Droplet Transfer in Two Phase Annular Mist Flow*

(Part 2, Prediction of Droplet Transfer Rate)

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The liquid droplet transfer onto the duct wall was studied experimentally in the previous paper, part 1⁽¹⁾. In the present paper, a method for predicting the droplet transfer rate is proposed. The droplet transfer coefficient is expressed theoretically in terms of the duct height, the eddy diffusivity and scale of turbulent motion of droplet based on an analytical model, in which are introduced a diffusion equation for droplet concentration in the turbulent core region of duct flow and a new boundary condition taking into account the behavior of droplets near the duct wall. The ratios of the eddy diffusivity and scale of turbulent motion of droplets to those respective values of gas phase are estimated from Tchen's theory by substituting empirical data of turbulent energy spectrum of single phase gas flow.

The trend of predicted values for the droplet transfer coefficient agrees well with the empirical results described in the previous paper when the effect of droplet concentration on the turbulence of gas phase can be neglected. A general semi-empirical equation for the coefficient including the effect of droplet concentration is also presented.

1. Introduction

Analytical studies of the particle transportation in turbulent gas flow onto the duct wall have been conducted chiefly for the particles smaller than 10 microns in diameter. For the particles in this range of diameters, the transfer coefficient has been confirmed empirically to show an opposite trend according to the size being larger or smaller than 1 micron⁽²⁾. In the case of particles smaller than 1 micron, the transportation phenomenon onto the wall may be caused by a process of the same kind as general mass transfer which is controlled by molecular diffusion, i.e. by Brownian motion, in the laminar sublayer of gas flow adjacent to the duct wall, and the transfer coefficient decreases with an increase of the particle size.

On the other hand, for particles larger than 1 micron the effect of the diffusion process by Brownian motion in the laminar sublayer becomes extremely low due to an inertia effect of the particles. However, such a large particle may penetrate the sublayer of a low level in turbulence and arrive at the wall surface with its large momentum given by the gas turbulence

in the core region of duct flow. Therefore, the transfer coefficient due to the momentum effect may be considered to increase with an increase of the particle size. For the particle in the latter range, some analytical models have been proposed by Friedlander et al.⁽³⁾, Davies⁽⁴⁾ and Beal⁽⁵⁾ assuming the thickness of gas layer through which the particle can pass with its momentum. Kondić⁽⁶⁾ has also discussed an effective force caused chiefly by a steep velocity gradient in the gas boundary layer in the case of condensed liquid droplets of 1 or 2 microns in size.

However, the liquid droplets in two phase mist flow discussed here are larger than 10 microns in diameter and have very large inertia effects. Hence, the models proposed by Friedlander and others under the assumption that the particles follow completely the turbulent motion of gas phase and have the same eddy diffusivity as that of gas phase can not be true in this case. Therefore, not the conditions of gas phase in the neighbourhood of the duct wall, but the magnitude of the droplet momentum given by the gas turbulence in the core region seems to have a significant effect on the droplet transfer onto the wall.

Nomenclature and Dimensions

a : parameter in Eq. (25)	1/sec
$a^2 = \bar{\epsilon}_d / \bar{u}_d$: in Eq. (7)	m
B : duct width	m

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C : droplet concentration	kg/m ³
d : particle (or droplet) diameter	m
E, E_L : Eulerian and Lagrangian energy spectrums (in y -direction)	
H : duct height	m
h : distance from center of duct	m
K : droplet transfer coefficient	m/sec
l : Prandtl's mixing length	m
$\dot{m}$: y -directional transfer rate of droplet per unit area	kg/m ² sec
n_d : number of droplets per unit volume	1/m ³
p : ratio of eddy diffusivities	
q : ratio of turbulence scales	
$\bar{S}$: ratio of mean velocities	
u : x -directional mean velocity	m/sec
u^* : friction velocity	m/sec
v' : y -directional velocity fluctuation	m/sec
$\sqrt{v'^2}$: root-mean square of velocity fluctuation	m/sec
W_d : total flow rate of droplet	kg/sec
x : axial distance from inlet of test section	m
y : distance from duct wall	m
A : Lagrangian macroscale of turbulence (in y -direction)	m
γ : specific weight	kg/m ³
δ : y -directional thickness of boundary region	m
Γ : number of droplets crossing unit area in unit time	1/m ² sec
ε : eddy diffusivity	m ² /sec
μ : viscosity, eigenvalue in Eq. (15)	kgsec/m ²
ν : kinetic viscosity	m ² /sec
ρ : density	kgsec ² /m ⁴
ω : angular frequency of turbulence	1/sec
Re_g : gas Reynolds number, $(2u_g H/\nu_g)$	

Subscript

d : droplet
g : gas stream
0 : duct wall
$-$: mean value
A : boundary plane
s : value in the case of low droplet concentration

2. Mechanism of droplet transfer

2.1 Analytical model for droplet transfer

The turbulent eddy of gas phase flow in the core region of a duct has been considered to consist of many eddy components of various sizes overlapping each other, and the sizes are usually larger than the diameters of liquid droplets in two phase mist flow. Hence, the droplets suspended in such a turbulent gas stream may have similar velocity fluctuation and

random motion in nature to those of a lump of gas phase. Therefore, it may be assumed that the liquid droplet is transferred toward the duct wall with its random motion. On this basis, an expression for the droplet transfer in turbulent core flow may be derived by defining the eddy diffusivity of droplets ε_d as follows:

$$\dot{m} = \varepsilon_d \frac{\partial C}{\partial y} \quad (1)$$

$$\varepsilon_d = \sqrt{v_d'^2} A_d \quad (2)$$

where $\dot{m}$ is the local net transfer rate of droplets through the unit area parallel to duct wall, C is the droplet concentration, y is the distance from duct wall, $\sqrt{v_d'^2}$ is the root-mean square of velocity fluctuation of droplets and A_d represents the macroscale of droplet turbulent motion. The value A_d means the y -directional average distance of droplet persisting a motion in a given direction, i.e. the mean free path of turbulent motion, and $\sqrt{v_d'^2}$ represents the y -directional mean velocity in the motion.

