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Analysis and Compensation of Machine-Tools Vibrations by Feedback Theory*

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For the analysis of the machine-tool dynamics, a mathematical model employing a signal flow diagram has been developed by the authors, through which a direct simulation and a precise analysis of the various nonlinear vibrations in the machine-tools, such as regenerative vibration, stick motion, etc. are made possible.

The point of this method lies in direct introduction of the analysis in the design of a real machine, making it possible for the machine-tool designer to take account of the vibration problem in his design work prior to drawing the conclusion.

In this paper the following problems are treated by the above method.

- (1) Vibration problem of machine-tools anchored to the ground.
- (2) Self-excited chattering taking place in the long ram of the milling head of horizontal boring machines.

1. Introduction

Authors developed a method to analyze the dynamic characteristics utilizing the mathematical modeling and the signal flow graph (denoted as S. F. G. hereafter) method, with which the direct simulation on the analog computer becomes possible and the precise consideration of the complex behaviour of a system including "back lash" and others becomes easy. They believe that this method would be a powerful means to treat the singular vibratory phenomena in machine-tools called the chatter vibration. Another advantage of this method in the real design work is that the results of analysis can be helpful to the designer in revising or determining his design.

This paper describes the application of this method to the next cases.

The analysis of a feedback vibration system taking into account the energy transfer between the system and the ground.

The countermeasure to meet the chatter vibration problem in a machine-tool having a long ram in its construction, for example, a vertical lathe.

2. Representation of the dynamic relations by the S. F. G.

Masses, springs and dampers are the constituent elements in the machine, but in addition to them machine tools comprise many long beams.

Strictly speaking, the dynamics of beams must be treated by the distributed parameter theory, but, the lumped parameter theory is substituted in this paper and thereby nothing comes into question in practice.

Generally the transverse and rotational displacements are predominant in the vibration of a beam, because the beam can be considered as a rigid body in the longitudinal direction. However, in this paper the problem is limited to a beam in which the transverse vibration is dominant.

In this case, the transverse vibration equation becomes:

$$EI \frac{\partial^4 Y}{\partial x^4}(x, t) - I\rho \frac{\partial^4 Y}{\partial x^2 \partial t^2}(x, t) + A\rho \frac{\partial^2 Y}{\partial t^2}(x, t) = 0 \dots\dots\dots (1)$$

Fig. 1 Symbolic representation of beam

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Here we define main symbols as follows

Y_i : transverse displacement at the end i

ϕ_i : slope angle at the end i

M_i : bending moment at the end i

F_i : lateral force at the end i

x : longitudinal displacement of the beam

ω : angular velocity

A_j : area of the section in the beam j

l_j : length of the beam j

m_j : mass of the beam j

ρ_j : density of the beam j

I_j : moment of inertia of the beam j

E : Young's modulus

The coordinate axes and their directions are illustrated in Fig. 1. Assume that the solution of Eq. (1) is separable in time and space and of the form

$$Y(x, t) = W(x)e^{i\omega t} \quad (2)$$

Substituting Eq. (2) in Eq. (1), the next relation about $W(x)$ is obtained.

$$\frac{d^4 W}{dx^4}(x) + \gamma^2 \frac{d^2 W}{dx^2}(x) - \theta^4 W(x) = 0 \quad (3)$$

Here $\theta^4 = A\rho\omega^2/EI$, $\gamma^2 = \rho\omega^2/EI$

By the use of Laplace transform, we obtain⁽¹⁾,

$$W(x) = \frac{1}{\alpha^2 + \beta^2} [(S + U\gamma^2)W(0) + (T + V\gamma^2)\dot{W}(0) + U\ddot{W}(0) + V\ddot{W}(0)] \quad (4)$$

where $\alpha = \sqrt{\sqrt{\frac{\gamma^4}{4} + \theta^4} + \frac{\gamma^2}{2}}$,

$$\beta = \sqrt{\sqrt{\frac{\gamma^4}{4} + \theta^4} - \frac{\gamma^2}{2}}$$

Differentiating the function $W(x)$ in Eq. (4), the slope angle, the bending moment and the shear force are respectively obtained,

$$\dot{W}(x) = \frac{1}{\alpha^2 + \beta^2} [(U\gamma^2 + V')W(0) + (S + U\gamma^2)\dot{W}(0) + T\ddot{W}(0) + U\ddot{W}(0)] \quad (5)$$

$$EI\ddot{W}(x) = \frac{EI}{\alpha^2 + \beta^2} [(S\gamma^2 + U')W(0) + (T\gamma^2 + V')\dot{W}(0) + S\ddot{W}(0) + T\ddot{W}(0)] \quad (6)$$

$$EI\ddot{W}(x) + \omega^2 I\rho\dot{W}(x) = \frac{\alpha^2\beta^2 EI}{\alpha^2 + \beta^2} \left[(T + V)W(0) + \left(U + \frac{S'}{\alpha^2\beta^2} \right) \dot{W}(0) + V\ddot{W}(0) + \frac{S'}{\alpha^2\beta^2} \ddot{W}(0) \right] \quad (7)$$

where

$$S = \beta^2 \cosh \beta x + \alpha^2 \cos \alpha x$$

$$T = \beta \sinh \beta x + \alpha \sin \alpha x$$

$$U = \cosh \beta x - \cos \alpha x$$

$$V = (1/\beta) \sinh \beta x - (1/\alpha) \sin \alpha x$$

$$S' = \alpha^2 \cosh \beta x + \beta^2 \cos \alpha x$$

$$T' = (\alpha^2/\beta) \sinh \beta x + (\beta^2/\alpha) \sin \alpha x$$

$$U' = \beta^4 \cosh \beta x - \alpha^4 \cos \alpha x$$

$$V' = \beta^3 \sinh \beta x - \alpha^3 \sin \alpha x$$

To determine the boundary conditions of $W(0)$, $\dot{W}(0)$, $\ddot{W}(0)$, and $\ddot{W}(0)$, we assume the next conditions at both sides of the beam.

