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Dynamic Characteristics of Non-Uniform Motion Mechanisms with Ball Bushings*

(2nd Report, On the Circulation of Balls and the Dynamic Characteristics of Mechanisms)

By Hiroaki FUNABASHI**, Kiyoshi OGAWA***, and Nozomu CHISAKI****

The influences of the bearing clearance and the eccentricity of loads upon the frictional characteristics of the ball bushing and upon the dynamic characteristics of non-uniform motion mechanisms have been analysed.

It has been clarified that the friction decreases when the bearing clearance is taken zero or negative, and that the amplitude and critical velocity for occurrence of stick-slip vibration may be reduced by making the clearance and the eccentricity of loads small. As well as the translational vibration, the angular vibration of the bushing and the bending vibration of the shaft have been analysed in relation to the circulation of the balls.

1. Introduction

It was pointed out in the 1st report⁽¹⁾ that the frictional characteristics of the ball bushings deteriorate and therefore the dynamic characteristics of mechanisms deteriorate also when light loads, eccentric loads or bending moments act on the bushings. However the characteristics of the bushings under light loads, eccentric loads and bending moments are quite important because the bushings in such non-uniform motion mechanisms as link mechanisms and X-Y recorders suffer always bending moments and fluctuating forces which become periodically small in magnitude.

Considering the deterioration of the frictional characteristics when light or eccentric loads act to be caused by unsmooth circulation of balls, the relations between the frictional characteristics of the bushings and the dynamic characteristics of mechanisms and the bearing clearances and eccentricities of loads which have influences upon the circulation are discussed in the present paper. The angular vibration of the bushing which occurs in the plane made by the axis of the bushing and the line of action of the load and the bending vibration of the shaft are also discussed here besides the translational vibration of the

bushing in the direction of the shaft described in the 1st report.

The manufacturing accuracies of the ball bushings and the shaft are the same as those in the previous report.

2. Bearing clearance and eccentricity of load and frictional characteristics of ball bushing

2.1 Coefficient of static friction

Let the difference between diameters of the ball bushing and the shaft be called the bearing clearance, and let it be indicated by c . To adjust the clearance, the bushing with a longitudinal slit is inserted into a tapered box as shown in Fig. 1. The values of the clearance are given in the following way.

The balls of the bushing roll in the direction of the shaft and the bushing moves smoothly even if a slight interference is given, but they slide and a con-

Fig. 1 Tapered box inserted a ball bushing with a longitudinal slit

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** Assistant Professor, Tokyo Institute of Technology, Meguro-ku, Tokyo.

*** Professor, Tokyo Institute of Technology.

**** Engineer, Nippon Electric Co., Ltd.

siderable torque is required to rotate the shaft even if the interference is less than 1μ . Therefore testing bars with diameters of (diameter of the shaft) + (bearing clearances) are inserted into the bushing, and the adjusting nut of the bearing box is turned up to the position where the hand feels resistance to rotate the bars.

Inclining slowly the shaft of the bushing with a dead weight W from the horizontal, we obtain the coefficient μ_s of static friction as the inclination λ of the shaft when the bushing starts its motion.

In general, ball bushings in non-uniform motion mechanisms suffer normal forces W and bending moments M at the same time like those in $[R-S-C-R]$ mechanism shown in Fig. 2(a). Putting the effective width as $2b$, the normal reaction N becomes as follows.

$$N = W + \frac{M}{b} \quad (1)$$

The reaction is equal to one when a normal load W with an eccentricity of

$$l = \frac{M}{W} \quad (2)$$

acts on the bushing. Therefore the frictional characteristics have been studied in the case where eccentric loads act on the bushing as shown in Fig. 2(b) instead of the case where normal forces and bending moments act at the same time.

