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The Effect of Thread Angle on Loosening by Impact*

(1st Report, Theoretical Analysis)

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The purpose of this research is to find out a certain angle of the thread unloosening under repeated impacts.

In the last report⁽²⁾, the author described the general theory of fastening and loosening process of screw threads qualitatively.

In this report, he induces theoretical formulae by using a three-dimensional analytical geometry to make clear the effect of thread angle on loosening quantitatively.

He performed numerical calculations on 10 metric screw threads used popularly by changing their thread angle from 65° to 55° and obtained the following conclusions.

(1) When the pitch-diameter ratio is 0.15 (M8, M10, M12, M14), 0.125 (M16, M8×1, M10×1.25) or about 0.1 (M12×1.25, M14×1.5, M16×1.5) and thread angles are 63°, 62.5° or 62° respectively, the strongest fastening action is obtained for each of them. These screw threads are insensitive to angle error.

(2) When the thread angle is 60° or 55°, a comparatively strong loosening action is obtained.

1. Introduction

The basic thread angle of a JIS (ISO) triangle screw thread for fastening is 60°. However, there is no engineering reason that it is the best angle.

Occasions of screw loosening had been explained in several ways⁽¹⁾. But the author uniquely found out that the most effective inducement is a rotation loosening caused by the impact which increases the bolt tension of a bolt-nut-fastener. He reported the mechanism of the above loosening with his own hypothesis and experiments in the last report⁽²⁾. The following is the summary of the report: (1) Compressive and impact stresses are produced on the pressure flank of an external thread and an internal thread. (2) The stress waves from the external pressure flank reflect on its clearance flank and the screw end, and change into tensile waves which reach the pressure flank. (3) The tensile waves decrease a compressive force on the pressure flank and cause loosening. (4) Therefore, when the thread angle and the dimension of screw end are changed in order to have smaller tensile waves, and also in order to change them to compressive waves, the fastener gets harder to loosen and rather tends to fasten.

However, in the last report the loosening and fastening actions were not explained quantitatively since the propagation of stress waves was interpreted graphically.

This time, the author focussed the study on the effect of thread angle and used a three-dimensional analytical geometry to explain his theory. The report became more theoretical than before. First of all, he clarified the formulae and equations to obtain the ratio of loosening or fastening energy to impact energy on the thread surface. Secondly, he identified the most desirable thread angle for ISO metric screw threads of which sizes are used popularly on the basis of calculated value.

2. Basic theory

2.1 Fundamental equation

Figure 1 shows how to determine the rectangular coordinates and the cylindrical coordinates. The z-axis stands for a screw axis and its plus direction is taken by the screw end. ABC in Fig. (a) shows the basic triangle, AB is the pressure flank and AC is the clearance flank. The equation for line AB and line AC is

$$z = (R_t - \rho) \tan \left\{ \begin{matrix} \alpha_p : AB \\ \alpha_c : AC \end{matrix} \right\} \dots\dots\dots (1)$$

where α_p is negative and α_c is positive. QQ' in Fig. (b) shows a helix, and it is formularized as

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$$\mathbf{r} = \{\rho \cos \theta, \rho \sin \theta, z + \rho \theta \tan \beta\} \dots\dots\dots (2)$$

Therefore, if α_p and α_c are expressed as α_{pe} , the general equation for thread surface becomes

$$\mathbf{R} = \{\rho \cos \theta, \rho \sin \theta, T\theta + (R_t - \rho) \tan \alpha_{pe}\} \dots\dots\dots (3)$$

where $T = P/2\pi = \rho \tan \beta$

The general equation for the normal of thread surface is derived from Eq. (3), referring to the vector formula for the normal of curved surface:

$$\begin{aligned} \mathbf{r}_n = & \{\rho \cos \theta + \lambda(T \sin \theta + \rho \tan \alpha_{pe} \cos \theta), \\ & \rho \sin \theta - \lambda(T \cos \theta - \rho \tan \alpha_{pe} \sin \theta), \\ & T\theta + (R_t - \rho) \tan \alpha_{pe} + \lambda\rho\} \dots\dots\dots (4) \end{aligned}$$

Also, the area of thread surface, which is made by θ' -rotation of the part, when $\rho'' \geq r \geq \rho'$, on the

helicoid generating line, referring to the general formula for the area of curved surface is given as

$$\left. \begin{aligned} S &= \frac{\theta'}{2 \cos \alpha_{pe}} \left[\Sigma \right]_{\rho'}^{\rho''}; \\ \Sigma &= \rho \sqrt{\rho^2 + K} + K \ln(\rho + \sqrt{\rho^2 + K}) \\ K &= T^2 \cos^2 \alpha_{pe} \end{aligned} \right\} \dots\dots\dots (5)$$

where $\ln$ is a natural logarithm.

Then, after the positive waves c_1 (dilatational waves) are propagated and reach the clearance flank, they are reflected there and are divided into the negative waves c_1 and the waves c_2 (distortional waves). The reflecting angle of waves c_1 is equal to its incident angle.