At $x=0$

$$W(0)e^{i\omega t} = Y_a, \quad \dot{W}(0)e^{i\omega t} = \phi_a, \quad EI\ddot{W}(0)e^{i\omega t} = M_a, \quad [EI\ddot{W}(0) + \omega^2 I\rho\dot{W}(0)]e^{i\omega t} = F_a$$

At $x=l$

$$W(l)e^{i\omega t} = Y_b, \quad \dot{W}(l)e^{i\omega t} = \phi_b, \quad EI\ddot{W}(l)e^{i\omega t} = M_b, \quad [EI\ddot{W}(l) + \omega^2 I\rho\dot{W}(l)]e^{i\omega t} = F_b$$

Above relations are reduced to the next transfer matrix between the sides "a" and "b".

$$\begin{bmatrix} Y_a \\ \phi_a \\ M_a \\ F_a \end{bmatrix} = \frac{1}{\alpha^2 + \beta^2} \begin{bmatrix} S & -T & U/EI & -V/EI \\ -\alpha^2\beta^2V & S & -T/EI & U/EI \\ EI\alpha^2\beta^2U & -EIV' & S & -T \\ -EI\alpha^2\beta^2T' & EI\alpha^2\beta^2U & -\alpha^2\beta^2V & S' \end{bmatrix} \begin{bmatrix} Y_b \\ \phi_b \\ M_b \\ F_b \end{bmatrix} \quad (8)$$

When the next relation has been assumed,

$$\frac{\gamma^4}{4} \ll \theta^4 \quad \text{or} \quad \frac{I}{A} \omega^2 \ll \frac{4F}{\rho} \quad (9)$$

α becomes equal nearly to β , and the Eq. (8) is simplified, but, as the relation (9) points out, the inertia force can not be neglected above a certain frequency range determined by Young's modulus and the density of material. Because the condition that the thickness of a steel beam is about 1m and the required frequency range is within 500 Hz conforms to the requirement (9), then the transfer matrix is reduced to what relates only to the bending moment. More discussions about this point have been made in the preceding paper⁽²⁾ written by authors, in which the next matrix is introduced where the variables M_a and F_b are exchanged in the both column vectors of Eq. (8).

$$\begin{bmatrix} Y_a \\ \phi_a \\ F_a \\ F_b \end{bmatrix} = \frac{1}{\sinh \theta l + \cosh \theta l} \begin{bmatrix} (\sinh \theta l \cosh \theta l + \cosh \theta l \sinh \theta l), & -2(\sinh \theta l \cosh \theta l)/\theta, & (\cosh \theta l \sinh \theta l - \sinh \theta l \cosh \theta l)/EI\theta^2, & (\sinh \theta l - \cosh \theta l)/EI\theta^2 \\ \theta(1 - \cosh \theta l \cosh \theta l), & (\cosh \theta l \sinh \theta l + \sinh \theta l \cosh \theta l), & -(\sinh \theta l \cosh \theta l)/EI\theta, & -(\cosh \theta l - \sinh \theta l)/EI\theta \\ -EI\theta^3(\sinh \theta l \cosh \theta l), & EI\theta^2(\cosh \theta l \sinh \theta l - \sinh \theta l \cosh \theta l), & \theta(\cosh \theta l \cosh \theta l + 1), & -\theta(\cosh \theta l + \sinh \theta l) \\ EI\theta^3(\cosh \theta l - \sinh \theta l), & EI\theta^2(\sinh \theta l - \cosh \theta l), & \theta(\cosh \theta l + \sinh \theta l), & -2\theta \end{bmatrix} \begin{bmatrix} Y_b \\ \phi_b \\ M_b \\ M_a \end{bmatrix} \quad (10)$$

where $sh = \sinh$, $ch = \cosh$, $s = \sin$, $c = \cos$.

In this equation each term consists of a transcendental function, so it is necessary to reform each term into a series expansion which is limited to the first three terms. By this procedure, we obtain the S. F. G. of Fig. 2 from Eq. (10), where "S" denotes the Laplace operator. If M_a and ϕ_b are exchanged in Eq. (8), the relationship is represented by the following Eq. (11) and Fig. 3. In this manner, the Eq. (8) is reformed to various equations, for example, Eq. (10) and Eq. (11).

$$\begin{bmatrix} Y_a \\ \phi_a \\ F_a \\ \phi_b \end{bmatrix} \parallel \frac{1}{(sh\theta l - s\theta l)} \begin{bmatrix} (sh\theta l c\theta l - ch\theta l s\theta l), & -(ch\theta l s\theta l + sh\theta l c\theta l)/EI\theta^2, & (2sh\theta l s\theta l)/EI\theta^3, & (sh\theta l + s\theta l)/EI\theta^2 \\ \theta(sh\theta l s\theta l), & (ch\theta l c\theta l + 1)/EI\theta, & -(sh\theta l c\theta l + ch\theta l s\theta l)/EI\theta^2, & -(ch\theta l + c\theta l)/EI\theta \\ EI\theta^3(1 - ch\theta l c\theta l), & \theta(sh\theta l s\theta l), & (sh\theta l c\theta l - ch\theta l s\theta l), & -\theta(ch\theta l - c\theta l) \\ \theta(ch\theta l - c\theta l), & (ch\theta l + c\theta l)/EI\theta, & -(sh\theta l + s\theta l)/EI\theta^2, & -2/EI\theta \end{bmatrix} \begin{bmatrix} Y_b \\ M_b \\ F_b \\ M_a \end{bmatrix} \quad \dots\dots\dots(11)$$

3. Analysis of mechanical vibration systems anchored to ground

Mainly the chatter vibration is considered to be due to the lack of a damping factor in the structure of machine-tools.

The machine tools are anchored ordinarily to the ground with anchor bolts.

In this section the possibility for improvement of dynamic characteristics in the structure is investigated when viscous or coulomb friction is assumed between the vibration system and the ground.