Putting the normal reaction and the friction between the ball bushing and the shaft as N and F respectively, the coefficient μ of the friction is given as $\mu = F/N$. But the ratio λ of the friction F and the load W , i.e. $\lambda = F/W$, is more practical than μ

(a) $[R-S-C-R]$ spatial link mechanism

(b) Ball bushing suffering eccentric load W

Fig. 2 Load acting on ball bushing in non-uniform motion mechanism

in the case where the eccentric load W acts on the bushing just as mentioned. Then the ratio is called the virtual coefficient of friction and is indicated by μ' . When the effective width of the bushing, that is the length of the shaft contacting the balls, is $2b$ and the distance between the center of the bushing and the point of action of the load is l , the eccentricity of the load is expressed by the dimensionless quantity $e = l/b$.

There exists the following relation between the eccentric load W and the normal reaction N .

$$N = \left(\frac{1+e}{2} + \frac{|1-e|}{2} \right) W \quad (3)$$

Therefore the following relation holds between the true coefficient of friction μ and the virtual one μ' .

$$\left. \begin{array}{l} \text{When } e \leq 1, \quad \mu' = \mu \\ \text{When } e > 1, \quad \mu' = e\mu \end{array} \right\} \quad (4)$$

in which the load is assumed to be supported by the balls which are positioned at both ends of the bushing when $e > 1$.

Some results of the experiment are shown in Figs. 3 and 4 in the case where $W = 3450$ g and the bushing is lubricated with machine oil #120. The relations between the virtual coefficients of friction and the eccentricities of the load in Fig. 3 are in accordance with Eq. (4). The true coefficient in Fig. 4 increases a little with the eccentricity e when the point of action of the load is within the effective width of the bushing, i.e. $e < 1$, and becomes constant independently of the eccentricity when $e > 1$.

Figures 3 and 4 show that the coefficient of static friction becomes smaller when a little interference is given than when the bearing clearance is given, though the fact was not distinct for light loads less than

Fig. 3 Relation between virtual coefficient of static friction and eccentricity of load

1 000 g because of the coefficient of friction being dispersive.

2.2 Coefficient of kinetic friction

As mentioned in the previous report, the coefficient μ_k of kinetic friction is obtained by measuring the acceleration of the bushing which moves on a horizontal shaft. Figure 5 shows the result when the load W is 3 450 g and the bushing is lubricated with machine oil #120. The coefficient increases rapidly with the velocity of bushing when a positive clearance is given, though it varies little with the bearing clearances at low velocities of the bushing. The tendency is remarkable when the bushing suffers eccentric loads. By the way, fairly large noises are observed and the balls are known to move irregularly in the case where the bearing clearance is positive.

In the 1st report the stick-slip vibrations were pointed out to occur in mechanisms with ball bushings just like in ones with sliding bushings. The amplitude and the maximum velocity for occurrence of the stick-slip vibration are known to increase with the difference between coefficients of static and kinetic frictions⁽²⁾⁽⁸⁾.

Fig. 4 Relation between true coefficient of static friction and eccentricity of load

Fig. 5 Alteration of coefficient of kinetic friction in regard to eccentricity of load and bearing clearance

Then the difference of coefficients is plotted in Fig. 6. The difference becomes great with the eccentricity of load, and the tendency is remarkable in the case when the bearing clearance is positive.

Therefore the eccentricity of load and the bearing clearance must be taken small in order to decrease the amplitude and the critical velocity for occurrence of the stick-slips.

3. Dynamic characteristics of a model mechanism

3.1 Translational vibration in the direction of shaft

The dynamic characteristics were analysed in

Fig. 6 Alteration of difference between coefficients of static and kinetic frictions in regard to eccentricity of load and bearing clearance

Fig. 7 Oscillograms of translational vibration of bushing when the velocity of shaft is constant

relation to the bearing clearance and eccentricity of load using the model mechanism shown in Figs. 8 and 9 in the 1st report. In the experiment, the total weight of the bushing and the dead weight was $W=3450$ g, the spring constant was $k=857$ g/cm, and the lubricant was machine oil #120.

As an example of oscillograms of the vibration when the shaft was driven at constant velocities, the result at the velocity of shaft $v=1.22$ mm/sec is shown in Fig. 7. A stick-slip vibration is observed, and its amplitude increases with velocity of the shaft as shown in Fig. 8. The maximum velocity for occurrence of stick-slip vibration is about 60 mm/sec.