The line which passes through a point (x, y, z) is expressed in general as

$$\mathbf{r} = \{x + \lambda l, y + \lambda m, z + \lambda n\} \dots\dots\dots (6)$$

And between each direction cosine—of incident waves c_1 , of helicoid normal at the incident point, and of reflected waves c_1 —and the incident angle, there are established the following two related formulae:

$$\cos \varphi = -(l_i l_n + m_i m_n + n_i n_n) \dots\dots\dots (7)$$

$$\left. \begin{aligned} l_r &= l_i + 2l_n \cos \varphi \\ m_r &= m_i + 2m_n \cos \varphi \\ n_r &= n_i + 2n_n \cos \varphi \end{aligned} \right\} \dots\dots\dots (8)$$

Since the velocity of the reflected waves c_1 and the velocity of the incident waves c_1 are equal, their energy ratio ε is equal to their amplitude ratio. Therefore, referring to reference books⁽⁸⁾⁽⁴⁾ ε is derived as

$$\left. \begin{aligned} \varepsilon &= \frac{4\kappa\mu\sqrt{1-\mu}\cos\varphi - (1-2\mu)^2}{4\kappa\mu\sqrt{1-\mu}\cos\varphi + (1-2\mu)^2} \\ \mu &= (\kappa \sin \varphi)^2 \end{aligned} \right\} \dots\dots\dots (9)$$

2.2 Loosening and fastening

It is assumed in this report that a screw thread has no error and its surface is perfectly smooth. But, the author's theory is fully applicable to a fine ground screw thread for reasons of (1) good agreement between his hypothesis and his experimental result in the last report and (2) no influence of surface roughness and fillet at the end of a cylindrical bar as proved in Mr. Matsumoto's impact experiment on cylindrical bars⁽⁵⁾.

Figure 2 shows an example of the propagation of stress waves in the region of external thread on the ground plane. Each number in mark $\bigcirc$ shows the flank numbers, $\textcircled{0}$ standing for AB in Fig. 1 and $\textcircled{1}$ for AC. Even numbers represent the pressure flanks and odd numbers represent the clearance flanks.

When the flank $\textcircled{0}$ in the figure receives compressive impact in z -direction⁽²⁾⁽⁶⁾⁽⁷⁾, it develops a normal compressive stress and a tangential stress. The former becomes compressive waves c_1 and the latter becomes waves c_2 , and both propagate in the direction of vertical arrow. But, the waves directed

Nomenclature

F : dynamic force	λ : parameter
H : height of basic triangle	μ : Eq. (9)
P : pitch	ρ, θ, z : circular cylindrical coordinates
Q : point on stress waves	Σ : Eq. (5)
R : radius of thread	σ : stress
$\mathbf{R}$: vector of thread surface	φ : incident angle
S : area	Suffix
T : reduced pitch	b : bottom of basic triangle
Z : effective height of nut	c : clearance side
e : energy ratio of stress wave packets	e : exceeding contact range
f : stress wave packet or the energy	ef : effect
i, j, k : unit vectors of rectangular coordinate	i : incident
l, m, n : direction cosines	ma : major
$\mathbf{r}$: vector of line	me : mean
x, y, z : rectangular coordinates	mi : minor
α : half angle of thread	n : normal
β : lead angle	p : pressure side
δ : angle of very small deflection	r : reflect
ε : ratio of reflected energy to incident energy of waves c_1	t : top of basic triangle
ζ : Eq. (11)	I : wave I, Fig. 3
η : Eq. (12)	II : wave II, Fig. 3
κ : velocity ratio of waves c_2 to waves c_1	0 : occurrence of stress waves
	1, 2, 3 : order of incidence and reflection

ABC: basic triangle, QQ': helix
Fig. 1 Coordinate systems

toward the screw end are not discussed here.

The propagation of stress waves is considered under the assumptions that the material for a screw is steel and the thread angles are $55^\circ \sim 65^\circ$.

The waves c_2 which reach ① has a bigger incident angle than 36° . So they become surface waves on the clearance flank⁽⁴⁾ and don't have any relation with a loosening.

On the plane illustration, it is understood that about 2/3 of the waves c_2 produced by the reflection of waves c_1 which arrive at ① propagate into the unthreaded part of a bolt, and that the rest reaches the pressure flank which is on the other side⁽²⁾. It is clear that the waves, which propagate into the unthreaded part of a bolt, don't have any relation with a loosening.

When the rest of waves c_2 reach the pressure flank, the incident angle is smaller than 40° and a vibration only occurs on the side of external thread and the internal thread has hardly any vibration⁽⁴⁾. Therefore, it can be said that these waves c_2 don't relate to loosening.

For the above reason, the figure only shows the propagation of waves c_1 . Wave I is the locus of waves which come from the intersection of ⑩ and the top line of the thread; and wave II is the locus of waves which are reflected at the contact point of ① and the bottom curve of the thread. Some or all of the waves which are reflected at ① reach the pressure flank or the bottom or the clearance flank or the top of thread on the other side. The destination depends on the difference in the thread angle.