3.1 Experimental device and its S. F. G. representation

The experimental device as shown in Fig. 4 is supposed to simulate the feedback vibration system, because the ordinary machine tools are J-shaped structures.

This system is driven by an electromagnetic dynamic driver having a constant relation between its electric current and the driving force. The relative displacement at the driving point is measured by an inductance type pick-up. The results

are obtained by the use of a servo analyzer. The experiment is carried out with this experimental device which is either anchored to ground at its bottom or suspended in the air by some wires and the results of both experiments are compared.

In order to analyze the system in this case, its structure is supposed to be partitioned into two concentrated masses and three beams, through which the S. F. G. are determined from their boundary conditions respectively. The coordinates of the variables are shown in Fig. 6. The S. F. G. of beams denoted by (I) and (III) should be

Fig. 4 Experimental apparatus as to closed-loop vibration

Fig. 2 S. F. G. representation of beam (Case 1)

Fig. 3 S. F. G. representation of beam (Case 2)

Fig. 5 Outline of J-shaped structure

so arranged as the moment is located at the input node in the S. F. G. because at the upper extremity of beams (I) and (III), M_1 and M_5 are equal to zero.

[Boundary condition]

$$\left\{ \begin{array}{ll} F_{in} = F_1' = F_6' & Y_1 + Y_{out} = -Y_6 \\ F_1 - F_1' + m_1 S^2 Y_1 = 0 & M_1 = M_6 = 0 \\ F_3 - m_2 S^2 Y_3 = 0 & \phi_2 = \phi_3, M_2 = M_3 \\ -F_4 + m_4 S^2 Y_4 = 0 & \phi_4 = \phi_5, M_4 = M_5 \\ F_2 + F_3 + m_3 S^2 Y_5 = 0 & Y_2 = -Y_5 \\ F_6' + F_6 - m_5 S^2 Y_6 = 0 & \end{array} \right.$$

[Data]

$$\left\{ \begin{array}{ll} E = 2.1 \times 10^9 \text{ kg/cm sec} & m_5 = 0.095 \text{ kg} \\ m_1 = 0.64 \text{ kg} & I_2 = 0.0531 \text{ cm}^4 \\ m_2 = 0.438 \text{ kg} & I_3 = 11.3 \text{ cm}^4 \\ m_3 = 2.34 \text{ kg} & I_4 = 0.0256 \text{ cm}^4 \\ m_4 = 0.33 \text{ kg} & \end{array} \right.$$

Unknown variables such as ϕ_1 and ϕ_6 must take place in output nodes. In this way the S. F. G. corresponding to this system is represented by Fig. 7 in which the necessary branches are denoted by thick lines.

(Real value)

$$\left\{ \begin{array}{ll} E = 2.1 \times 10^9 \text{ kg/cm sec} & m_5 = 0.095 \text{ kg} \\ m_1 = 0.64 \text{ kg} & I_2 = 0.0531 \text{ cm}^4 \\ m_2 = 0.438 \text{ kg} & I_3 = 11.3 \text{ cm}^4 \\ m_3 = 2.34 \text{ kg} & I_4 = 0.0256 \text{ cm}^4 \\ m_4 = 0.33 \text{ kg} & \end{array} \right.$$

Fig. 6 Division into three fundamental elements and their notation

Fig. 7 S. F. G. representation of closed-loop vibration system

In general the stiffness at the bottom of the beam is much higher than that of other beams, so that the slope angles at the both sides of this beam, ϕ_2 , ϕ_3 , ϕ_4 , and ϕ_5 are negligible. In this way Fig. 7 is reduced to Fig. 8, where the transfer characteristic to the ground is denoted by z .

3.2 Examination of the results

Comparison of experimental with theoretical data is shown in Figs. 9 and 10 where the results of experiments with anchored and suspended bodies are indicated respectively by the Bode diagram. Scale factors in the theoretical analysis are shown in Fig. 8. In this figure the condition of $m_3 \rightarrow \infty$, that is $1/m_3 \rightarrow 0$, corresponds to fixing the bottom of a machine tool body to the ground.

This approximation produces good results for this case, so that hereafter the transfer characteristics between a body and the ground are investigated by the use of Fig. 8.

The analytical results when the Coulomb friction is assumed are given in Fig. 11. In this figure, it is clear that corresponding to input magnitude, the damping effect differs between the rigidly anchored body and the suspended one.

The effect of the viscous damping is shown by Fig. 12. According to this result, the dynamic characteristics of a machine tool are improved to some extent without direct application of a damper in the vicinity of the cutting point.

(Scale factor)

$$\left[\begin{array}{l} S = 5 \times 10^2 P, Y = \bar{Y}, F = 10^2 \bar{F} \\ \phi = 10^{-1} \bar{\phi}, M = 10^2 \bar{M} \end{array} \right]$$

Fig. 8 Simplified S. F. G. of the one shown in Fig. 7

4. A countermeasure for the chatter vibration of a vertical lathe

The method of analysis employed in this paper can be applied to develop a countermeasure for elimina-

Fig. 9 Dynamic compliance in the case of fixed boundary condition at the structure bottom

Fig. 10 Dynamic compliance in the case of free boundary condition at the structure bottom

Fig. 11 The computed dynamic compliance depends on harmonic exciting force of different amplitudes (In the case of coulomb friction as z)

Fig. 12 The computed dynamic compliance
(In the case of linear damping as z)

tion of the chatter vibration which occurs in a horizontal boring machine and a vertical lathe. In these cases, the shape of work is complex and requires a long movement of the cutter head, so the ram length must be long, resulting in an unpreferable condition from the standpoint of the chattering. The method described above is applied to these cases. Herein, an experimental model of a vertical lathe is constructed for the sake of comparison between theory and experiment.

4.1 Experiments and measurements

Figures 13 and 14 show the experimental structure of a vertical lathe in which violent

Fig. 13 Experimental structure supposed as the real vertical lathe

Fig. 14 Outline of the structure

chattering characteristic is given deliberately. The relative displacement corresponding to the cutting depth is measured by an inductance-type pick up.