On the other hand, we may classify the droplets into two groups according to the direction of each droplet motion. One group moves toward the duct wall and the other in the opposite direction. Denoting the number of droplets per unit volume of the gas phase as n_d , let us consider here the number of droplets $\bar{\Gamma}$ crossing the unit area parallel to duct wall from right to left per unit time as shown in Fig. 1(a). Since the droplets have a mean free path A_d and a mean fluctuating velocity $\sqrt{v_d'^2}$, we may assume that the droplets come, on the average, from the plane 1 at a distance A_d penetrating the gas phase as shown in the figure. Therefore, the droplet transfer rate $\bar{\Gamma}$ may be expressed as follows:

(a) Droplet behavior (b) Analytical model
Fig. 1

$$\bar{\Gamma} = \sqrt{\bar{v}_d'^2} \frac{1}{2} \left(n_d + A_d \frac{\partial n_d}{\partial y} \right) \quad \dots\dots\dots (3)$$

where $\{n_d + A_d(\partial n_d / \partial y)\}$ represents the concentration of droplets at the plane 1 and the value of 1/2 means the probability of droplet motion toward the unit area, i.e. in the negative direction of y . The droplet transfer rate $\bar{\Gamma}$ in the opposite direction is expressed in the same way as,

$$\bar{\Gamma} = \sqrt{\bar{v}_d'^2} \frac{1}{2} \left(n_d - A_d \frac{\partial n_d}{\partial y} \right) \quad \dots\dots\dots (4)$$

From Eqs. (3) and (4), the net transfer rate of droplets toward duct wall Γ is now given by,

$$\Gamma = \bar{\Gamma} - \bar{\Gamma} = \sqrt{\bar{v}_d'^2} A_d \frac{\partial n_d}{\partial y} \quad \dots\dots\dots (5)$$

When both sides of Eq. (5) are multiplied by the mean weight of droplet, Γ and n_d may be rewritten as $\dot{m}$ and c respectively. Therefore, the relations of Eqs. (1) and (2) are obtained again.

On the fundamental concept for the droplet motion in turbulent gas stream mentioned above, an analytical model of the droplet transfer is proposed by dividing the gas stream in the duct into two flow regions, i.e. the turbulent core region and the other region near the duct wall, as shown in Fig. 1(b). In the core region, the turbulent diffusion of droplets is assumed to take place in the uniform turbulence over the region. Neglecting the turbulent diffusion in the x -direction, the mass balance of droplets in the duct is expressed as,

$$\frac{\partial \dot{m}}{\partial h} = - \frac{\partial (C u_d)}{\partial x}$$

where u_d is the droplet velocity in the x direction and h is a distance from the center of duct. Substituting $\dot{m}$ in Eq. (1) into the above equation and equalizing ∂h with $-\partial y$, the following diffusion equation for droplet concentration is obtained as,

$$\frac{\partial}{\partial h} \left(\varepsilon_d \frac{\partial C}{\partial h} \right) = \frac{\partial (C u_d)}{\partial x} \quad \dots\dots\dots (6)$$

From the experimental results of the droplet velocity distribution described in the previous report⁽¹⁾, the droplet velocity u_d in Eq. (6) is assumed to be uniform over the core region as,

$$u_d = \bar{u}_d = \text{constant}$$

Furthermore, for the first approximation, the value of eddy diffusivity of droplets ε_d is also assumed to be constant in the core region as,

$$\varepsilon_d = \bar{\varepsilon}_d = \text{constant}$$

Though the value decreases with a decrease of distance from duct wall, as well known, the approximation may be appropriate at least for the core region. By using these approximations, the differential equation for droplet diffusion in the core region, Eq. (6), may be rewritten as follows:

$$a^2 \frac{\partial^2 C}{\partial h^2} = \frac{\partial C}{\partial x}, \quad a^2 = \frac{\bar{\varepsilon}_d}{\bar{u}_d} \quad \dots\dots\dots (7)$$

Next, consider the second region near the duct wall. The turbulence of gas phase in this region including the laminar sublayer and buffer layer is of small scales and high frequencies. Hence, the droplet in this region can not follow such a high frequency gas turbulence due to the large inertia of droplet, and keeps its motion insensitive to the gas turbulence. Therefore, the eddy diffusivity of droplets in this region may be approximated as zero. Here, we shall call the region the "boundary region" and denote the thickness and the edge plane of the boundary region as δ and plane A respectively as illustrated in Fig. 1(b). Then, all of the droplets transferred across the plane A from the core region to the boundary region may reach the duct wall with the momentum. On the contrary, no droplet can return through the plane A from the boundary region to the core, that is,

$$\bar{\Gamma}_A = 0$$

where the subscript A represents a value at the plane A.

From the relation described above, a new boundary condition for the droplet transfer will be derived.

Substituting the relation $\bar{\Gamma}_A = 0$ into Eq. (4),

$$n_{dA} = \left[A_d \frac{\partial n_d}{\partial y} \right]_A \quad \dots\dots\dots (8)$$

Then, from Eqs. (3) and (5), the net transfer rate of droplets at the plane A can be expressed as follows:

$$\Gamma_A = \bar{\Gamma}_A = \left[\sqrt{\bar{v}_d'^2} n_d \right]_A = \left[\sqrt{\bar{v}_d'^2} A_d \frac{\partial n_d}{\partial y} \right]_A \quad \dots\dots\dots (9)$$

Multiplying both sides of the above equation by the mean droplet weight, the following relations for the droplet transfer rate $\dot{m}_A$ and droplet concentration C_A at the plane A are derived as,

$$C_A = \left[A_d \frac{\partial C}{\partial y} \right]_A \quad \dots\dots\dots (10)$$

$$\dot{m}_A = \left[\sqrt{\bar{v}_d'^2} C \right]_A = \left[\varepsilon_d \frac{\partial C}{\partial y} \right]_A \quad \dots\dots\dots (11)$$

In the equation, $\dot{m}_A$ is equal to $\dot{m}_0$, the droplet transfer rate onto the duct wall, since all droplets crossing the plane A have been supposed to fall on the wall, i.e.,

$$\dot{m}_A \equiv \dot{m}_0$$

Furthermore, we may assume the thickness of boundary region δ to be zero in the case of estimating the droplet flow rate through the duct area in the x direction since the value of δ is small as compared with the duct height. Then, the boundary condition of droplet concentration and the droplet transfer rate onto the duct wall may be expressed respectively as,

$$C_{y=0} = \left[A_d \frac{\partial C}{\partial y} \right]_{y=0} \quad \dots\dots\dots (12)$$

$$\dot{m}_0 = \left[\varepsilon_d \frac{\partial C}{\partial y} \right]_{y=0} \quad \dots\dots\dots (13)$$

After all, the new analytical model for droplet transfer is represented by the equation for droplet diffusion in the core region, Eq. (7), and the boundary condition, Eq. (12).