The waves which reach the pressure flank cause a loosening action⁽²⁾. The waves which reach the bottom are reflected and spread out at the bottom curve, and they don't relate to the loosening. The waves which reach the clearance flank are reflected, or they are reflected at the top of thread again and reach the pressure flank again. The former causes a fastening action and the latter causes a loosening action. However, when the top is round the latter is reflected and spreads, and when it is flat, the latter is reflected many times, and the width of the top is

small. Thus, both loosening actions get much weaker and they can be ignored.

The waves which are reflected at ① and reach directly the top can be ignored too, when the top is round. However, when the top is flat, they are reflected and reach the pressure flank again as shown in the left side of Fig. 2, and cause a fastening action. But, the analysis for this is done by plane illustration.

Waves c_1 , which propagate from the whole surface of pressure flank, grow up into a wave packet which is made by Z/T helicoid motion of the locus of the above mentioned waves in the positive direction of axis z . Each f in the figure means the energy of each packet. Now, we denote the ratio of energies as follows:

$$\left. \begin{aligned} e_0 &= \frac{f_{1i}}{f_0}, & \varepsilon_1 &= \frac{f_{1r}}{f_{1i}}, & e_{2p} &= \frac{f_{2pi}}{f_{1r}} \\ e_{2c} &= \frac{f_{2ci}}{f_{1r}}, & \varepsilon_2 &= \frac{f_{2r}}{f_{2ci}}, & e_{3p} &= \frac{f_{3pi}}{f_{2r}} \end{aligned} \right\} \dots\dots\dots (10)$$

Each e in Eq. (10) can be obtained as an area ratio from Eq. (5), for example, $e_0 = [\Sigma]_{\rho\rho_0 Rma} / [\Sigma]_{Rmi Rma}$. And each ε in Eq. (10) can be obtained from Eq. (9) by assuming φ_{me} —the average incident angle of the stress wave packet—as φ . Next, if the amount z , of which part of f_{2pi} or f_{3pi} doesn't relate to loosening and fastening and which is in excess of thread contact, is put as z_{2pe} or z_{3pe} , the efficiency of f_{2pi} or f_{3pi} is given as

$$\zeta_{2p} = 1 - \frac{z_{2pe}}{Z}, \quad \zeta_{3p} = 1 - \frac{z_{3pe}}{Z} \dots\dots\dots (11)$$

Therefore, the energy ratio, of loosening or fastening action to compressive energy which was produced rectangularly on the pressure flank, becomes as follows by using Eqs. (10) and (11).

$$\eta = e_0 \varepsilon_1 (e_{2p} \zeta_{2p} \cos \varphi_{2me} + \varepsilon_2 e_{2c} e_{3p} \zeta_{3p} \cos \varphi_{3me}) \dots\dots\dots (12)$$

It can be thought that wave c_1 interferes with the wave which has been generated on the other side during its propagation, and also its phase on the pressure flank where it reaches, is different from that of the other. However, as long as there is no loss of total energy during the propagation, the calculation for η is done by Eq. (12).

Thus, when a normal dynamic compressive force F_0 is produced on the pressure flank, a turning force $\eta F_0 \sin \beta$ is caused on the internal thread side as in the last report, and a loosening action appears when $\eta < 0$ and a fastening action when $\eta > 0$.

3. Procedure of calculation

In the following, we put $\alpha_c = |\alpha_p|$ and show it as α .

3.1 Three-dimensional analysis

If Q_1 in Fig. 2 is fixed on a paper (x - z plane) to compare the results of three-dimensional analysis with the results of two-dimensional analysis, other

Fig. 2 An example of the propagation of stress waves

real points Q cannot be shown on the paper because of lead angle.

3.1.1 Generation of the waves and 1st incidence and reflection

Here, coordinates of Q_0 and Q_1 , φ_1 , direction cosines of $\overrightarrow{Q_0Q_1}$, $\overrightarrow{Q_1Q_2}$, etc. are obtained. Since Q_1 is on x - z plane and on ①, its position is written as follows by using Eq. (3):

$$R_1 = \{\rho_1, 0, (R_t - \rho_1) \tan \alpha\} \quad (13)$$

And, since Q_1 is on $\overrightarrow{Q_0Q_1}$, the formula is obtained by using Eq. (4) as follows:

$$\begin{aligned} r_{0n} &= \{\rho_0 \cos \theta_0 + \lambda_1 (T \sin \theta_0 - \rho_0 \tan \alpha \cos \theta_0), \\ &\quad \rho_0 \sin \theta_0 - \lambda_1 (T \cos \theta_0 + \rho_0 \tan \alpha \sin \theta_0), \\ &\quad T \theta_0 + \lambda_1 \rho_0 - (R_t - \rho_0) \tan \alpha\} \quad (14) \end{aligned}$$

Since x , y , z -components in Eq. (13) are assumed respectively to be equal to x , y , z -components in Eq. (14), the following equations are derived:

$$T(\rho_0 - \rho_1 \cos \theta_0) = \rho_0 \rho_1 \tan \alpha \sin \theta_0 \quad (15)$$

$$T^2 \theta_0 + \rho_0 \rho_1 \sin \theta_0 - (2R_t - \rho_0 \rho_1) \tan \alpha = 0 \quad (16)$$

The lead angle of fastening screw thread is 3° at most, and θ_0 is of course very small. Therefore, we can assume that $\sin \theta_0 = \theta_0$ and $\cos \theta_0 = 1$. Then, from the calculation (as in Table 1) $\theta_0 < 34'$, and the relative error of the assumption is only a number of the fifth decimal place.

