strong effect at A line, because initially the atmospheric pressure is held.

In this sense, when air is forcibly fed into A line, the author calculates the surge by assuming that air content ratio at P line is equal to that of A line.

Exactly, the author should calculate the surge by changing air content ratio at P and A line, and compare it with experiment.

Further discussion will be needed.