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Frictional Moment and Pressure Drop of the Flow through Co-Axial Cylinders with an Outer Rotating Cylinder*

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We studied experimentally the frictional moment (torque resistance) and the pressure drop (loss of head) of the flow in co-axial cylinders of which the outer cylinder rotates, using water and a glycerine-water solution. Comparing these experimental results with those of the inner rotating cylinder, we clarified the behaviors of shearing stresses of the flow in the co-axial cylinders.

When the clearance ratio β is large, the coefficient of the frictional moment C_M and the coefficient of the loss of head λ is extremely smaller than that of the inner rotating cylinder in a wide range of rotating Reynolds numbers R_ω and axial Reynolds numbers R_e for the turbulent flow. When R_ω is slightly larger than the critical value from laminar to turbulent flow for $R_e=0$, the values of C_M and λ approach the theoretical values of the laminar flow as R_e increases from zero to a certain value. In the case of $\beta=0.03$, λ shows a good agreement with the theoretical value for turbulent flow.

1. Introduction

About an axial flow through co-axial cylinders with rotating inner cylinders, we have already reported the experimental results on the pressure drop of the flow (loss of head)^{(1) (2)} and the frictional moment (torque)^{(2) (3)}. In the case of an outer cylinder only rotating, on the other hand, recently we have also reported the frictional moment⁽⁴⁾ and the pressure distribution around the eccentric cylinders⁽⁵⁾ when there is no axial flow. However there is no experimental result of the frictional moment and the pressure drop of the flow through co-axial cylinders with an outer rotating cylinder.

When the inner cylinder only rotates, Taylor vortices as well known are formed due to instability of the flow, and the values of a loss of head and a torque become larger than the theoretical values. When the outer cylinder only rotates, on the other hand, the flow is more stable than in the case with an inner rotating cylinder, because the peripheral velocity of the fluid near the outer cylinder is larger than that near the inner cylinder. Therefore in the case of no axial flow the experimental values of a torque when the outer

cylinder rotates are extremely smaller than when the inner cylinder rotates.

On the loss of head of a flow through co-axial cylinders with an outer rotating cylinder, Tomita⁽⁶⁾ experimented the flow through co-axial cylinders with a very wide clearance, and he reported the value of total loss of head included the effect of an inlet and an outlet. Furthermore he did not show the temperature of water or a dimensionless expression, so we can not compare his paper with our data.

In this report, we measured the loss of head and the torque of the flow through an annulus, with the outer cylinder rotating and the inner cylinder stationary, under fully developed conditions, using water and glycerine-water solutions. And we clarify the effects of a few parameters, such as axial flow Reynolds number, rotating Reynolds number and clearance ratio, on the loss of head and the torque. Moreover we compare the results with those of inner rotating cylinder.

2. Nomenclature

- r_o : inside radius of outer cylinder
- r_i : outside radius of inner cylinder
- $\delta=r_o-r_i$: clearance
- $\beta=\delta/r_i$: clearance ratio
- v_m : axial mean velocity
- ω : angular velocity of outer cylinder
- ν : kinematic viscosity

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$R_c = \delta v_m / \nu$: axial flow Reynolds number

$R_w = \omega(r_o^2 - r_i^2) / 2\nu$: rotating Reynolds number

l : length of the section to measure the torque or the pressure drop

M : frictional moment (torque) acting on the measuring section

$C_M = M / 2\pi\rho r_o^2 r_i^2 \omega^2 l$: coefficient of the frictional moment

Δp : pressure drop over distance l

$\lambda = \frac{\Delta p}{\gamma} / \frac{l}{2\delta} \frac{v_m^2}{2g}$: coefficient of the loss of head

3. Experimental apparatus and method

As the experimental apparatus is very similar to the one in the previous report^{(4) (7)}, we give its outline briefly. A general arrangement of the apparatus is shown in Fig. 1. The fluid is supplied from a tank ① to the rotating cylinders, passing through a valve ②, which controls the quantity of fluid. To control the temperature, there are installed a heater and a cooling coil pipe in the tank. An orifice ③ is used to measure the comparatively large quantity of flow. For small flow rates, we used a balance to weigh the flow rate. In such a case, valves ④, ⑤ are used to turn over the flow.