By the way it was pointed out in the 1st report that the amplitude of the stick-slip vibration increases remarkably with the eccentricity of load in the case where $W=280$ g and $c \approx 10\mu$. However distinct differences in amplitudes cannot be observed in the case where $W=3450$ g, $c=-4, 0, 4\mu$ and $e=0, 1.4$

Fig. 8 Relation between the amplitude of translational vibration of ball bushing and the velocity of shaft when the velocity of shaft is constant

as shown in Fig. 8.

When the shaft was driven non-uniformly by a slider-crank mechanism, the relation between the velocity of shaft and the amplitude became Fig. 9 in which there are no large differences in amplitudes with bearing clearances and eccentricities of loads.

These results are different from the inference in Chapter 2, which is thought to be caused by the fact that the frictions in Fig. 6 are mean values. The friction of the ball bushing is very dispersive as mentioned in the 1st report, and therefore the friction must be grasped as a phenomenon of probability to discuss exactly the characteristics of mechanisms. But it is expected that the amplitude and the maximum velocity for occurrence of stick-slip vibration may be decreased by making the bearing clearances and the eccentricities of loads small according to the inference mentioned in Chapter 2, if the manufacturing accuracies of the ball bushings and shafts are improved, because the dispersion of the friction is largely dependent upon the geometry⁽⁴⁾ of contact surfaces of balls, races and shafts.

Fig. 9 Relation between the amplitude of translational vibration of ball bushing and the crank speed when the shaft is driven by a crank mechanism

3.2 Angular vibration of ball bushing

While the ball bushing vibrates in the direction of the shaft, it vibrates at the same time in the plane made by the line of action of the load and the axis of the shaft as shown in Fig. 10. By this angular vibration, for example, there occur bending vibrations with high frequencies in the coupler link of $[R-S-C-R]$ mechanism besides the stick-slips and the forced vibration due to inertia forces as shown in Fig. 20 of the 1st report. Thus the angular vibrations of the bushings have bad influences upon the dynamic characteristics of mechanisms. Then the vibration was analysed using the model mechanism shown in Fig. 8 of the previous report.

Figures 11 and 12 are oscillograms obtained respectively in the cases where the shaft was driven at a constant velocity $V=51.3$ mm/sec and where it was driven by a crank mechanism at a rotational frequency $\omega=24.4$ rpm under the condition that the load was 3450 g and the lubricant was machine oil #120. In both figures the vibrations with low frequencies are

Fig. 10 Angular vibration of ball bushing

Fig. 11 Oscillograms of translational and angular vibrations of bushing when the velocity of shaft is constant

translational ones in the direction of the shaft and those with high frequencies are the angular vibrations.

The amplitudes of the angular vibrations are shown in Fig. 13 in reference to the bearing clearances and the eccentricities of loads. In the figure, (a) is the result when the shaft was driven at a constant velocity $V=74.5$ mm/sec, and (b) is the result when it was driven by the crank mechanism at $\omega/\omega_n=0.4$. In the experiment, the moment of inertia of the bushing and dead weight was $J=0.47$ kg·mm·sec² and the lubricant was machine oil #120. The figures show that the amplitudes are very large when the load is eccentric and this tendency is distinct when the bearing clearance is positive. Therefore the bearing clearances must be zero or negative and the eccentricities of loads must be taken small in order to suppress the angular vibrations just like the translational ones.

In Fig. 10, putting the moment of inertia of the bushing and dead weight, the displacement angle of the bushing, the restoring torque, and the exciting torque as J , ψ , $T(\psi)$ and $T_0(t)$ respectively, and neglecting the viscous damping, the equation of motion is given as follows.

$$J \frac{d^2 \psi}{dt^2} + T(\psi) = T_0(t) \quad \dots\dots\dots (5)$$

The restoring torque $T(\psi)$ was measured in regard to the displacement angle ψ . It became Fig. 14, for example, when the shaft was horizontal. The characteristic of the restoring torque varies with

Fig. 12 Oscillograms of translational and angular vibration of bushing when the shaft is driven by the crank mechanism

(a) In the case $V=\text{constant}$ (b) In the case $V \neq \text{constant}$

Fig. 13 Alteration of amplitude of angular vibration in regard to bearing clearance and eccentricity of load

(a) $e=2.3, c=4\mu$ (b) $e=0.6, c=-4\mu$

Fig. 14 Restoring torque of ball bushing

values of the bearing clearance and the eccentricity of the load. When the clearance is positive, it becomes a complex function of the gravitational force, the eccentricity of load and the clearance between the bushing and the shaft as shown in Fig. 14(a). The torque has the characteristic of a hard spring in all conditions.