2.2 Droplet transfer coefficient

Equation (7) can be easily solved by the separation of variables and the droplet concentration is expressed by assuming the symmetrical distribution against the center plane of duct as follows:

$$C = \sum c_\mu e^{-a^2 \mu^2 x} \cos \mu h \quad (14)$$

In this equation, an eigenvalue μ is the solutions, $\mu = \mu_j (j=1, 2, \dots)$, of the following equation,

$$\mu A_d \tan \mu \frac{H}{2} = 1 \quad (15)$$

which was derived by substituting Eq. (14) into the relation of boundary condition, Eq. (12) rewritten with variable h instead of y , i.e., $C_{h=H/2} = -[A_d(\partial C/\partial h)]_{h=H/2}$. Mean while, the coefficient c_μ is determined by the initial condition of the droplet concentration. Therefore, the distribution of droplet concentration across the duct area is

$$C = \sum_{j=1}^{\infty} c_j e^{-a^2 \mu_j^2 x} \cos \mu_j h \quad (16)$$

On the other hand, the droplet transfer coefficient K has been defined in the previous paper⁽¹⁾ as,

$$K = \frac{\dot{m}_0}{\bar{C}} = \frac{H \bar{u}_g}{2 \bar{S}} \frac{d(\ln W_d)}{dx} \quad (17)$$

where H is the duct height and $\bar{S}$ is the ratio of mean gas velocity over the duct area, $\bar{u}_g$, to that of droplet, $\bar{u}_d$. The term in the right hand side of Eq. (17), $d(\ln W_d)/dx$, is rewritten as,

$$\frac{d(\ln W_d)}{dx} = \frac{d(W_d/B)}{dx} \frac{1}{(W_d/B)}$$

where B is the width of duct and W_d/B denotes the droplet flow rate per unit duct width and may be obtained by integrating the product of droplet velocity, $u_d = \bar{u}_d$, and droplet concentration C , namely

$$\left. \begin{aligned} \frac{W_d}{B} &= 2 \int_0^{H/2} C \bar{u}_d = 2 \bar{u}_d \sum_{j=1}^{\infty} F_j(x) \\ \text{and} \quad \frac{d(W_d/B)}{dx} &= -2 \bar{u}_d a^2 \sum_{j=1}^{\infty} \mu_j^2 F_j(x) \end{aligned} \right\} \quad (18)$$

where

$$F_j(x) = c_j e^{-a^2 \mu_j^2 x} \frac{1}{\mu_j} \sin \mu_j \frac{H}{2}$$

Therefore, putting these terms into Eq. (17), the transfer coefficient is expressed generally as a function of the axial distance x , namely

$$K = \frac{H \bar{u}_g}{2 \bar{S}} a^2 \frac{\{\mu_1^2 F_1(x) + \mu_2^2 F_2(x) + \dots\}}{\{F_1(x) + F_2(x) + \dots\}} \quad (19)$$

For the practical purpose, the following approximation is permitted except at the entrance part of test section⁽⁷⁾, i.e. at $x=0$,

$$F_1(x) \gg F_2(x), \dots, \quad \mu_1^2 F_1(x) \gg \mu_2^2 F_2(x), \dots \quad (20)$$

Applying this approximation and the relations $\bar{S} = \bar{u}_g/\bar{u}_d$ and $a^2 = \bar{\epsilon}_d/\bar{u}_d$ to Eq. (19), the resultant expression for the droplet transfer coefficient is obtained as follows:

$$K = \frac{H}{2} \bar{\epsilon}_d \mu_1^2 = \frac{2}{H} \bar{\epsilon}_d (\mu_1^+)^2 \quad (21)$$

where μ_1^+ is the first root of the modified equation of Eq. (15), i.e.,

$$\mu^+ A_d^+ \tan \mu^+ = 1 \quad (22)$$

and μ^+ and A_d^+ represent nondimensional forms of μ and A_d , i.e.

$$\mu^+ = \frac{H \mu}{2} \quad \text{and} \quad A_d^+ = \frac{2 A_d}{H}$$

Figure 2 shows the relation of Eq. (22), and from the figure the value of μ_1^+ can be obtained immediately for an assumed value of A_d and the duct height H .

2.3 Eddy diffusivity of droplets

With respect to the relation between the eddy diffusivity of droplets ϵ_d , which is included in Eq. (21) as an unknown factor, and that of gas phase and also relation between Lagrangian macroscale of the droplet turbulent motion A_d and that of gas phase, there are some theoretical studies by Tchen⁽⁸⁾ and Peskin⁽⁹⁾. In this report, the unknown factors are estimated from Tchen's theory as described below.

Tchen⁽⁸⁾ has introduced a momentum equation for the spherical particle suspended in the gas flow of uniform turbulence assuming Stokes force on the droplet given by the surrounding gas phase fluctuation and derived a ratio p of the eddy diffusivity of droplets to that of gas phase. According to his estimation, the ratio is equal to the ratio of the mean square of turbulent velocity fluctuations of droplets and gas phase and can be expressed as,

$$p = \frac{\epsilon_d}{\epsilon_g} = \frac{\overline{v_d'^2}}{\overline{v_g'^2}} = \frac{\int_0^\infty d\omega E_{Lg}(\omega) \eta^2(\omega)}{\int_0^\infty d\omega E_{Lg}(\omega)} \quad (23)$$

$$\eta^2(\omega) = \{1 + f_1(\omega)\}^2 + \{f_2(\omega)\}^2 = \frac{a^2}{a^2 + \omega^2} \quad (24)$$

Fig. 2 Relation between μ_1^+ and A_d^+

$$\left. \begin{aligned} f_1(\omega) &= -\frac{\omega^2}{a^2 + \omega^2}, & f_2(\omega) &= \frac{-a\omega}{a^2 + \omega^2} \\ a &= \frac{36\mu_g}{(2\rho_d + \rho_g)d^2} \end{aligned} \right\} \dots (25)$$

where v_d' and v_g' are the turbulent velocity fluctuations of particles (or droplets) and gas phase respectively, ω is an angular frequency of the turbulence, E_{Lg} is Lagrangian spectrum of gas phase, ρ_d and ρ_g are the densities of particle and gas phase respectively, μ_g is the gas viscosity and d is the particle diameter. Furthermore a is a parameter defined in Stokes equation, and $\eta^2(\omega)$ denotes an intensity of the particle turbulence as compared with that of gas phase for each value of angular frequency ω . Mean while, the ratio of the macroscale of droplets to that of gas phase q is derived from Eqs. (23) and (2) as,

$$q = \frac{A_d}{A_g} = \frac{\varepsilon_d}{\varepsilon_g} \frac{\sqrt{v_g'^2}}{\sqrt{v_d'^2}} = \sqrt{p} \dots (26)$$