The arrangement of the co-axial rotating cylinders is shown in Fig. 2. An inner cylinder shown in Fig. 2 is used to measure the torque. In order

to measure the pressure drop, we replace it with another inner cylinder having pressure holes. Inner cylinder to measure the torque is assembled from three parts ①, ② and ③, and each part is supported by ball bearings: ① has a measuring section of torque, whose length is 180 mm, ② and ③ have an inlet and an outlet section in order to eliminate the end effect whose lengths are 150 mm and 20 mm respectively. The annular clearance between ① and ② or ③ is made very small in order to reduce the leakage, but they do not touch each other because of a precision machine work.

The inside diameter of the outer cylinder is 64.110 mm. The outside diameter of the inner cylinder is changed by inserting three parts of inner cylinder into the three brass pipes having same diameters respectively. Table 1 shows the dimensions of co-axial cylinders and radius ratio. Inner cylinder H in Table 1 was used to measure the torque only when there was no axial flow. The temperature of fluid is measured by thermometers fitted up at the entrance and the exit. The range of the rotating speeds of the outer cylinder is 0~4 000 rpm. The working fluids are water, 32.4%, 42.7% and 54.0% glycerine water solutions.

In our experiments, we measured the torque at first when there was no axial flow at various rotating Reynolds numbers, and then we measured the loss of head and the torque at various flow quantities (axial velocities) for the constant rotat-

Table 1 Dimensions of rotating cylinders

Singn of inn. cyl.	Inside rad. of out cyl. r_o mm	Outside rad. of inn. cyl. r_i mm	Clear-ance δ mm	Radius ratio r_o/r_i	Clear-ance ratio δ/r_i
A	32.055	31.734	0.321	0.010	0.010
B	"	31.569	0.486	1.015	0.015
C	"	31.353	0.702	1.022	0.022
D	"	31.065	0.991	1.032	0.032
E	"	30.570	1.485	1.049	0.049
F	"	29.979	2.077	1.069	0.069
F	"	28.966	3.089	1.107	0.107
H	"	27.000	5.055	1.187	0.187

Fig. 1 General arrangement of apparatus

Fig. 2 Co-axial rotating cylinders

ing Reynolds number. When there is an axial flow, it is very remarkable that the inlet length of co-axial rotating cylinders for the fully developed peripheral velocity is completely different from the inlet length for the fully developed axial velocity. Especially in the case of large axial velocity, the inlet for the peripheral velocity is extremely long. Therefore we set up a rectifying apparatus at the entrance of outer and inner cylinders respectively, whose shape is similar to a gear⁽²⁾.

4. Frictional moment when there is no axial flow

The frictional moment M for laminar flow is shown as

$$M=4\pi\mu\omega l r_0^2 r_i^2/(r_0^2-r_i^2)$$

if we define the coefficient of the frictional moment as

$$C_M = M / 2\pi\rho r_0^2 r_s^2 \omega^2 l$$

the theoretical value of C_V is given by

$$C_M = \frac{1}{R} \dots \dots \dots (1)$$

Fig. 3 Relation between C_M and R_n for $R_s=0$

Fig. 4 Relation between C_M and R_e (Part 1)

The relationship between the coefficient of the frictional moment C_M and the rotating Reynolds number R_ω for eight radius ratios within 1.010~1.187 is shown in Fig. 3. The data of previous papers^{(2) (3)} on the inner rotating cylinder are shown for comparison. In this figure, the value of

$$C_M = 0.00759 R_\omega^{-0.24} \quad (2)$$

is obtained by assuming the law of 1/7 power in the velocity distribution, neglecting the curvature of the pipe walls and using an empirical formula of the coefficient of pressure drop λ for the stationary co-axial cylinders⁽²⁾

$$\lambda = 0.26 R_e^{-0.24} \quad (3)$$

The value of C_M for outer rotating cylinder, on the contrary to the inner rotating cylinder, decreases with an increase of radius ratio r_o/r_i . The straight line of Eq. (2) is the boundary of the value of C_M for two cases when r_o/r_i is very small. The difference of the value of C_M with different values of radius ratio is larger than that for the inner rotating cylinder.