By the way, the bushing inclines suddenly by the circulation of balls and the shaft suffers impacts when the balls begin or finish contacting the shaft. Putting the velocity of the bushing and the diameter of the balls as V mm/sec and d mm respectively, the frequency of the impacts f_T is given as follows.

$$f_T = \frac{\varepsilon V}{2d} \text{ Hz} \quad (6)$$

in which ε is a factor determined by the condition whether the balls in each cage begin and finish contacting the shaft at the same time or not. When the number of cages is n and there are no slips in rolling of balls, the factor is given as follows.

$$1 \leq \varepsilon \leq 2n \quad (7)$$

As the impact is small when $\varepsilon=2n$ and is great when $\varepsilon=1$, the values of f_T at $\varepsilon=1$ and 2 are important for the angular vibration of the bushing.

In the ball bushings used in the experiments, $d=2.38$ mm and $n=4$. Then, putting $V=10$ mm/sec for example, f_T with $\varepsilon=1$ becomes as follows.

$$\frac{V}{2d} = \frac{10}{2 \times 2.38} = 2.1 \text{ Hz}$$

On the other hand the frequencies of the angular vibrations while the bushing moved on the shaft in the experiments reached up to $f=50 \sim 100$ Hz and the natural frequencies obtained by percussion became $f=30 \sim 70$ Hz.

Therefore, when the bushing moves at a low speed, the exciting torques by the circulation of balls act impulsively at large intervals to the angular vibration, and the vibration is thought to have the same characteristics as the natural vibration caused by percussion, and it may be approximated by the following equation instead of Eq. (5).

$$J \frac{d^2 \psi}{dt^2} + T(\psi) = 0 \quad (8)$$

When the velocity of the bushing is high enough, that is when $f_T=f$, Eq. (5) must be adopted, though the velocity $V \neq 2df$, at which $f_T=f$, should be avoided in designing mechanisms.

Substituting the restoring torque in Fig. 14 into Eq. (5) or (8), the angular vibrations may be analysed theoretically with digital or analog computers. Here the relation between the frequency f and the amplitude ψ_0 is obtained from Eq. (8) by way of Lord Kelvin's construction⁽⁵⁾.

In order to construct the figure $\psi=\psi(t)$ of Eq. (8), t sec and ψ rad in Eq. (8) are replaced with

$\tau = \gamma_t t$ mm and $y = \gamma_\phi \phi$ mm (9)
in which γ_t and γ_ϕ are diagram factors with dimensions of mm/sec and mm/rad respectively.

Thus the solution of Eq. (8) is constructed as $y = y(\tau)$ and its radius of curvature ρ is given as follows.

$$\rho = \frac{\gamma_t^2 J}{\cos^3 \alpha \gamma_\phi T(y)} \quad \text{..... (10)}$$

in which

$$\tan \alpha = \frac{dy}{d\tau} \quad \text{..... (11)}$$

Therefore the relation between the amplitude ϕ_0 and the frequency f can be obtained if the initial conditions are given. Figure 15 shows the construction to obtain the frequency when $\phi_0 = 2 \times 10^{-3}$ rad., i.e., $y_0 = \gamma_\phi \phi_0 = 40$ mm, in the case where the restoring torque is Fig. 14(b) and the diagram factors are $\gamma_t = 10^4$ mm/sec and $\gamma_\phi = 2 \times 10^4$ mm/rad. Putting the point where $y = y(\tau)$ cuts the axis of abscissa as $\tau = a$, $a = 51.2$ mm expresses a quarter of the period. Then the frequency f is given as follows.

$$f = \frac{\gamma_t}{4a} \quad \text{..... (12)}$$

Fig. 15 Lord Kelvin's construction to obtain the relation between the amplitude and frequency of the angular vibration

Fig. 16 Relation between the amplitude and frequency of angular vibration of ball bushing

$$\begin{aligned} &= \frac{10^4}{4 \times 51.2} \\ &= 48.8 \text{ Hz} \end{aligned}$$

Figure 16 is the result obtained in this manner. The theoretical and experimental values are in good coincidence.