From Eqs. (15) and (16), and also from the definitions of wave I and wave II, the following equations are derived:

$$I\rho_0 = R_{ma}, \quad I\rho_1 = \frac{TR_{ma}}{T + R_{ma}I\theta_0 \tan \alpha} \quad (17)$$

$$II\rho_0 = \frac{TR_{mi}}{T - R_{mi}II\theta_0 \tan \alpha}, \quad II\rho_1 = R_{mi} \quad (18)$$

$$(T^2 + \rho_0 \rho_1) \theta_0 - (2R_t - \rho_0 - \rho_1) \tan \alpha = 0 \quad (19)$$

Also, $I\theta_0$ is derived from Eq. (17) and Eq. (19), and $II\theta_0$ is derived from Eqs. (18) and (19) as follows:

$$\begin{aligned} \theta_0 &= \frac{1}{2a_0} \left\{ \begin{aligned} &(-b_0 + \sqrt{b_0^2 + 4a_0c_0}); \text{ wave I} \\ &(b_0 - \sqrt{b_0^2 - 4a_0c_0}); \text{ wave II} \end{aligned} \right\} \\ a_0 &= T \tan \alpha \{R_{ma}; R_{mi} \text{ wave I; wave II}\} \\ b_0 &= T^2 + \left\{ \begin{aligned} &R_{ma}^2 \sec^2 \alpha - 2R_{ma}R_t \tan^2 \alpha; \\ &R_{mi}^2 (1 - \tan^2 \alpha) + 2R_{mi}R_t \tan^2 \alpha; \end{aligned} \right\} \\ c_0 &= 2T \tan \alpha \{(R_t - R_{ma}); (R_t - R_{mi})\} \quad (20) \end{aligned}$$

Thus, $I\rho_1$ and $II\rho_1$ are obtained too. The direction cosines of $\overrightarrow{IQ_0Q_1}$ and $\overrightarrow{IIQ_0Q_1}$ are obtained from the following equation (21) which is derived from Eq. (4):

$$\left. \begin{aligned} l_{0n} &= \frac{1}{K} (T \theta_0 - \rho_0 \tan \alpha) \\ m_{0n} &= -\frac{1}{K} (T + \rho_0 \theta_0 \tan \alpha) \\ n_{0n} &= \frac{\rho_0}{K}, \quad K = \sqrt{T^2 + \rho_0^2 \sec^2 \alpha} \end{aligned} \right\} \quad (21)$$

Also, the direction cosines of ①'s normal which has inward direction from IQ_1 and IIQ_1 are obtained

from the following equation (22) which is derived from Eq. (4):

$$\left. \begin{aligned} l_{1n} &= -\frac{\rho_1 \tan \alpha}{K}, \quad m_{1n} = \frac{T}{K} \\ n_{1n} &= -\frac{\rho_1}{K}, \quad K = \sqrt{T^2 + \rho_1^2 \sec^2 \alpha} \end{aligned} \right\} \quad (22)$$

Therefore, $I\varphi_1$ and $II\varphi_1$ are obtained from Eqs. (7), (21) and (22), and the direction cosines of $\overrightarrow{IQ_1Q_2}$ and $\overrightarrow{IIQ_1Q_2}$ are obtained from Eqs. (7), (8) and (22).

3.1.2 2nd incidence and reflection

Since Q_2 is on a thread surface and on $\overrightarrow{Q_1Q_2}$, its position is written as follows by using Eqs. (3), (1) and (6):

$$R_2 = \{\rho_2 \cos \theta_2, \rho_2 \sin \theta_2, T \theta_2 + (R_t - \rho_2) \tan (\pm \alpha)\} \quad (23)$$

$$r_{1r} = \{\rho_1 + \lambda_2 l_{1r}, \lambda_2 m_{1r}, (R_t - \rho_1) \tan \alpha + \lambda_2 n_{1r}\} \quad (24)$$

And, in the same way as the last paragraph, the following equations are derived:

$$\rho_2 = \rho_1 \frac{m_{1r}}{m_{1r} \cos \theta_2 - l_{1r} \sin \theta_2} \quad (25)$$

$$T \theta_2 + \rho_1 \left\{ \frac{n_{1r} \sin \theta_2 + m_{1r} \tan (\pm \alpha)}{l_{1r} \sin \theta_2 - m_{1r} \cos \theta_2} + \tan \alpha \right\} - R_t \{ \tan \alpha - \tan (\pm \alpha) \} = 0 \quad (26)$$

The rotation coordinate of Q_2 is generally shown as:

$$\theta_2 = (\pm n)\pi + (\pm \delta_2) \quad (27)$$

$(\pm n)\pi$ is an angle which rotates as far as Q_2 , which gives ② or ③ on the plane illustration like Fig. 2; n is positive and odd, $(\pm \delta_2)$ is a correction angle as far as a real point, and δ_2 is positive.