Measurement of the dynamic characteristics of the whole body is done by suspending it in the air, and an unpredictable transfer from the ground can be avoided and the measured data become reliable. By inserting an electrohydraulic vibration generator and a spring type force pick up, a feedback vibration system is realized.

The vertically moving ram is supported on a gate-shaped column by way of washer springs.

4.2 S. F. G. of the experimental structure

The structure shown by Figs. 13 and 14 comprises the following five fundamental elements.

The S. F. Gs. of each of these elements are constructed as in Fig. 15, where the boundary conditions are taken into account for each element, and their combination develops the S. F. G. representation of the whole system.

The boundary conditions are represented by the following equations. Here the two columns are supposed to vibrate symmetrically so that they are represented theoretically by one column.

[Boundary condition] cf. Figs. 14 and 15

At the lower end of the element (1)

$$F_1 + m_0 S^2 Y_1 = F_{in}, \quad Y_1 = Y_{out}, \quad M_1 = 0$$

At the connecting point between the elements (1) and (2)

$$Y_2 = Y_3, \quad \phi_2 = \phi_3, \quad M_2 = M_3, \quad F_{k1} = F_2 - F_3,$$

$$Y_2 = (1/k) F_{k1} - Y_7$$

At the connecting point between the elements (2) and (3)

$$M_4 = 0, \quad F_{k2} = F_4,$$

$$Y_4 = (1/k) F_{k2} - Y_5,$$

$$M_5 = 0, \quad -F_{k2} = F_5$$

At the connecting point between the elements (3) and (4)

$$M_6 = M_7, \quad \phi_6 = \phi_7,$$

$$F_7 = F_6 - F_{k1}, \quad Y_6 = Y_7$$

At the connecting point between the elements (4) and (5)

$$Y_8 = (1/m_5 S^2)(F_8 + F_{10}),$$

$$\phi_8 = 0, \quad Y_8 = Y_9, \quad F_8 = F_9$$

At the setting point of the slider on the bed

$$Y_{10} = -Y_9$$

The S. F. Gs. of all elements must be put together in the manner in which the configuration of variables satisfies the boundary condition. For example, consider the unknown variables, ϕ_1 , ϕ_4 , and ϕ_5 .

If these variables are considered as the input signals and arranged at the input nodes in S. F. G., the magnitudes of input signals are undetermined, making the computation impossible. In these cases care must be taken in dealing with these variables so that they do not influence the solution. According to this consideration, these unknown variables must be arranged in output nodes and the S. F. G. representing a vertical lathe is given by Fig. 16.

Fig. 15 Division into five fundamental components and their notation

Fig. 16 S. F. G representation of the vertical lathe structure shown in Fig. 14

Figure 16 is fairly complex as it is, so when the capacity of the analog computer is relatively small, or the requirement for the computing accuracy is not severe, the following assumptions are possible.

(1) Stiffness of the column is much larger than that of the spring supporting the ram.

(2) Vice versa of the above.

The analysis of these two cases is useful in considering a real structure, in which the characteristic lies between the above mentioned two assumptions.

4.3 Dynamic stiffness of a ram suspended by an elastic element

When the stiffness of the column is larger enough than that of the spring supporting the ram, the displacements of the column, Y_5 and Y_7 are negligible and the signals from those nodes are not supposed to appear, so the lower half of the S. F. G. can be eliminated. The reason why we take this assumption is to know how the

spring constant K influences the dynamic characteristics of the ram. The analog computer circuit which simulates the displacement Y_{out} corresponding to the input force F_{in} at the lowest point of the ram, is given in Fig. 17.

Figure 18 presents the natural frequency of the ram when the ram length l_1 and the spring constant K are varied. From this relation, in the case of $K > 5 \times 10^6 \text{ kg}^*/\text{cm}$ it may be possible for the ram to be rigidly fixed to the column. Further Fig. 18 is useful in designing the ram system with a dynamic damper provided in the ram which moves vertically.

The value of K is given in engineering units, though other constants are given in physical units.

4.4 Analysis of self excited vibration in a vertical lathe

In this section, the analysis of the self excited vibration which takes place in the actual cutting action, and a countermeasure for this phenomenon are described employing the mathematical model-

Fig. 17 Computer circuit to analyze dynamic stiffness of the ram supported by elastic elements

Fig. 18 Natural frequency of the ram versus spring constant for various ram lengths

Pot. No	Transfer function	Pot. No	Transfer function
P1	l_1	P13	$6l_1/m_2l_2^2$
P2	$1/m_0$	P14	l_1, l_2
P3	m_0	P15	$m_2/6$
P4	$l_1^3/3EI_1$	P16	$m_2/3$
P5	$0.01\sqrt{(P8)}$	P17	$3l_1/l_2$
P6	$m_1l_1/2$	P18	2
P7	$m_1l_1/2$	P19	$3m_2/4$
P8	$8EI_1/m_1l_1^3$	P20	1
P9	$m_1l_1/6$	P21	l_1
P10	$m_1/2$	P22	$l_1^3/3EI_1$
P11	$3/l_2$	P23	$1/k$
P12	$6/m_2l_2$	P24	$1/k$

Fig. 19 Constitution of feedback oscillation system

ing method. The self excited vibration is regarded as a nonlinear feedback vibration⁽³⁾ and this vibration system is illustrated by Fig. 19. This system comprises the electrohydraulic vibration generator fixed on the bed and the spring type force detector inserted between the actuator and the tool box provided at the lowest end of the ram, where the spring deflection is considered to represent the force applied to the ram.

This force detector consists of a spring and a displacement pick up by which the cutting impedance can be measured. The relation between cutting force and cutting depth is considered similar to the spring stiffness and this is called the cutting stiffness.