Although Lagrangian energy spectrum of gas phase E_{Lg} should be known in order to obtain the values of p and q from the above equations, the empirical investigations on E_{Lg} are not sufficient for performing such a calculation. Hence, Rauhiainen et al.⁽¹⁰⁾ have estimated the values by substituting the empirical values of Eulerian energy spectrum E_g instead of Lagrangian. Figure 3 shows the empirical result of E_g obtained by Comte-Bellot⁽¹¹⁾. Though the value by Comte-Bellot has been measured at a fixed position of $(2y/H)=0.4$ in a rectangular duct for three gas Reynolds numbers Re_g , the value may be assumed to be a representative one in the turbulent core region since the value of eddy diffusivity is supposed to be constant over the core region. The dotted lines in Fig. 3 represent the data obtained at various distances from duct wall for a constant gas Reynolds number. Figure 4 shows the distribution of $\eta^2(\omega)$ calculated from Eq. (24) for the droplets in a range of 10~90 microns in diameter.

Then, the value of p may be calculated, in the same way as Rouhiainen did, by substituting the

Fig. 3 Distribution of energy spectra (by G. Comte-Bellot⁽¹¹⁾)

values $E_g(\omega)$ in Fig. 3 and $\eta^2(\omega)$ in Fig. 4 into Eq. (23) and performing a numerical integration. The results obtained are indicated in Fig. 5. Since the gas Reynolds number in the data of Comte-Bellot is high comparing with that in our experiment, the extrapolated values of p are also shown for a low gas Reynolds number taking into account the results that the values vary as an exponential function of Re_g .

3. Consideration of droplet transfer coefficient

3.1 Modified expression for droplet transfer coefficient

According to the previous analytical result, Eq. (21), the droplet transfer coefficient K has been expressed as a function of the duct size H , the eddy diffusivity of droplet ε_d and the scale of droplet turbulent motion A_d . The values of ε_d and A_d are derived from multiplying ε_g and A_g respectively by the ratios p and q which are determined from the gas Reynolds number and droplet diameter [or the parameter a in Eq. (25)] as mentioned above. Hence, the gas Reynolds number Re_g , duct size H , droplet diameter d and droplet concentration c , which will be discussed later, may be considered the controlling factors for the droplet transfer coefficient. Though the factors affect the coefficient through the above

Fig. 4 Distribution of function $\eta^2(\omega)$

Fig. 5 Ratio of eddy diffusivities

mentioned variables ε_g , A_g , p and q , it may be more convenient to express the coefficient as a simple function of the factors instead of Eq. (21).

First, we shall discuss the case of low droplet concentration where the effect of suspended droplets on the turbulence of gas stream described in the previous paper⁽¹⁾ can be neglected. Here, the values in this case will be distinguished by adding a subscript "s". Then, the droplet transfer coefficient in the case of low droplet concentration, $K=K_s$, can be calculated by substituting the eddy diffusivity and scale of turbulent motion for single gas phase, ε_{gs} and A_{gs} , respectively into ε_g and A_g of Eqs. (23) and (26). As for the value of ε_{gs} , Hinze has proposed the following equation for a range of $0.2 < 2y/H < 1.0$ which is modified in the case of duct flow,

$$\frac{\varepsilon_{gs}}{\nu_g} = 0.07 \frac{H^+}{2} \dots \dots \dots (27)$$

where $H^+ = (Hu_0^*/\nu_g)$ and u_0^* is the friction velocity. Figure 6 shows the values calculated from Eq. (27) and from the 1/7 power law velocity distribution and the empirical results obtained in the previous paper⁽¹⁾. Though the empirical values are somewhat scattered around the results of Eq. (27), the equation is assumed to represent the value of ε_{gs} as the first approximation. In the case of mist flow with liquid film flow on the duct wall, the friction velocity based on the shear at gas-liquid interface should be used for u_0^* in Eq. (27). As for Lagrangian macroscale of single phase gas flow, A_{gs} , the following value may be also proposed as the first approximation since Lagrangian scale A_{gs} is same order of magnitude as Prandtl mixing length l_{gs} though A_{gs} can not be identified with l_{gs} ,

$$\frac{2A_{gs}}{H} \cong \frac{2l_{gs}}{H} \cong 0.10 \dots \dots \dots (28)$$

Then, using ε_{gs} and A_{gs} of Eqs. (27) and (28) the values of $\bar{\varepsilon}_d$ and A_d may be determined from the relations presented in Eqs. (23), (26) and Fig. 5.

Fig. 6 Eddy diffusivity of single phase gas flow (with and without liquid film flow on duct wall)

Consequently, the droplet transfer coefficient K_s can be calculated from Eqs. (21) and (22). Figure 7 indicated the calculated results plotted for the droplets in a range of 10 to 90 microns in diameter. Since the logarithmic plots of the coefficient K_s are found to vary linearly with the gas Reynolds number Re_g as shown in Fig. 7, we may express the coefficient in the following form,

$$K_s = \frac{\nu_g}{H} f_1(Re_g) f_2(d), \quad f_1(Re_g) = Re_g^{k_1} \dots \dots (29)$$

The exponent k_1 in this equation can be determined from the slope of the curves. Examining Fig. 7 leads to the value $k_1 = 0.70$ insensitive to the droplet diameter.

By applying $f_1(Re_g)$ determined above, the function $f_2(d)$ which represents the effect of droplet diameter is derived as $f_2(d) = K_s H / \{\nu_g f_1(Re_g)\}$. Figure 8 represents the function f_2 against the parameter a of Eq. (25) instead of the droplet diameter d . It suggests

Fig. 7 Effect of gas Reynolds number on droplet transfer coefficient

Fig. 8 Effect of droplet diameter

that f_2 increases and approaches a constant value as the value of parameter a increases, i.e. as the droplet becomes smaller in size.

Hence, a resultant expression for the droplet transfer coefficient in the case of low droplet concentration is obtained as follows.

$$K_s = \frac{\nu_g}{H} R_{eg}^{0.70} f_2(a) \quad \dots\dots\dots (30)$$

where $f_2(a)$ is indicated in Fig. 8.