Also, from the calculation, $\theta_2 = (+n)\pi + (+\delta_2)$ when $\alpha > 30^\circ$ because $m_{1r} < 0$ and $n_{1r} > 0$, $\theta_2 = (\pm n)\pi + (-\delta_2)$ when $\alpha \leq 30^\circ$ because $m_{1r} > 0$ and $n_{1r} < 0$, and δ_2 becomes very small because m_{1r} is also very small. Therefore, we can assume $\sin \theta_2 = -(\pm \delta_2)$ and $\cos \theta_2 = -1$, and Eqs. (25) and (26) become respectively

$$\rho_2 = \rho_1 \frac{m_{1r}}{(\pm \delta_2) l_{1r} - m_{1r}} \quad (28)$$

$$T \{ (\pm n)\pi + (\pm \delta_2) \} + \rho_1 \left\{ \frac{n_{1r} (\pm \delta_2) - m_{1r} \tan (\pm \alpha)}{(\pm \delta_2) l_{1r} - m_{1r}} + \tan \alpha \right\} - R_t \{ \tan \alpha - \tan (\pm \alpha) \} = 0 \quad (29)$$

Hence, $(\pm \delta_2)$ of IQ_2 and IIQ_2 is obtained from the following equation (30) which is derived from Eq. (29):

$$\left. \begin{aligned} (\pm \delta_2) &= \frac{b_2 \mp \sqrt{b_2^2 - 4a_2c_2}}{2a_2}, \quad b_2 = T - \rho_1 \frac{n_{1r}}{m_{1r}} \\ &\quad - \frac{l_{1r}}{m_{1r}} \left\{ \begin{aligned} &(\pm n) \frac{P}{2} - (2R_t - \rho_1) \tan \alpha; \text{ pressure side} \\ &(\pm n) \frac{P}{2} + \rho_1 \tan \alpha; \text{ clearance side} \end{aligned} \right\} \\ a_2 &= T \frac{l_{1r}}{m_{1r}}, \quad c_2 = -(\pm n) \frac{P}{2} + \left\{ \begin{aligned} &2R_t \tan \alpha; \\ &-2\rho_1 \tan \alpha; \end{aligned} \right\} \end{aligned} \right\} \quad (30)$$

$$\left. \begin{aligned} \cos \varphi_{3me} = & \left\{ \begin{aligned} & \cos \varphi_{3p} + C(\rho_{2c} - R_{mi}); \text{ Fig. 4(a)} \\ & \cos \varphi_{3p} - D(R_{ma} - \rho_{3p}); \text{ (b) (c)} \\ & \cos \varphi_{3p} + C(\rho_{2c} - R_{mi}) + D(\rho_{3p} - R_{ma}); \text{ (d)} \end{aligned} \right\} \\ C = & \frac{\cos \varphi_{3p} - \cos \varphi_{3p}}{2(\rho_{2c} - \rho_{2c})} \\ D = & \frac{\cos \varphi_{3p} - \cos \varphi_{3p}}{2(\rho_{3p} - \rho_{3p})} \end{aligned} \right\} \dots\dots\dots (41)$$

(3) When ${}_I Q$ or ${}_II Q$ is an assumption, the calculations for z_e are also made by assuming z to be in direct proportion to ρ :

$$\left. \begin{aligned} z_{2e} = & \left\{ \begin{aligned} & {}_I z_{2p} + A(R_{ma} - \rho_{2p}); \text{ Fig. 3(a), } {}_II z_{2p} > 0 \\ & {}_II z_{2p}; \text{ (b) (c), } {}_II z_{2p} > 0 \\ & {}_II z_{2p} + A({}_II \rho_{2p} - R_{mi}) - 0.5P; \text{ (c), } {}_II z_{2p} > 0 \\ & -{}_I z_{2p} - 0.5P; \text{ (b) (a), } {}_II z_{2p} > 0 \end{aligned} \right\} \\ A = & \frac{{}_II z_{2p} - {}_I z_{2p}}{{}_II \rho_{2p} - {}_I \rho_{2p}} \end{aligned} \right\} \dots\dots\dots (42)$$

$$\left. \begin{aligned} z_{3e} = & \left\{ \begin{aligned} & {}_I z_{3p}; \text{ Fig. 4(a), } {}_I z_{3p} > 0 \\ & {}_II z_{3p} + \frac{({}_I z_{3p} - {}_II z_{3p}) R_{ma} - {}_II \rho_{3p}}{{}_I \rho_{3p} - {}_II \rho_{3p}}; \text{ (b), } {}_I z_{3p} > 0 \\ & {}_II z_{3p} + \left\{ {}_II z_{2c} - {}_II z_{3p} - \frac{({}_II z_{2c} - {}_I z_{2c}) R_t - {}_II \rho_{3p}}{{}_I \rho_{2c} - {}_II \rho_{2c}} \right\} \\ & \quad \times \frac{R_{ma} - {}_II \rho_{3p}}{R_t - {}_II \rho_{3p}}; \text{ (c), } {}_I z_{3p} > 0 \\ & -{}_I z_{3p} + \frac{({}_I z_{3p} - {}_II z_{3p}) {}_I \rho_{2c} - R_{mi}}{{}_I \rho_{2c} - {}_II \rho_{2c}} - 0.5P; \\ & \quad \text{(d)(a), } {}_I z_{3p} < 0 \end{aligned} \right\} \end{aligned} \right\} \dots\dots\dots (43)$$

3.2 Two-dimensional analysis

Here, the propagation loci of stress waves are obtained in a two-dimensional analysis, in order to simplify the calculation of energy ratio η .