For the analysis the characteristics of the

Fig. 20 S. F. G representation of apparatus shown in Fig. 19

Fig. 21 Computer circuit of system shown in Fig. 19

Fig. 22 Computer circuit of vertical lathe structure

vibration generator and the force detector must be known in advance. The displacement, velocity and acceleration feedback servo control is employed in this vibrator⁽⁴⁾. The S.F.G. of this feedback vibration system is shown by Fig. 20. All constants are expressed in physical units. The scale factors for the time, force, moment, slope angle, displacement, hydraulic pressure and the electric voltage are as follows respectively:

$$F=10^7 \bar{F}, \quad M=10^6 \bar{M}, \quad \phi=10^{-2} \bar{\phi}, \quad Y=\bar{Y}, \\ P=10^6 \bar{P}, \quad V=10^2 \bar{V}$$

$$\text{Here } G(S) = \frac{0.48 \times 10^6}{\left(\frac{1}{300}S+1\right)\left(\frac{1}{(1800)^2}S^2 + \frac{0.8}{1800}S+1\right)} \quad \text{kg/sec}^2\text{cmV}$$

(Transfer function of servo valve)

$A=11 \text{ cm}$(Area of cylinder bore)

$m_a=6 \text{ kg}$(Mass of actuator)

$K=7.7 \times 10 \text{ kg/sec}$ (Spring constant of force pick up)

$K_p=0.164 \times 10 \text{ V/cm}$ (Position feedback constant)

$K_v=0.52 \times 10 \text{ V/cm sec}$ (Velocity feedback constant)

$r_p=0.9 \times 10 \text{ kg/cm sec}$ (Internal resistance of servo valve)

Pot. No.	Transmittance	Real value
P1	$l_1^2/3EI$	3.41
P2	l_1	0.65
P3	$1/m_0$	1.25
P4	m_0	0.8
P5	$m_1 l_1/3$	0.232
P6	$m_1/2$	0.054
P7	$8EI_1/m_1 l_1^3$	0.725
P8	$m_1 l_1/6$	0.116
P9	$m_1/2$	0.054
P10	m_3	2
P11	$m_1 l_1^2 l_1/6EI_1$	0.486
P12	m_4	1.6
P13	$0.01\sqrt{8EI_4/m_4 l_1^3}$	0.1
P14	$8EI_4/m_4 l_1^3$	1.97
P15	$1/m_3$	0.37
P16	$6/l_1$	0.67
P17	$6/m_1 l_1$	0.417
P18	$6/m_4 l_1$	0.417
P19	$3/m_4$	1.88
P20	$3/m_4$	1.88
P21	l_1	0.65
P22	$l_1^2/3EI_1$	3.41

The computation circuit of the analog computer using the above scale factors is shown in Fig. 21. The computation circuit for the structure dynamics is given by Fig. 22, where the same scale factors are employed. In this case the ram length l_1 is 65 cm. This circuit corresponds to the case where the spring stiffness supporting the ram is larger enough than that of the column. In this case the transmittances including k are omitted owing to the condition, $1/k \rightarrow 0$. As the result of above discussion, at upper side of the ram, the conditions of $Y_2 = -Y_7$ and $\phi_2 = \phi_7$ are satisfied, so the right hand half of S.F.G. is eliminated except in the next relations,

$$F_7 = -m_3 S^2 Y_7 - F_2$$

$$M_7 = \frac{l_3}{l_2} M_2 + \frac{(m_3 l_3) 3EI}{(m_4 l_4 + 3m_3 l_3) l_4} \times \left(\frac{m_4 l_4^2 l_3}{9EI_4} S^2 - 1 \right) \phi_7$$

Fig. 23 Input force characteristics shown by Bode diagram at driving point

Fig. 24 Displacement characteristics shown by Bode diagram at driven point

Figure 22 is obtained after the terms of S^2 are properly arranged. In this circuit, the results of theory and experiment are compared when the ram length is 65 cm. Figure 23 indicates the input force and Fig. 24 the relative displacement between the ram and the bed in the Bode diagram.

These frequency characteristics are obtained with the standard frequency oscillator. The dynamic stiffness can be obtained by subtracting the Bode diagram of Fig. 24 from that of Fig. 23, where we must be careful about the gain and phase characteristics. This method has the advantage of using the Bode diagram.

In Fig. 23 and Fig. 24 the first natural frequency in the experiment is lower than that in the theory. This may be attributed to the neglect of the spring effect in the theory. At higher frequencies several higher modes are observed in the experiment, and this phenomenon can be explained even with a fairly simple assumption. If calculation of the dynamic stiffness at the base of the ram is required, the values of $\bar{F}_2$ and $\bar{M}_2$ should be calculated by Fig. 21 and 22. In this analysis the solution was obtained by assuming the structure as suspended so that the effect of the earth is avoided.

On the contrary if the solution for the body anchored to ground is required, the mass of bed m_3 must be made infinite so that the condition $1/m_3 \rightarrow 0$ is applied to S.F.G. As shown by the

Fig. 25 Modeling and its S. F. G. representation of the simplest cutting mechanism

Fig. 26 Ratio of u to u_e depends on K_c

mathematical modeling of Fig. 22, the optimum design of a column is made possible. Using the computation circuit described in this section, it is possible to discuss the cutting process including the machine dynamics.

4.5 Analysis of the cutting process

In the previous section the analog computation circuit is given for the case where the vibration is included in the closed loop. In this section the same modeling is obtained except that the cutting process is included instead of the vibration generator.

Taking into consideration the desired cut u_0 , the real cut u , the cutting stiffness K_c and the cutting force F_c , the S.F.G. in this case is shown as in Fig. 25 where the effects of the regenerative vibration, negative-damping and backlash are not considered. From Fig. 25 the dynamic characteristics between the required cut and the real cut

Fig. 27 Chatter observed during cutting

Fig. 28 Outline of dynamic damper

Fig. 29 S. F. G representation of the dynamic damper

are given by Fig. 26. In this case an abrupt change of the cutting force is observed at about 80 Hz. Therefore a slight change of the frequency gives rise to a violent change of the cutting force. However, the phase lag at the driving point is within 180° so that self excitation does not occur even though the influence of the external force is great. In general the regenerative effect, negative-damping⁽⁵⁾, and backlash make this system quite unstable, that is, self excitation is apt to occur. The dynamic mechanism of self excitation is explained by the application of various effects⁽⁶⁾ to the fundamental S.F.G. of Fig. 25. But the details about this point will be given in the papers in future.