5. Frictional moment when there is axial flow

Relations between the coefficient of the frictional moment and the axial flow Reynolds number for seven radius ratios

$$r_o/r_i = 1.010 \sim 1.107,$$

with the rotating Reynolds number as parameter, are shown in Figs. 4(a)~(g). When the flow is laminar, the theoretical value of C_M is given by Eq. (1), and it is independent of the axial flow⁽²⁾. When radius ratio $r_o/r_i = 1.010$ and 1.015, which correspond to Fig. (a) and (b) respectively, we can see from Fig. (3) that the value of C_M is in good agreement with the laminar theoretical value $1/R_\omega$ and the flow is laminar, when R_ω is smaller than 1600 and there is no axial flow. For $R_\omega = 1000$ and 1500, therefore, the flow state is laminar and the value of C_M is independent of axial flow (R_e) and keeps a constant value, but beyond about 800 of R_e the flow state changes to turbulent and C_M increases with an increase of R_e . This is similar to previous papers for the inner rotating cylinder^{(2) (3)}.

When R_ω is larger than 1600 in Fig. 3, the value of C_M increases sharply, and this is due to the transition from laminar to turbulent flow. In this case, the increasing rate of C_M is extremely larger than that of the case of an inner rotating cylinder which is due to the formation of Taylor vortices. Hence for $R_\omega = 1700$ and

Fig. 4 Relation between C_M and R_e (Part 2)

2000, which are close values to critical rotating Reynolds number from laminar to turbulent flow, the value of C_M is fairly larger than the theoretical value for laminar flow. When we keep R_ω at these values and increase R_e from zero, the value of C_M decreases once and settles to a certain value. When R_e is larger than about 800, the value of C_M increases with an increase of R_e again. This C_M - R_e curve for the outer rotating cylinder is very similar to one for the inner rotating cylinder.

For the inner rotating cylinder, when R_ω is even five times as large as $R_{\omega c}$ at which Taylor vortices begin to form, owing to the adding of an axial flow, Taylor vortices disappear and the flow becomes purely laminar, and the value of C_M coincides with the theoretical value for laminar flow. For the outer rotating cylinder, however, the value of R_ω , at which C_M decreases by axial flow, is limited to the neighborhood of $R_{\omega c}$, and the minimum value of C_M due to axial flow is slightly larger than the theoretical value for laminar flow. For $R_\omega=3000$ in Figs. 4(a) and (b), it is only twice the number of $R_{\omega c}$ in Fig. 3, the flow already develops into a fully turbulent state, and the decrease of C_M due to axial flow doesn't occur.

The fact that the value of C_M decreases due to axial flow can be explained by disappearance of Taylor vortices for the inner rotating cylinder. For the outer rotating cylinder, however, the flow receives a stabilizing effect and vortices such as Taylor vortices cannot exist. Thus it can be considered a peculiar phenomenon. For $R_\omega=2000$ in Fig. 4(c), C_M decreases once and then increases with an increase of axial flow, similarly to Figs. 4(a) and (b).

When the clearance δ is comparatively large, Figs. 4(d)~(g), when R_e increases from zero, the many C_M - R_e curves do not decrease but increase linearly with a further increase of R_e .

Fig. 5 Relation between C_M and R_e of Eq. (4)

The value of C_M for turbulent flow with axial flow may be expressed as⁽³⁾

$$C_M = 0.00759 R_\omega^{-0.24} \left\{ 1 + \left(\frac{8}{7} \right)^2 \left(\frac{2R_e}{R_\omega} \right)^2 \right\}^{0.38} \quad \dots (4)$$

by means of a similar method to Eq. (2). Relations between C_M and R_e for various values of R_ω , calculated from Eq. (4), are shown in Fig. 5. When the radius ratio r_0/r_i is large in Figs. 4(a)~(g), the tendency of the C_M - R_e curves is similar to that of the theoretical curves in Fig. 5. However the flow has a stabilizing effect due to the rotation of the outer cylinder, so the experimental value of C_M is smaller than the value in Fig. 5. Furthermore, when R_e is extremely larger than R_ω , Eq. (4) shows that C_M increases in proportion to $R_e^{0.76}$ and experimental results also show a similar tendency.

When the quantity of flow is constant ($R_e = \text{constant}$), the relations between C_M and R_ω obtained from Fig. 4 are shown in Figs. 6(a) and (b). For $R_e=200, 500$ and 1000 in Fig. (a), the value of C_M within the transient region is smaller than the value for $R_e=0$. When R_e is large in Fig. (b), C_M - R_ω curves are almost straight lines inclined at 45 degrees and show the same tendency of Eq. (4).