The coordinates of Q_0 and Q_1 are derived from the equations of $\odot$, $\overrightarrow{Q_0 Q_1}$ and $\textcircled{1}$ as

$$\left. \begin{aligned} x_0 = & \{x_1(1 - \tan^2 \alpha) + 2R_t \tan^2 \alpha\} \cos^2 \alpha \\ z_0 = & -(R_t - x_0) \tan \alpha \end{aligned} \right\} \dots\dots\dots (44)$$

$$\left. \begin{aligned} x_1 = & (x_0 \sec^2 \alpha - 2R_t \tan^2 \alpha) \frac{1}{1 - \tan^2 \alpha} \\ z_1 = & (R_t - x_1) \tan \alpha \end{aligned} \right\} \dots\dots\dots (45)$$

and by substituting ${}_I x_0 = R_{ma}$ and ${}_II x_1 = R_{mi}$, each coordinate of wave I and wave II is obtained. Graphically,

$$\varphi_1 = 2\alpha \dots\dots\dots (46)$$

Also, angle ω_1 between $\overrightarrow{Q_1 Q_2}$ and x -axis (x is negative) is

$$\omega_1 = 90^\circ - 3\alpha \dots\dots\dots (47)$$

Next, the coordinates of Q_2 are derived in the same way as Q_1 as follows:

$$\left. \begin{aligned} x_2 = & \frac{-x_1 \tan \omega_1 + z_1 - z_b - R_b(\pm \tan \alpha)}{(\pm \tan \alpha) - \tan \omega_1} \\ z_2 = & z_b + (R_b + x_2)(\pm \tan \alpha) \end{aligned} \right\} \dots\dots\dots (48)$$

where z_b is z -coordinate of the bottom of the basic triangle on the flank which includes Q_2 obtained graphically, the double sign is positive in Q_{2c} and is negative in Q_{2p} . Graphically,

$$\varphi_2 = \begin{cases} 2\alpha & ; \varphi_{2p} \\ 180^\circ - 4\alpha & ; \varphi_{2c} \end{cases} \dots\dots\dots (49)$$

Also, angle ω_2 between $\overrightarrow{Q_2 Q_3}$ and x -axis (x is negative) is

$$\omega_2 = 5\alpha - 90^\circ \dots\dots\dots (50)$$

In the same way, the following is derived for Q :

$$\left. \begin{aligned} x_3 = & \frac{-x_2 \tan \omega_2 + z_2 - z_b - R_b(\pm \tan \alpha)}{(\pm \tan \alpha) - \tan \omega_2} \\ z_3 = & z_b + (R_b + x_3)(\pm \tan \alpha) \end{aligned} \right\} \dots\dots\dots (51)$$

$$\varphi_3 = 6\alpha - 180^\circ \dots\dots\dots (52)$$

Next, we denote z -coordinate of any point of the incidence as z_2' when reflected waves at $\textcircled{1}$ reach directly the flat threadtop as described in paragraph 2.2. It is understood now graphically that the waves which reach z_2' are reflected with a reflecting angle $-\omega_1$, and that they reach $\textcircled{2}'$ or $\textcircled{8}'$ depending on the angle of thread. The z -coordinate z_3' of the reaching point is obtained from the following equation:

$$z_3' = \frac{z_2' \tan \alpha + z_b \tan \omega_1 - (R_{ma} + R_b) \tan \alpha \tan \omega_1}{\tan \alpha + \tan \omega_1} \dots\dots\dots (53)$$

And also, graphically,

$$\varphi_3' = 2\alpha \dots\dots\dots (54)$$

4. Calculated result and discussion

In this report, a coarse thread and a fine thread of ISO M8 16 which is popularly used have been investigated by changing their thread in several angles from 65° to 55° .

Three-dimensional analysis and two-dimensional analysis were used for the calculation of M12 coarse thread, but, the effect of thread angles on the others was only assumed from the positions of ${}_I Q_2$ and ${}_II Q_2$ which were derived from two-dimensional analysis. Also, the calculations were done by rounding off to six places of decimals.

4.1 Propagation of stress waves

Table 1 shows the propagating course of waves c_1 which were obtained by three-dimensional analysis of M12 coarse thread. However, wave I is omitted since the differences between wave I and wave II in their direction cosine and φ are smaller than 0.001 and smaller than $2'$. ρ_2 and ρ_3 are the ρ coordinate of Q_2 and Q_3 respectively which exist in the $\odot$ described above, and Q is an imaginary point when $\rho > 6$ mm and $\rho < R_{mi}$. The sign for φ_3 is the same as the sign for angle from the incident waves to the

surface normal. The table shows ρ_2 in two-dimensional analysis at the same time.