The exciting torque increases with the velocity of the shaft, and the amplitude of the angular vibration is conjectured to increase also with the velocity. But the amplitude was fairly dispersive even if the shaft was driven at a constant velocity. This is because the circulation of balls is irregular, resulting in variations of ϵ in Eq. (6) and of the exciting torque.

4. Bending vibration of shaft

Bending vibrations are caused in the shafts by the angular vibrations of the ball bushings. Then the bending vibration was analysed experimentally by use of a capacitance detector in the case where the bushing moved on a horizontal shaft both ends of which were simply supported as shown in Fig. 17.

Fig. 17 Equipment for measuring the bending vibration of shaft

Fig. 18 Oscillograms of bending vibration of shaft

Fig. 19 Relation between amplitude of bending vibration of shaft and velocity of bushing

Figure 18 shows the oscillograms at $V=120$ mm/sec. The frequency in Fig. 18(d) is about 30 Hz which is nearly equal to the natural frequency of the shaft with the bushing at its center. The other frequencies in Fig. 18 are about 50 Hz which is equal to the frequency of the angular vibration of the bushing.

The amplitude of the bending vibration of the shaft is shown in Fig. 19 in reference to the velocity of the bushing. The figure shows that the amplitude increases nearly in proportion to the velocity of the bushing, though the amplitude is fairly dispersive in the same manner as the angular vibration of the bushing because the balls of the bushing circulate irregularly as mentioned in section 3.2. The amplitude increases remarkably in the case where $e > 0$ and $c > 0$.

In order to know the influences, upon the amplitudes, of the bearing clearances and the eccentricities of loads, the amplitude of the bending vibration of the shaft at the velocity of the bushing $V=200$ mm/sec is plotted in Fig. 20. The figure shows that the amplitude is great and increases in proportion to the clearance in the case where the load is eccentric, though it is small and independent of the clearance when the eccentricity is zero. These facts are the same as the angular vibration of the bushing.

5. Conclusions

Varying the bearing clearance and the eccentricity of the load which have influences upon the circulation of balls of the ball bushing, the characteristics of friction, translational and angular vibrations of the bushing and the bending vibration of the shaft have been analysed. The results obtained here are summarized as follows.

(1) The coefficient of static friction of the ball bushing increases with the eccentricity of the load when the point of action of the load is within the effective width of the bushing, and becomes constant when the point is out of the effective width. The coefficient becomes smaller when the clearance is zero or negative than when it is positive.

Fig. 20 Alteration of the amplitude of bending vibration in regard to bearing clearance and eccentricity of load

(2) The coefficient of kinetic friction increases remarkably with the velocity of the bushing with a positive clearance, though it is independent of the clearance at low velocities.

(3) The difference between the coefficients of static and kinetic frictions increases with the eccentricity of the load. The tendency is distinct when the bearing clearance is positive. Therefore it was thought that the amplitude and the critical velocity for occurrence of the stick-slip vibration might decrease if the eccentricity of the load was made small and a little interference was given, however they did not decrease due to the dispersion of the friction.

(4) The ball bushing vibrates in the plane made by its axis and the line of action of the load. The angular vibration is a non-linear vibration with a hard spring, and its amplitude increases with the frequency. The amplitude is great when the load is eccentric, but it can be decreased remarkably by making the bearing clearance zero or negative, that is by giving a little interference.

(5) There is excited a bending vibration in the shaft by the angular vibration of the bushing. Though the amplitude of the vibration increases with the velocity of the bushing, it is decreased by making the eccentricity of the load and the bearing clearance small and zero or negative respectively in the same manner as the angular vibration.

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