(a) $r_0/r_i=1.015$ (b) $r_0/r_i=1.049$ Fig. 6 Relation between C_M and R_ω

6. Loss of head

For seven radius ratios r_0/r_i within 1.010~1.107 which are the same as Fig. 4, the relations between the coefficient of pressure loss λ and the axial Reynolds number Re for various values of R_w are shown in Figs. 7 (a)~(g). In the case of a laminar flow, the theoretical value of λ for stationary co-axial cylinders (for $R_w=0$) is shown by

$$\lambda = \frac{48}{Re} \quad \dots\dots\dots (5)$$

For rotating co-axial cylinders, the theoretical value of λ is independent of the value of R_w and usually equal to Eq. (5) whenever the flow state is laminar.

In Figs. 7 (a) and (b) where $r_0/r_i=1.010$ and 1.015 respectively, the value of λ for $R_w=0$,

500, 700 and 1000 is in good agreement with the theoretical value of Eq. (5), because when R_w is smaller than 1600 for $Re=0$ the flow is laminar as mentioned above. When $R_w=1700$ and 2000, the flow state belongs to the transient region for $Re=0$. The value of λ is in good agreement with the theoretical value $48/Re$ within a certain range of Re . When Re is smaller or larger than this range, λ increases from the theoretical value. The range of Re , in which the value of λ coincides with the theoretical value, almost agrees with the range, in which the value of C_M becomes small from the value for $Re=0$ and keeps a certain constant value in Fig. 4. In such a range of Re , the value of C_M is a little larger than the theoretical value for laminar flow, so we may consider that the flow state is almost laminar but not purely laminar. When R_w is larger than 3000, the experimental curves of $\lambda \cdot Re$ run parallel with the

(a) $r_0/r_i=1.010$

(c) $r_0/r_i=1.022$

(b) $r_0/r_i=1.015$

(d) $r_0/r_i=1.032$

Fig. 7 Relation between λ and Re (Part 1)

Fig. 7 Relation between λ and R_e (Part 2)

curve for the theoretical value of the laminar flow.

In Figs. 7 (c) and (d), when R_ω is smaller than 2000 in which the flow state is laminar or transient in Fig. 3, λ is in good agreement with Eq. (5). When R_ω is larger than 3000, all of the experimental values of λ are larger than Eq. (5) and $\lambda \cdot R_e$ curves run parallel with the curve of a laminar flow. And when R_e is extremely large, experimental curves get nearer to the curve of turbulent flow for $R_\omega = 0$, i. e. curve of $\lambda = 0.26 R_e^{-0.24}$. In Figs. 7 (e) and (f), the value of λ for $R_\omega = 3000$ is in good agreement with Eq. (5), while the value of C_V is a little larger than Eq. (1) and slightly decreases once with an increase of R_e [see Figs. 3, 4 (e) and (f)]. When we compare Figs. 7 (a)~(g) with the papers reported on the inner cylinder rotating^{(1) (2)}, $\lambda \cdot R_e$ curves for the outer cylinder rotating are simple and run parallel with the theo-

Fig. 8 Relation between λ and β

tical curve for laminar flow Eq. (5), while for the inner cylinder rotating they are complicated owing to occurrence of Taylor vortices. When we accept the same assumption as that for Eq. (2), the theoretical value for turbulent flow is⁽¹⁾

$$\lambda = 0.26 R_e^{-0.24} \left\{ 1 + \left(\frac{7}{8} \right)^2 \left(\frac{R_w}{2R_e} \right)^2 \right\}^{0.38} \dots\dots\dots (6)$$

It is seen easily from this equation that λ is inversely proportional to R_e for $R_w \gg R_e$.

The relations between λ and clearance ratio β , for $R_e = 4\,000$, $1\,000$ and $3\,000$ are shown in Figs. 8 (a), (b) and (c) respectively, in which R_w is a parameter. And the λ - β curves for the inner cylinder rotating are also indicated in the figure by dotted lines for comparison. The value of λ for $\beta = 0$ which is on the left side in the figure is calculated from Eq. (6). When we obtain Eq. (6) the curvature of walls are neglected, so λ of Eq. (6) corresponds to the value for $\beta = 0$ ($\epsilon = 1$).