The difference of 2α gives a great influence on n_{1r} in the table and decides the flank number which has Q_2 , and loosening or fastening is decided thereby.

The table distinctly shows m , θ_0 and δ which aren't clarified by two-dimensional analysis. Since $m_{0n} < 0$ and $\theta_0 > 0$, a nut rotates in the direction of loosening when the nut vertically jumps on the surface of external thread. This fact justifies the inference of "reaction loosening" which was mentioned in the last report. Also, it is found by the variation of $(m_{1r}, \pm \delta_2)$ and $(m_{2r}, \pm \delta_3)$ that the real point of Q_2 and Q_3 in Fig. 2 goes away from the back of a paper

when $2\alpha > 60^\circ$ and from the surface of it when $2\alpha \leq 60^\circ$. But the amount is small and permits use of close approximation as mentioned in the last paragraph.

It is found by the variation of ρ_2 , φ_2 , ρ_3 , etc. that the smaller 2α is, the larger φ_{2c} becomes and the 2nd reflected waves which arrive the top of thread are more increased. These facts should be noted in the estimation of fastening action in paragraph 4.3.

Now we consider the errors in two-dimensional analysis of φ , ρ and z which are the functions of η . The error of φ is very small, since the round angle of each in the table is equal to φ in two-dimensional analysis. ρ and z have the largest errors in Q_2

Table 1 The calculation results of the propagation of stress waves (when M12×1.75)

unit of ρ and z : mm											
2α	65°	64°	63°	62°	61°	60°	59°	58°	57°	56°	55°
$11l_{0n}$	-0.537	-0.530	-0.522	-0.515	-0.507	-0.500	-0.492	-0.484	-0.477	-0.469	-0.461
$11m_{0n}$	-0.032	-0.032	-0.032	-0.032	-0.032	-0.032	-0.033	-0.033	-0.033	-0.033	-0.033
$11n_{0n}$	0.843	0.848	0.852	0.857	0.861	0.866	0.870	0.874	0.878	0.883	0.887
$11\rho_0$	5.667	5.658	5.648	5.639	5.629	5.620	5.610	5.601	5.591	5.582	5.572
$1\theta_0$	6°54''	7°21''	7°47''	8°12''	8°44''	8°58''	9°21''	9°41''	10°01''	10°21''	10°39''
$11\theta_0$	32°13''	32°24''	32°35''	32°46''	32°56''	33°07''	33°17''	33°26''	33°35''	33°44''	33°53''
$11z_0$	-0.285	-0.294	-0.302	-0.310	-0.318	-0.326	-0.334	-0.341	-0.348	-0.355	-0.362
$11\varphi_1$	65°-6'	64°-5'	63°-6'	62°-6'	61°-5'	60°-6'	59°-9'	58°-6'	57°-6'	56°-6'	55°-5'
$11l_{1r}$	-0.992	-0.995	-0.997	-0.999	-1.000	-1.000	-1.000	-0.999	-0.997	-0.994	-0.991
$11m_{1r}$	-0.007	-0.006	-0.004	-0.003	-0.001	0.001	0.002	0.003	0.004	0.006	0.007
$11n_{1r}$	0.128	0.102	0.076	0.050	0.024	-0.003	-0.030	-0.055	-0.081	-0.107	-0.132
$1Q_2$	③	③	③	②	②	②	⑦	⑦	⑦	⑧	⑧
$11Q_2$	④	③	③	③	③	②	②	②	⑦	⑦	⑦
$1\rho_2$	5.008 4.977	5.379 5.351	5.797 5.765	6.040 6.082	5.525 5.554	5.016 5.052	4.804 4.748	5.306 5.273	5.914 5.884	5.994 6.019	5.540 5.556
$11\rho_2$	5.116 5.117	4.833 4.797	5.209 5.168	5.627 5.582	6.089 6.045	5.764 5.810	5.268 5.329	4.837 4.872	4.920 4.882	5.539 5.492	6.269 6.224
$\pm 1\delta_2$	46°07''	35°27''	25°21''	16°20''	8°19''	-36''	-10°35''	-18°39''	-26°09''	-34°37''	-44°54''
$\pm 11\delta_2$	48°34''	39°50''	28°27''	17°54''	8°17''	-53''	-11°53''	-21°12''	-31°00''	-38°53''	-45°41''
$1z_2$	1.586	1.348	1.091	0.805	0.494	0.198	-0.084	-0.370	-0.702	-1.016	-1.260
$11z_2$	1.990	1.690	1.451	1.196	0.926	0.629	0.344	-0.106	-0.162	-0.498	-0.877
$1\varphi_2$	50°+8'	52°+7'	54°+7'	62°-4'	61°-4'	60°-3'	62°+14'	64°+8'	66°+6'	56°-4'	55°-4'
$11\varphi_2$	65°-6'	52°+10'	54°+11'	56°+11'	58°+9'	60°-7'	59°-9'	58°-6'	66°+9'	68°+10'	70°+18'
$11l_{2r}$		-0.345	-0.386	-0.426				-0.577	-0.612	-0.646	
$11m_{2r}$		-0.058	-0.053	-0.047				-0.048	-0.039	-0.031	
$11n_{2r}$		-0.937	-0.921	-0.904				-0.815	-0.790	-0.763	
Q_3		②	②	②				⑧	⑧	⑧	
$1\rho_3$		5.659	5.945	6.218				5.820	6.104	6.403	
$11\rho_3$		5.317	5.587	5.855				5.426	5.698	5.982	
$\pm 1\delta_3$		59°30''	35°13''	13°39''				3°54''	-19°36''	-39°23''	
$\pm 11\delta_3$		88°54''	58°13''	32°21''				35°30''	3°20''	-23°44''	
$1z_3$		0.584	0.755	0.911				-1.097	-0.949	-0.799	
$11z_3$		0.373	0.538	0.695				-1.312	-1.168	-1.022	
$1\varphi_3$		+12°-12'	+9°-12'	+6°-11'				-6°-13'	-9°-9'	-12°-9'	
$11\varphi_3$		+12°-18'	+9°-17'	+6°-16'				-6°-20'	-9°-14'	-12°-13'	