3.2 Comparison of the predicted values with experimental data

Experimental data of the coefficient K presented in the previous report⁽¹⁾ are shown again in Fig. 9. The values marked with an arrow in the figure are the extrapolated coefficients corresponding to the condition that the droplet concentration becomes low and the eddy diffusivity of gas phase suspending droplets ϵ_g is equal to that for single phase flow ϵ_{gs} . The experimental data for the lowest droplet concentration in the figure coincides approximately with the condition. In Fig. 10, the extrapolated coefficients are shown in comparison with the values calculated by Eq. (30) (or by Fig. 7). The solid lines in the figure represent the calculated coefficients for the droplets of 30 and 50 microns in diameter. The

Fig. 9 Correlation between droplet transfer coefficient and eddy diffusivity of gas phase

Fig. 10 Comparison of droplet transfer coefficient calculated with that measured

trend for the gas Reynolds number coincides fairly well with that of extrapolated experimental values.

In order to discuss the above results quantitatively, the effect of the distribution of droplet diameters should be taken into account since the droplets in two phase mist flow have a fairly wide distribution as shown in Fig. 11. Consider the droplets in a range of diameters d to $d + \Delta d$, and denote the number of droplets per unit volume as Δn_d , the concentration as $\bar{C}(d)$ and the transfer coefficient as $K_s(d)$ respectively. $K_s(d)$ can be calculated from Eq. (30). Then, the total droplet transfer rate onto the wall may be expressed as, $\dot{m}_0 = K_s \bar{C} = \sum_d K_s(d) \bar{C}(d)$, since the total transfer rate is a summation of the transfer rate for each droplet size. Therefore, the coefficient K_s can be calculated by the following equation:

$$K_s = \frac{1}{C} \sum_d K_s(d) \bar{C}(d) = \frac{\sum_d \{K_s(d) d^3 \Delta n\}}{\sum_d (d^3 \Delta n)} \quad \dots\dots (31)$$

For example, the calculated value of Eq. (31), taking into account the distribution of droplet diameters shown in Fig. 11 under the condition of gas velocity 43 m/sec ($R_{eg} = 5.61 \times 10^4$) which is corresponding to the condition of data No. 21 in Fig. 9, is $K_s = 0.08$ m/sec. This value is nearly equal to the analytical value obtained from Eq. (30) for a droplet diameter of 50 microns, which corresponds approximately to the root-mean square of actual diameter in Fig. 11. Hence, it is supposed that the root-mean square may be considered a representative value of droplet diameter for comparison with the analytical value since the flow condition seems to have little effect on the form of droplet diameter distribution in this experimental range.

On the other hand, the experimental result for

Fig. 11 Example of distribution of droplet diameters

the above condition was $K_s = 0.118$ m/sec as shown in Fig. 10. Though the analytical value including the effect of droplet diameter distribution is about 30% lower than the experimental result, a qualitative agreement between the two values seems to be fairly good considering many approximations used in the analysis.

In Fig. 10, the experimental data measured by Friedlander et al.⁽⁸⁾ for solid particles of 32 microns in size and 0.621 in density through a circular tube of 2.5 cm in inner diameter are also presented. The theoretical equation for the transfer coefficient in the case of circular tubes becomes as follows⁽⁷⁾ instead of Eqs. (21) and (22),

$$K = \frac{1}{D} \bar{\epsilon}_d \mu_1^{+2} \dots (32)$$

$$\mu^+ A_d^+ J_1(\mu^+) = J_0(\mu^+) \dots (33)$$

where D is the tube diameter, μ_1^+ is the first root of Eq. (33), J_0 and J_1 are the zero order and first order Bessel functions of first kind and nondimensional parameters μ_1^+ and A_d^+ are equal to $\mu_1 D/2$ and $2A_d/D$ respectively. The calculated values by Eq. (32) for the experimental conditions of Friedlander et al. also agree quantitatively with their experimental data as shown in Fig. 10. Though the experimental data are fairly lower than the predicted value K_s , this fact might be caused by the effect of particle concentration in the experiment.

3.3 Semi-empirical expression including droplet concentration effect

The droplet transfer coefficient K is fairly influenced by the droplet concentration C and decreases with an increase of $\bar{C}$ as indicated in the previous report⁽¹⁾. This phenomenon was related empirically in the report to a decrease of the eddy diffusivity in the gas phase ϵ_g with an increase of $\bar{C}$. The fact is also explained theoretically by the analytical result of Eq. (21) since $\bar{\epsilon}_d$ in the equation is equal to $p \times \epsilon_g$. Meanwhile, the coefficient p may be also influenced by the droplet concentration, since the energy spectrum of gas turbulence, which determines the value of p , is affected by the droplet concentration. However, the magnitude of the effect of the concentration on p is not yet clarified theoretically. Therefore, it will be necessary to determine the effect of the concentration experimentally. Here, the droplet transfer coefficient K obtained by the experiments in the previous report is expressed with K_s as,

$$K = f_3(\bar{C}) K_s \dots (34)$$

The obtained values of $f_3(\bar{C})$ vary from 1.0 to 0.3 for the experimental conditions Fig. 12(a) in the previous report as shown in Fig. 12(a). The results indicate that $f_3(\bar{C})$ may be expressed as the following function of concentration $\bar{C}$,

$$f_3(\bar{C}) = e^{k_3 \bar{C}} \dots (35)$$

and the value of k_3 falls between -0.32 and -0.66 m³/kg in accordance with the gas velocity. The term $f_3(\bar{C})$ may be also expressed as a function of the droplet flow rate per unit cross area of duct, W_d/BH , instead of as a function of $\bar{C}$, i.e.,

$$f_3(\bar{C}) = e^{k'_3 (W_d/BH)} \dots (36)$$

Figure 12(b) shows the correlation between $f_3(\bar{C})$ and W_d/BH . From this result, it is seen that an appropriate value for the constant $k'_3 = -0.12$ m²sec/kg, and data plots fall within a range of $\pm 12\%$.

Then, the final expression for the droplet transfer coefficient including the effect of droplet concentration is semi-experimentally obtained by substituting K_s of Eq. (30) into Eq. (34), i.e.,

$$\frac{HK}{\nu_g} = Re_g^{0.70} f_2(a) f_3(\bar{C}) \dots (37)$$

where $f_2(a)$ is presented in Fig. 8 and $f_3(\bar{C})$ in Fig. 12. The ranges of variables for which Eq. (37) is applicable are $2 \times 10^4 \sim 10^5$ for the gas Reynolds number, 10 ~ 90 microns for the droplet diameter and 0 ~ 1.0 kg/m³ for the droplet concentration.