In Figs. 8 (a) and (b), where $R_e = 400$ and $1\,000$, the flow state is laminar when $R_w = 0$. The horizontal line, which is minimum value of λ in the figure, shows the theoretical value for laminar flow $48/R_e$ [$= 0.12$ in Fig. (a), $= 0.048$ in Fig. (b)], and in this case the flow is in a laminar state. For the outer cylinder rotating of this experiment, the value of λ decreases remarkably with an increase of clearance ratio β and approaches the value of laminar flow. And the value of λ is smaller than that for the inner cylinder rotating, and the difference of the value between two cases becomes striking with an increase of β . For the inner cylinder rotating, on the other hand, the value of λ does not change too much with β when R_w is comparatively large. However, when R_w is in the transient region, in which Taylor vortices occur and λ increases sharply from the value of laminar flow, the value of λ increases extremely with an increase of β , and this is the opposite tendency to the case for the outer cylinder rotating.

When we compare the value of λ for the outer cylinder rotating with Eq. (6), the experimental value is found in good agreement with Eq. (6) for only about $\beta = 0.03$. When β is nearly equal to zero, the curvature of cylinder walls can be neglected against clearance, and hence the case of outer cylinder rotating becomes the same as the case of inner cylinder rotating, and the experimental values of λ for both cases also approach each other, but they are a little larger than the value of Eq. (6).

For $R_e = 3\,000$, λ - β curves are shown in Fig. 8

(c). In this case, the flow is always in a turbulent state, so the minimum value of λ is equal to the value of the empirical formula Eq. (3), namely $\lambda = 0.26 R_e^{-0.24}$, and substituting the value $R_e = 3\,000$, we have $\lambda = 0.0381$. For the outer cylinder rotating, the value of λ decreases remarkably with an increase of β , which is similar to Figs. (a) and (b), while for the inner cylinder rotating the difference of λ due to a change of β is small. The experimental value of λ is in good agreement with Eq. (6) for about $\beta = 0.03$ similarly to Figs. (a) and (b). From the experimental results of λ described above, we can see that for $\beta \approx 0.03$ the value of λ is in good agreement with Eq. (6) over a wide range of R_e and R_w ; for large R_e the value of λ always coincide with Eq. (3); and for small R_e , λ - R_e curves of constant R_w run parallel with the curve for laminar flow. Therefore, we change the second term in the bracket of Eq. (6),

$$\lambda = 0.26 R_e^{-0.24} \left\{ 1 + C \left(\frac{R_w}{2R_e} \right)^2 \right\}^{0.38} \dots\dots\dots (7)$$

The value of C may be regarded as the function of β only. Accordingly from Eq. (7)

$$C = \left\{ \left(\frac{\lambda}{0.26 R_e^{-0.24}} \right)^{1/0.38} - 1 \right\} \left(\frac{2R_e}{R_w} \right)^2 \dots\dots\dots (8)$$

we sought the value of R_w and λ for $R_e = 200, 400, 600, 1\,000$ and $2\,000$ in the turbulent region from Figs. 7 (a)~(g) respectively. Substituting the value of λ , R_e and R_w into Eq. (8), we obtain many values of C , which are almost constant in each figure. Then we obtain each mean value of C , and show the relation between C and β in Fig. 9. In the figure, $(7/8)^2$ is the value of C for Eq. (6) and it agrees with the value of C for $\beta = 0.03$ in Fig. 9. The broken lines in Figs. 7 (b) and (e) shown above, indicate the value of Eq. (7), using the value of C in Fig. 9. And the broken lines in Fig. 7 (d) indicate the value of Eq. (6). These broken lines are in good agreement with the experimental results.

Fig. 9 Relation between C and β

7. Conclusions

The torque and the loss of head of flow through the coaxial cylinders when the outer cylinder rotates are measured and compared with the experimental results for the inner rotating cylinder. Obtained results are as follows.

(1) When there is no axial flow, the coefficient of frictional moment C_M is smaller than the value for the inner rotating cylinder, especially for a large clearance ratio β .

(2) When the rotating Reynolds number is a little larger than the critical value at which the laminar flow breaks down and the axial Reynolds number increases from zero, a reduction of C_M is observed. This phenomenon is similar to the case of disappearance of Taylor vortices for the inner rotating cylinder.

(3) When the value of C_M decreases with an increase of R_e , the coefficient of head loss λ also sharply decreases at same R_e , and then coincides with the theoretical value for laminar flow.

(4) When the flow is turbulent, the value of λ is smaller than the value for the inner rotating cylinder, and it is extremely small for a large clearance ratio β .

(5) When $\beta = 0.03$, the experimental results

are in good agreement with Eq. (6).

(6) Equation (7) is in good agreement with the experimental results, provided the value of C is taken from Fig. 9.

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