The chatter traces at the cutting depth 0.2 mm and 0.4 mm are shown in Fig. 27. The natural frequency of Fig. 27 is about 70 Hz and this frequency can be predicted by the analysis of Fig. 26. In this way Fig. 25 and its development make possible the prediction of the natural frequency of the structure and the method to improve the design of the structure prior to the final design.

Fig. 30 Polar coordinate plotting of resultant displacement to exciting force

Fig. 31 Comparison of resulting traces both with and without additional damper

4.6 Countermeasure for self excitation in a vertical lathe

Among many factors which cause self excitation the most fundamental one is poor dynamic stiffness. the countermeasure for this is to eliminate the undamped resonances from the frequency range under consideration.

For this purpose, the resonant frequency of the structure must be made high or the mode control theory must be applied, but these methods are difficult and the problem is left for future solution. In practice a realizable countermeasure is to adopt a dynamic damper having a viscous element. Figure 28 presents the detail of this damper, where k_d is the spring constant of the cantilever retaining the sliding mass m_d . The damping factor D is given by the oil within this device. Assuming F_d is the reaction force, we have next equation.

$$F_1 + F_d + m_0 S^2 Y_{out} = F_{in}$$

S. F. G. of this damper is shown by Fig. 29. By inserting this S. F. G. between F_{in} and Y_{out} of Figs. 16 and 22, we can obtain the S. F. G. representing the whole system which incorporates the dynamic damper. Using this method a suitable damper is designed. In Fig. 30, the Nyquist diagrams of the dynamic characteristics are given for the experiments with and without this damper. The dynamic stiffness is the ratio of the displacement and the force, and the improvement of the dynamic stiffness is clarified by Fig. 30.

Figure 31 shows the results of actual cutting.

Fig. 32 Surface of test workpiece where the dynamic damper is activated off and on

With this damper a good surface is obtained even though the dynamic characteristics of the structure is unsatisfactory. Figure 32 shows the surface when the dynamic damper has been operated intermittently. On account of that the dynamic stiffness of a ram can be adjusted easily by sliding the mass, this damper is intended for practical use.

5. Conclusion

In this paper it was ascertained that the analog computer method for the simulation of a vibratory system can be conveniently applied to the analysis of the gate type machine-tools. By a slight modification of this method, the effect of the rotational motion on the column dynamics can be treated. If the computation accuracy needs to be improved, some skill is necessary in application of this method. Authors are grateful to the help in experiments given by Mr. Kamiyama and Mr. Murai.

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Discussion

M. KURODA (Seikei University):

(1) In the extent of the practical use, in

spite of the magnitude of θl being far larger than unity, you say that good. precise approximation

can be obtained even by putting as follows:

$$e^{\theta l} = 1 + \theta l + \frac{(\theta l)^2}{2} + \dots \approx 1 + \theta l$$

On what ground do you dare say that?

(2) In the explanation of the authors, it has been stated below Eq. (10) that "a lumped parameter system is expressed by taking no account of above three terms of the series expansion".

However, depending on the occasion, it seems the system may well be approximated up to the terms containing S^4 .

(3) In computing an approximate value of the natural frequency, I wish to keep an error to within a few per cent. In the case of Fig. 23, there is a difference of about 20% in quantity between the theoretical and experimental values. To compute the natural frequency or to obtain the level of frequency response in a frequency range beyond the fundamental one, the application of the S.F.G. of Figs. 7 and 16 seems theoretically impossible.

H. SATO (University of Tokyo):

(4) About Figs. 9 and 10, the way to suspend is not known from the paper, so that this question might be out of the point. However, examining the figure of the structure, it seems the beam in the component (II) has too large a mass and too big a moment of inertia of the cross-section comparing with other components.

Besides, considering the point that the whole figure is in a tonometer-shaped structure it seems that one of the two peaks in Fig. 9 is shifted too much in Fig. 10. I would like to know what is your interpretation of it.

In the condition of the suspension, I would also like to know the reason that the difference between the theoretical value and the experimental one in higher frequency is so great.

(5) I would like to know how authors deal with the damping in the theoretical solution.

(6) In Fig. 11, authors consider the damping factor depending on the friction characteristic. However, in an actual problem it is generally considered that a damping effect of vibrating the ground around the bottom when the machine-tool vibrates is greater than that.

H. TOMITA (Toshiba Electric Co.):

(7) Thanks to the improvement of measuring technics, the dynamic analysis of structures from the viewpoint of mechanical impedance has become generally possible.

In this case there is a general tendency to use a versatile digital computer for analysis since in many cases, objects with complex structures

have to be treated. Despite this tendency, the authors' suggestion to reconsider the advantage of analogue computer is valid.

The point authors are insisting upon in this paper seems to be the validity of the analysis they devised.

However, as far as the questioner comprehends, it seems there cannot be seen in this paper any progress in any of the expressions of the beam property by transfer matrix, utilization of the signal flow graph, and simulation by analogue method. In the sense of expanding the limitation of the possibility of analogue methods carried out up to this time, what kind of a new proposal do the authors suggest in this paper?

S. OHNO (University of Tokyo):

(8) I suppose that Fig. 26 is obtained by applying the sinusoidal input U_0 to the cutting point. What does it mean in actual cutting process that U_0 changes sinusoidally?

(9) How and where was the structure supported when the vibration test was carried out?

(10) In Fig. 9 the experimental value and the theoretical value agree rather well with each other. It seems that this agreement resulted from the condition that the structure was tested using a model with the base fixed rigidly. But I think that in an actual machine tool the evaluation of the stiffness of the base is difficult and affects much the result of calculation.