On the other hand, Paleev⁽¹²⁾ has proposed the following empirical equation for droplet transfer rate $\dot{m}_0$ in two phase mist flow through a horizontal rectangular duct of 4 mm in height by 48 mm in width,

$$\dot{m}_0 = 0.022 \gamma_g \bar{u}_g \left(\frac{\bar{C}}{\gamma_g} \right)^{0.74} \frac{1}{Re_g^{0.25}} \dots (38)$$

(a) Effect of droplet concentration on the transfer coefficient

(b) Effect of droplet flow rate on the transfer coefficient

Fig. 12

where the experimental range of gas Reynolds numbers was from 3.0×10^4 to 8.5×10^4 and that of droplet concentrations from 0.1 to 1.3 kg/m^3 . Equation (38) can be rewritten in the form of transfer coefficient as,

$$K = \frac{\dot{m}_0}{\bar{C}} = \frac{\nu_g}{H} R_{eg}^{0.75} f(\bar{C}), \quad f(\bar{C}) = 0.011 \left(\frac{\bar{C}}{\gamma_g} \right)^{-0.26} \quad (39)$$

which is approximately the same expression as Eq. (37) except a term of the droplet diameter effect. The value of $f(\bar{C})$ in Eq. (39) was in a range from 0.011 to 0.020 according to the experimental conditions by Paleev. Meanwhile, the value of $f_2(a) \times f_3(\bar{C})$ in Eq. (37) is found in a range from 0.01 to 0.03 for the experimental conditions of droplet diameter and concentration in the previous report⁽¹¹⁾.

3.4 Consideration on the analytical model

In the present model, the gas stream in ducts is divided into two regions, the turbulent core region and the boundary region. Here, we shall estimate the thickness of the boundary region δ using the empirical values of energy spectrum of gas phase for the various distances from the duct wall which are shown by dotted lines in Fig. 3. Figure 13 shows the ratio of the eddy diffusivity of droplets to that of gas phase p calculated in the same way as Fig. 5 for droplet diameters of 1, 10 and 50 microns. The ordinate of the figure represents the ratio p in comparison with that in the center region p_B , while the abscissa represents a nondimensional distance $y^+ = yu_*^*/\nu_g$. In the model, the eddy diffusivity of droplets ε_d and also that of gas phase ε_g are assumed to be uniform over the core region, and the boundary plane $\bar{A}$, i.e. plane at $y = \delta$, is defined as a position where ε_d becomes low rapidly and droplets begin to move as if they were not influenced by the gas turbulence. This may be caused by a steep reduction of the ratio p . Then, it is suggested from the result in Fig. 13 that the trend of p -curves changes depending on the droplet diameter and the position where a steep reduction of p takes place approaches the duct wall rapidly as the particle diameter becomes smaller than 10 microns. In this case, the

Fig. 13 Distribution of p in y direction (calculated from data by Comte-Bellot⁽¹¹⁾)

eddy diffusivity of gas phase ε_g near the position may be very low in comparison with that in the center region, hence, the assumption of ε_g being constant proposed in this paper becomes unreasonable and the model must be modified.

On the other hand, for the particles larger than 10 microns such as the droplets in two phase mist flow described here, the droplet size has little effect on the value p/p_B and a steep reduction of p takes place at the location of y^+ equal about 10^2 . The location corresponds to $2y/H = 0.1 - 0.2$ for the gas Reynolds number in the previous paper.

Since the eddy diffusivity ε_g at $y^+ \approx 10^2$ has a fairly high value and is near the value of center part. Therefore, we may consider, as the first approximation, the existence of the above mentioned core region, and the thickness of the boundary region may be supposed to be the same order as $y^+ = 10^2$.

The relations between the transfer coefficient K and the particle diameter calculated from the present analysis are shown in Fig. 14 for a diameter larger than 10 microns. In the figure, the data obtained by Wells et al.⁽²⁾ for the particles smaller than 10 microns are also plotted to show a qualitative trend of the transfer coefficient for a wide range of particle diameters.

4. Conclusions

An analytical model is presented for predicting the droplet transfer rate in two phase annular mist flow in a duct in which the flow is assumed to consist of two regions, the turbulent core region and the

Fig. 14 Effect of particle diameter on the transfer coefficient

boundary region adjacent to the duct wall. By introducing the eddy diffusivity of droplets, a diffusion equation for droplet concentration is derived under the assumption that in the core region the droplets move with a velocity fluctuation of the same quality as that of gas phase. A new boundary condition is proposed with the consideration of the behavior of droplets in the boundary region that all droplets crossing the boundary plane from the core region to the boundary region may be transferred onto duct wall with a momentum, while no droplet may move in the opposite direction since the scale of gas turbulence in the boundary region is not large enough to influence the motion of droplets.

On the basis of the model, an expression is derived for the droplet transfer coefficients onto duct wall as Eq. (21) in terms of the duct height H , the eddy diffusivity of droplets ϵ_d and turbulence scale of droplets Λ_d . As for ϵ_d and Λ_d , the values are estimated from Tchen's theory by substituting the turbulent energy spectrum of single phase gas flow.

From this procedure, an analytical expression for the droplet transfer coefficient, Eq. (30), is introduced in the case of low droplet concentration in which the effect of suspended droplets on gas turbulence may be neglected. The predicted values from Eq. (30) agree fairly well with the experimental data of the previous report qualitatively and quantitatively.

Furthermore, a final expression for the droplet transfer coefficient including a term of the droplet

concentration effect obtained empirically from the data in the previous report is presented as a semi-empirical equation, Eq. (37).

It is suggested from the discussion on the thickness of boundary region that the present model is reasonable for droplets larger than 10 microns in diameter such as in two phase mist flow.

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Discussion

M. NAKASATOMI (Kyushu University):

(1) In this report, the authors have clarified a concept of the thickness of gas phase adjacent to the duct wall through which particles can penetrate, i.e. the "stopping" distance defined by Friedlander et al.⁽³⁾ and Beal⁽⁵⁾, and proposed an analytical model for the transfer of the particles larger than 10 microns. Though they assumed the model to be also applicable to the droplet deposition phenomenon in two phase mist flow, the discussor supposes that the droplet deposition is closely connected with the droplet generation phenomenon from the liquid film on the duct wall. How do they consider the point?

The form of distribution of droplet diameters seems to change with the duct axis. Hence, the distribution at the end of droplet-generating section might not be the same as that observed at the 1.5 m downstream part of the test section. In view of the considerations above, the discussor asks the authors to give the axial distance of the test section from the

end of droplet-generating section where Eq. (37) is applicable.

Y. MORI and K. HIJIKATA (Tokyo Institute of Technology):

(2) In the analysis, the authors have regarded the behavior of droplets to be quite the same as that of solid particles except under the boundary condition at the duct wall. However, the discussors think the break-up and the coalescence of droplets are the important factors in droplet transfer phenomena in the case of large liquid droplets as treated in this analysis.