H. SATO (University of Tokyo):

(11) In the explanation of the authors, "the natural frequency of the cantilever becomes lower as the restriction at the retained end increases...", I cannot understand how authors used the meaning of "restriction increases", but I think it is not "becomes lower" but "becomes higher".

(12) About the damping factor $\zeta = 0.05$. I suppose the value of ζ is generally smaller in a smaller model as in Fig. 5 and then I would like to know whether the experimental value of the gain diagram in Figs. 9 and 10 or the damping factor obtained otherwise becomes such a value.

Authors' closure

(1) First let me add a few more words on this point which caused misunderstanding.

In regard to our approximative method, Mr. Kuroda interprets as approximate the first two terms of Taylor's expansion, that is,

$$e^{\theta l} = 1 + \theta l + \frac{(\theta l)^2}{2} \dots \approx 1 + \theta l$$

but our method depends on the series expansion of the triangular and the hyperbolic functions as fol-

lows,

$$\sin \theta l = \theta l - (\theta l)^3/3! + (\theta l)^5/5! \dots = \theta l - (\theta l)^3/6$$

$$\cosh \theta l = 1 + (\theta l)^2/2! + (\theta l)^4/4! \dots = 1 + (\theta l)^2/2$$

Hence its approximation corresponds to employing the first four terms in Taylor's expansion.

(2) Though in this paper the S.F.G was arranged in the term S^2 , arrangement in the term S^4 , as Mr. Kuroda pointed out, can be realized by taking up to the first six terms of Taylor's expansion.

However, it does not necessarily follow that the precision of eigen values is improved as the result. Let us prove it with a cantilever as a typical example. Solving the precise solution, we find

$$\omega_1 = \sqrt{\frac{12.2EI}{ml^3}}, \quad \omega_2 = \sqrt{\frac{485EI}{ml^3}},$$

$$\omega_3 = \sqrt{\frac{3780EI}{ml^3}}$$

Against this, we obtain an approximate solution from each term of the series expansion as follows,

(1) up to the first two terms (corresponding to S^2)

$$\omega_1 = \sqrt{8EI/ml^3}$$

(2) up to the first three terms (corresponding to S^4)

$$\omega_1 = \sqrt{6.5EI/ml^3}, \quad \omega_2 = \sqrt{90EI/ml^3}$$

It is found that the former is closer to the precise solution than the latter at the fundamental frequency, and it seems the error in the latter originates from exhibiting two natural frequencies ω_1 and ω_2 by only three terms.

Therefore, for obtaining a solution closer to the precise solution at the fundamental frequency, it is necessary to take at least the first eight terms in Taylor's expansion. The resultant transmittances in the S.F.G are represented by the function of S^8 , but such representation needs a complicated analogue computer circuit.

In view of the above-mentioned fact, we doubt the advisability of the plan. On the other hand, according to our approximate method, the S.F.G

is constituted by the computer circuit consisting of simple units as shown in Append. Fig. 1, and higher precision is obtained by increasing the partitioning number of the beam under consideration. Now consider partitioning into two sections in the same problem. The resultant natural frequencies are

$$\omega_1 = \sqrt{\frac{11.7EI}{ml^3}}, \quad \omega_2 = \sqrt{\frac{170EI}{ml^3}}$$

With respect to precision, it is found that increasing the partitioning number is more effective than the other method.

(3) Mr. Kuroda calls into question the statement "there is difference of about 20% in quantity between the theoretical ...". However by considering that our method naturally makes the solution smaller than the exact solution in the eigen value, it is recognized that the error depends on mainly the above-mentioned assumption (1), that is, assuming as infinite the stiffness of spring supporting the ram in the theoretical solution.

Actually, by employing the spring with high stiffness, the experimental solution has increased to 74 Hz at natural frequency. Accordingly if the effect of this spring is taken into account in the theoretical analysis, the error is reduced without fail.

(4) Usually, the machine tools are fixed on the base with connecting elements. We first mentioned that the purpose in this section was to investigate whether the damping can be effectively increased or not by insertion of a damping material between the machine tools and their base, that is, by positive use of the machine-tool suspension method,

For realization of the free boundary condition, the structure shown in Fig. 4 is suspended by strings from all sides of its beam (II). As the resonant frequency consisting of the strings and the structure is about 3 Hz, a coupled vibration effect between the structure and the suspended setup can be neglected.

Append. Fig. 1

Append. Fig. 2

The natural frequency of the cantilever decreases when restricted heavily at the retained end, and when the motion is completely restricted at the end, it indicates the lowest value. As the restriction is unfixed by supporting freely, the natural frequency of the structure becomes higher than under the fixed boundary condition.

Moreover the coupled frequencies always lie outside the uncoupled ones, that is, above the higher uncoupled frequency and below the lower uncoupled one. The fixed structure corresponds to an uncoupled vibration whereas the free boundary condition corresponds to a coupled vibration.

For the above-mentioned reason, it is illustrated that the higher frequency under the fixed boundary condition (c.f. Fig. 9) becomes higher with the restriction unfixed (c.f. Fig. 10).

Next, let us explain the great difference between the theoretical value and the experimental value at higher frequencies as shown in Fig. 10. Because of error the eigen values in the energy method are apt to be larger than the true values, whereas our method makes the solution smaller than the exact solution and the difference of the two is great at higher frequencies. Moreover as compared with the result shown in Fig. 9 involving relatively slight errors due to the uncoupled condition, the theoretical value in Fig. 10 has been computed from a more complicated S.F.G. due to the coupled condition, that is, as finite m_3 . For this reason, the difference between both values occurs to be great at higher frequencies.

(5) In the vibration analysis employing an analog computer, an undamped system cannot be dealt with. However the transmittances of the S.F.G. resulting from the fundamental equation (I) do not include the terms of internal energy loss, so they take on an undamped system.

In practice, mechanical elements have a slight damping factor of about $\zeta=0.05$, so that by adding the term of S the following modification in the main transmittance $G(s)=1/(1+ml^3S^2/8EI)$ must be made in the case of calculus with the analog computer.

$$G(S)=1/(1+0.1\sqrt{ml^3/8EI}S+ml^3S^2/8EI)$$

(6) Commonly the machine tools are fixed to the base by bolts or other suitable method without quantitative examination of damping effect from the base.

In this paper, supposing the base is immobile, it has been theoretically investigated whether the structure damping can be increased or not by insertion of a damping material between the machine and the base, as positive employment of the ma-

chine-suspension method.

However, since the dynamic characteristic of the base may not be neglected as Mr. Sato pointed out, the characteristics must be known in designing the damping material.

(7) Recently, it has been definitely shown by many papers that machine-tool chatter is closely related with its structure dynamics. For example, according to Merritt's paper, the machine tool cutting process can be represented by a closed-loop diagram as shown in Append. Fig. 2 which enables us to apply control techniques to the formulation of the problem. Thereafter, his theory is developed to what permits calculation of borderlines of stability for a structure having n -degrees of freedom, and it has been highly appreciated by other persons as a suitable stability criterion method. However, it is a fact that structure dynamics which is the most principal term in the closed-loop diagram is obtained by taking frequency-response data of ready-made machine tools or using a rough mathematical model.

To predict the dynamics of the machine-tool

Append.-Fig. 3

Append.-Fig. 4

Append.-Fig. 5

structure in the preliminary design and to investigate more complicated phenomena in the cutting process, the authors have developed a mathematical model of the structure by employing a signal flow graph which helps direct simulation of the dynamic system by the analog computer.

As Mr. Tomita pointed out, as far as utilization of the signal flow graph, expression of the beam property by transfer matrix, and simulation by manner of analog method are concerned, individually they are not anything out of the ordinary.

Even if the analog computer is useful in analysis of the dynamic systems with nonlinearity and damping, it has not been well employed due to lack of a suitable programming method for the complicated structure dynamics. Concerning this point, this paper gives some useful techniques for programming like the subroutine method in the digital computer, by which a simple and systematic model can be devised. So this analytical method has an advantage over the normal method in direct comparison between theory and real physical phenomena, and this method is considered to supplement the matrix method utilizing the digital computer. Therefore the S.F.G. has wide application in addition to the examples described in this paper. In regard to the S.F.G. representation method of fundamental component and its application, the authors have shown some examples in the foregoing paper⁽²⁾.

(8) As is generally known in the linear control theory, the characterization of a linear system by a transfer function depending on the complex frequency "s" maintains a specific correlation between time domain and frequency domain, and permits the designer to consider simultaneously both transient and frequency characteristics.

Accordingly, if the relationship between the real cutting thickness U and the commanded cutting depth U_0 is arranged as the frequency characteristics, we can advantageously presume the transient response of U to U_0 employing this result.

In this case the authors would like to think that this means any disturbance which occurs in the cutting process besides the commanded cutting depth.

(9) Since a scaffold was assembled with steel pipes to a height of about 2 m, the experimental apparatus shown in Figs. 13 and 14 was suspended on four wires hanging from the upper end of the scaffold.

(10) The purpose in section 3 was to investigate whether the damping can be increased or

not by insertion of a damping material between the bottom of the machine tool and its base, that is, by positive employment of the machine-tool suspension method. As a result, it was found that the suspension method introduced appropriate damping elements which could contribute to improvement of the dynamic stiffness of the structure. As Mr. Ohno pointed out, it also seems that installation of the machine tools must be theoretically evaluated and examined as to whether it can contribute to a better dynamic stiffness of the machine tools hereafter.

(11) The term "heavily restricted at the retained end of the cantilevers" means that the motion at its end is heavily restricted. Let us explain the term employing the Append.-Fig. 2, where the end "b" corresponds to the retained end.

Now by assuming that the mass M which is movable only along the longitudinal axes is connected to the end "b", the degree of restriction should be evaluated by the size of the mass, for example, assuming M to be infinite, the fixed boundary condition is imaginarily realized here. Therefore by introducing M as a parameter, we illustrate the meaning of the foregoing.

Referring to the Append.-Fig. 4, the boundary conditions in this case are $\phi_b = 0$, $M_a = 0$ and $F_b = MS^2 Y_b$. Introducing these boundary conditions into Fig. 2, a mathematical model of the structure shown in the Append.-Fig. 4 is obtained as in Append.-Fig. 5. From this S.F.G., the transfer function between F_a and Y_a is as follows,

$$\begin{aligned} \frac{Y_a}{F_a} &= \frac{1}{MS^2} \left[1 + (4M + 3m) \left(\frac{l^3}{12EI} \right) S^2 \right] \\ &\quad \left(1 + \frac{m}{M} \right) + \left(1 + \frac{m}{2M} \right) \left(\frac{ml^3}{8EI} \right) S^2 \\ &= \frac{1}{MS^2} \frac{(1 + S^2/\omega_d^2)}{1 + S^2/\omega_n^2} \end{aligned}$$

where

$$\begin{aligned} \omega_n &= \sqrt{\frac{1+m/M}{1+m/2M}} \sqrt{\frac{8EI}{ml^3}}, \\ \omega_d &= \frac{1}{\sqrt{4M+3m}} \sqrt{\frac{12EI}{ml^3}} \end{aligned}$$

It is found that the resonant frequency ω_n of the cantilever is shifted with the size of mass M and its frequency becomes minimum in the fixed boundary condition, that is, when the restriction is the heaviest.

By this, it was proved that the natural frequency in the free boundary condition became higher than that in the fixed boundary condition.

(12) The authors gave about $\zeta=0.05$ in the foregoing answer; however, as Mr. Sato pointed out, it seems the damping factor differs with material, size of structure, etc., so we consider

that the value ζ must be selected in accordance with a given problem; in the present state where no theory has been established for the structure damping factor.