(3) The authors have mentioned on the 1st line from the bottom in the left-hand column of page 753 that the droplets suspended in the turbulent gas stream may have similar velocity fluctuation and random motion in nature to those of a lump of gas phase. The discussors suppose that since the droplet velocity fluctuation is caused by the viscous resistance force, the force due to the "Basset" term and so on in

the equation of droplet motion, the droplet motion must be essentially different from that of the gas phase.

(4) The discussers suppose that it may not be correct to estimate the probability for a droplet to move in from the plane 1 in Eq. (3) at $1/2$. For instance, the directional probability of mean velocity (v) obtained from Boltzman velocity distribution in the kinetic theory of gases is $1/4$.

(5) It seems that the magnitudes δ and A_d should be considered when the boundary condition on page 754 is introduced. The authors have assumed the thickness of the boundary region to be negligible in the final stage of the analysis, hence the value A_d^+ may be small enough in the range of the assumption above. In such a case, it seems more appropriate to suppose that $n_{dA}=0$ and that $A_d^+=0$ and $\mu^+=\pi/2$ in Fig. 2 instead of the complicated boundary condition such as Eq. (8). By this simplification, the transfer coefficient of Eq. (21) may be modified as,

$$K = \frac{2}{H} \bar{\epsilon}_d \left(\frac{\pi}{2} \right)^2 = \frac{\pi^2}{2H} \bar{\epsilon}_d$$

(6) The authors have estimated the eddy diffusivity of droplets based on Tchen's theory, which was derived for a small spherical particle in the homogeneous gas turbulence by applying Stoke's law of resistance. However, the liquid particles in this report are suspended in the gas turbulence of shear flow and are large in size. The Reynolds number R_{ed} for a droplet of 90 microns, for instance, may become larger than unity. The discussers suppose there may be some problems in applying the theory to this case. Furthermore, they wish the authors to show the reason why Tchen's theory was used in this report instead of Peskin's theory.

(7) The discussers suppose that there is no reason to express K_s as Eq. (31) and that K_s should be expressed in terms of the controlling factors for K_s , for instance, the effective terms in the equation for droplet motion as described in 'Turbulence' by Hinze [Eq. (5-112) on page 354].

Though the authors have stated that by considering the root-mean square of droplet diameter as a representative value the effect of droplet size distribution can be excluded, it seems to the discussers that a general conclusion can not be drawn from the result in a special case of this experiment. Since the function f_2 is found to be approximately proportional to d^{-2} as shown in Fig. 8, from Eq. (31), $d_m = \sqrt{\bar{d}^3/\bar{d}}$ might be preferable as a representative diameter.

(8) It seems that Eq. (21) may be obtained from Eqs. (12), (13) and the definition $K=m_0/\bar{C}$ in Eq. (17) without using δ and W_d in the previous report.

(9) How do the authors think about the

relation between p and S ?

(10) What value did they use for u_0^* ?

K. AKAGAWA (Kobe University):

(11) The authors have stated on the 4th line from the top in the left-hand column of page 759 that the analytical value K_s is about 30% lower than the experimental result. Does the semi-empirical Eq. (37) based on the value K_s involve an error of the same magnitude as the above?

(12) From the discussion of the result in Fig. 13, they have stated that the existence of the core region over which ϵ_g and ϵ_d are approximately uniform may be assumed in the case of droplets larger than 10 microns. However, the p/p_B distribution for a droplet of 1 micron in size is more uniform except near the duct wall than that for droplets larger than 10 microns. Hence, the discussers ask the authors to clarify the connection with above ϵ_g distribution.

(13) The discussers also ask them to indicate the numerical data of each term in Eq. (20), i.e. $F_1(x) \gg F_2(x)$, and so on.

Authors' closure

(1) In the analysis of this paper, only the droplet transfer from the turbulent core region onto the duct wall was discussed and the effect of droplets supplied to the core region was not considered. It seems that the effect pointed out in the question might be taken into account in the case of the droplet transfer in an actual mist flow. However, in the experimental result by Cousins^(*), for instance, any remarkable difference can not be recognized between the case where only the deposition occurs and the case where the deposition and the generation take place at the same time, though the droplet transfer rate is found to be a little higher in the latter case than in the former.

Strictly speaking, the distribution of droplet diameters and accordingly the value of transfer coefficient may vary with the axial distance of the duct since the transfer rate is affected by the droplet diameter as indicated in the analytical result of this report. However, the above effect is considered to be small in the experimental value of duct length 1.5 m since the experimental results of the transfer coefficient do not vary along the duct axis as shown in Fig. 4 of the first report⁽¹⁾. Therefore, the range of duct lengths for which Eq. (37) is applicable was not indicated in this paper. For extremely long ducts, there may be a case where the effect of variation of the droplet diameter distribution must be taken into account.

The boundary condition, Eq. (12), and the

(*) Cousins, L. B. and Hewitt G. F., *Symp. on Two-Phase Flow*, Vol. 2 (1965), C 401.

thickness of boundary region δ in the present analytical model are proposed from the consideration of droplet behavior at the edge of the turbulent core region. Hence, the concept of this model is a little different from that by Friedlander and others in which the stopping distance for droplet motion is defined taking into account the resistance of gas phase adjacent to a duct wall.

(2) The volumetric fraction of the droplets treated in the theoretical part of the present analysis corresponds to the lowest condition of the experiment described in the first report⁽¹⁾ and the value is about 0.02% as indicated in Fig. 11 of the report. Hence, the coalescence of droplets may not be the controlling phenomenon for droplet transfer. In the experiment, a fairly long entrance section was provided upstream of the test section as described in the first report. Therefore, it seems that the droplets have approximately finished the break-up process in the gas turbulence of the entrance section. Hence, the break-up phenomenon in the test section is also considered to have little effects on the droplet transfer.

(3) The factors of the velocity fluctuation of droplets might be different from those of the gas turbulence as pointed out in the question. The meaning of "the similar velocity fluctuation in nature" mentioned in the paper is that, regardless of the factors, the droplet motion may have a random nature with the velocity fluctuation $\sqrt{v_d'^2}$ and turbulence scale A_d of Eq. (2) if the droplets were assumed to move following the gas turbulence, and hence a concept of the eddy diffusivity represented by Eq. (1) may be introduced.

(4) For simplicity, we decided to express the model one-dimensionally in this report by using the y -directional components of the velocity fluctuation, the turbulence scale and so on after comparing the two methods for expression, i.e., one-dimensional expression and the other expression indicated in the question. Consequently, the probability in Eq. (3) is expressed as 1/2, which means one of the two directions of y -positive and y -negative, though the value is found to be 1/4 when the model is expressed three-dimensionally by using the absolute values instead of the y -components. By substituting $\sqrt{3}\sqrt{v_d'^2}$ and $\sqrt{3}A_d$ into the three-dimensional expression assuming the uniformities of the velocity and turbulence scale, we may obtain an expression for $\bar{I}$ replacing $\sqrt{v_d'^2}$ and A_d of Eq. (3) by $\sqrt{3/2}\sqrt{v_d'^2}$ and $(2/\sqrt{3})A_d$ respectively. The above constants, $\sqrt{3}/2$ and $2/\sqrt{3}$, are not so different from unity.

(5) Since the values of A_d and δ are approximately $0.1 \times H/2$ and $(0.1 \sim 0.2) \times H/2$ respectively, it may include some errors in the calculation of the

droplet transfer coefficient to ignore the thickness of boundary region in the final stage of the present analysis as pointed out in the question.

The droplet concentration C_A at the boundary surface A of the model is not so high and about 20~30% of the mean concentration $\bar{C}$ in the case of the large droplet as in the experiment. Therefore, the difference may not be so large between the result obtained by assuming $n_{dA}=0$ and the result of the present analysis. However, the authors suppose there is no physical reason to suppose, in general, that $n_{dA}=0$.

(6) The assumption of homogeneity of the gas turbulence in Tchen's theory may be justified to some extent, since the theory has been applied chiefly to the core region of duct. The actual value of drag coefficient for a sphere is only 50% lower than that calculated by Stoke's law even for an extreme condition that $R_{ed} \cong 10$. Hence, the theory can be used by modifying only the value of drag coefficient in the viscous resistance term, i.e. the value of parameter a in the equation of droplet motion. However, the variation of the value a by the above modification is considered a secondary one in comparison with that by a variation of the denominator d^2 in Eq. (25). Then, as the first approximation, the theory of Tchen was used without any modification in the present analysis.

It is not easy in the present stage to conclude which theory, Tchen's or Peskin's, is applicable for this analysis, since there has been no theoretical comparison between the two theories yet and also the experimental verification is not sufficient for the theories. Nevertheless, Tchen's theory was used for the present analysis for the reason that although in the theory of Peskin the effect of gas turbulence is presented as a ratio of Lagrangian and Eulerian turbulence scales, experimental data of the ratio have not been obtained sufficiently yet. For the results of Peskin, it is shown from the estimation by Soo^(*) that the value of p decreases from 1.0 in proportion to the gas velocity and d^2 , and the result does not greatly contradict that by Tchen's theory described in Fig. 5.

(7) Equation (31) is the definition of the transfer coefficient in the case of no interaction between particles and represents the relation between the droplet transfer rate for each droplet diameter and that for an actual droplet distribution. With respect to the controlling factors for K_s , it is indicated from the estimation by Soo^(*) that in the equation for particle motion the terms, like Basset term and so on, can be neglected in comparison with the viscous

(*) Soo, S.L., *Fluid Dynamics of Multiphase Systems*, (1967), p. 59, Blaisdell.

term in the case of particles of very high density as in the present experiment. Hence, only the viscous force term was taken into account for calculating the value p as a controlling factor and the result obtained in this way has been expressed as $f_2(a)$ in Fig. 8.

The description of representative value for droplet diameter is not a general conclusion just as pointed out in the question, and the indicated value of $\sqrt{\bar{d}^3/\bar{d}}$ may be more reasonable than $\sqrt{\bar{d}^2}$ in the report.

(8) Though the last term of Eq. (17) was used in connection with the experimental results in the first report, Eq. (21) can, of course, be derived from Eqs. (12) and (13) and the droplet concentration of Eq. (16).

(9) The value of S is concerned with the ratio of axial mean velocities, while p is concerned with the ratio of the turbulent motions. The authors did not assume any correlation between them in this report.

(10) The question on u_0^* seems to concern Eq. (27). The value was obtained from the experimental result of mist flow with a liquid film on the duct wall as presented in the first report⁽¹⁾.

(11) The error of about 30% may be involved in Eq. (37) as pointed out in the question. In this paper, $f_3(\bar{C})$ was determined from Eq. (34) by substituting the extrapolated experimental result of Fig. 10 into K_s . Although the error could be absorbed in $f_3(\bar{C})$, if the analytical value of K_s were applied in Eq. (34) for calculating the value of $f_3(\bar{C})$, the meaning of $f_3(\bar{C})$ becomes indistinct in this expression. Hence, the above expression for $f_3(\bar{C})$ was used.

(12) For the particles larger than 10 microns, the position where the steep reduction of p takes place corresponds to a location $y^+ \approx 10^2$, i.e. $2y/H = 0.1 \sim 0.2$,

hence the value of ϵ_g at the location is fairly high as shown in Fig. 6. Since the value of ϵ_d is equal to $\epsilon_g \times p$, we may consider that ϵ_d in the range of $y^+ > 10^2$ has fairly high values even though the value p becomes small at the location $y^+ \approx 10^2$. Therefore, we may divide the gas stream by a boundary plane into two regions: the core region over which ϵ_d is roughly constant and the boundary region in which ϵ_d is approximately equal to zero. On the other hand, for the particles smaller than 10 microns the position where the steep reduction of p takes place comes close to the duct wall. The eddy diffusivity of gas phase ϵ_g near the position may be some order of magnitude lower than that in the center region. Hence, the assumption of ϵ_g and also ϵ_d being constant may become unreasonable even though the value p is fairly constant.

(13) In order to obtain numerical examples of Eq. (20), it is necessary to assume the distribution of droplet concentration at the entrance of test section as an initial condition. Let us show here an example at the section of 0.5 m downstream from the entrance by assuming a uniform distribution of droplet concentrations as an extreme case. The result becomes:

$$\begin{aligned} F(\mu_2)/F(\mu_1) &\cong 0.01, \\ \mu_2^2 F(\mu_2)/\mu_1^2 F(\mu_1) &\cong 0.1, \dots \end{aligned}$$

In the above calculation, representative values in the experimental range of the first report⁽¹⁾ were used for $\bar{u}_d$ and a^2 in Eq. (18). From this result, the approximation of Eq. (20) is found to include an error of about 10% maximum. However, the actual distribution of droplet concentrations at the entrance is not so different from the distribution in the downstream part of test section in the experiment. Hence, the actual error seems to be much smaller than the above value.