Notes: (1) $R_{mi}=5.0528$, $R_{ef}=5.4315$

(2) ○ shows flank number as Fig. 2

(3) the lower of $1\rho_2$ and $11\rho_2$ shows the result of 2-dimension analysis

when $2\alpha=59^\circ$. However, their largest errors are considerably small as errors. For example, (1) the error is -0.056 mm for ${}_I\rho_{2c}$ and 0.061 mm for ${}_II\rho_{2p}$, and the ratio of each error to the depth of thread engagement is 3.65% and 3.92% respectively and, (2) the error is 0.033 mm for ${}_IZ_{2c}$ and 0.035 mm for ${}_IIZ_{2p}$, and the ratio of each error to $Z-z$ is 0.33% and 0.36% respectively when $Z=10$ mm. Thus it is clear that the errors of ρ and z are not large either.

The same thing can be said for $M12 \times 1.25$ but here we just show partial results of analysis in Table 3. The analytical results for other screw threads are shown in Fig. 5 and will be described later.

4.2 Loosening and fastening

Table 2 shows the energy ratio η of $M12$ coarse thread which has a round top, and the calculation for the ratio is done on conditions of $c_1=5940$ m/sec, $c_2=3220$ m/sec and $Z=10$ mm. The ratio η' of the

thread with a flat top is also shown in the table, but it will be described later.

The loosening action occurs when $\eta < 0$ and the fastening action occurs when $\eta > 0$. And it is clear from Table 2 that (1) a strong loosening action occurs when $2\alpha=60^\circ, 55^\circ$. (2) fastening action occurs when $2\alpha=62^\circ \sim 64^\circ$ and $2\alpha=56^\circ \sim 58^\circ$, and that the strongest action occurs in the case of 63° . Therefore, the recommendable angle is 63° in this case.

Next, we consider the general tendency of the change of η . The changes of e_{2p} and $e_{2c}e_{3p}$ are irregular and those of the other factors of η are regular. $e_0\varepsilon_1\varphi_{2p}\cos\varphi_2$ gradually changes from -0.021 when it is 65° to -0.066 when it is 55° , and the smaller 2α is, the stronger the loosening action becomes when each e_{2p} or each ρ_{2p} is equal at these 2α . $e_0\varepsilon_1\varepsilon_2\zeta_{3p}\cos\varphi_3$ is 0.017 when $2\alpha=64^\circ$ and 56° , and it slightly

Table 2 The calculation results of the energy ratio η (when $M12 \times 1.75$)
upper: 3-dimension analysis, lower: 2-dimension analysis

2α	65°	64°	63°	62°	61°	60°	59°	58°	57°	56°	55°
e_0	37.130 37.004	38.124 37.996	39.139 39.011	40.159 40.032	41.166 41.039	42.185 42.069	43.265 43.141	44.239 44.125	45.263 45.149	46.306 46.195	47.341 47.226
ε_1	-16.304 -16.274	-16.697 -16.664	-17.229 -17.182	-17.882 -17.820	-18.633 -18.571	-19.504 -19.427	-20.512 -20.385	-21.520 -21.430	-22.650 -22.565	-23.881 -23.781	-25.165 -25.102
e_{2p}	8.727 16.895				62.047 57.050	95.456 99.919	30.109 38.370				61.255 58.164
ζ_{2p}	80.099 79.746				90.664 92.276	93.708 93.440	96.557 96.204				96.153 96.271
$\cos\varphi_2$	42.422 42.262				48.632 48.481	50.129 50.000	51.717 51.504				57.453 57.358
ε_2		-29.168 -29.353	-26.233 -26.436	-23.572 -23.781				-16.602 -16.664	-15.999 -16.020	-15.983 -15.961	
$e_{2c}e_{3p}$		55.091 54.534	100.000 99.975	39.183 44.924				28.716 25.134	60.616 60.562	4.091 8.556	
ζ_{3p}		94.156 94.321	92.453 92.639	92.190 92.210				97.174 96.939	97.369 98.681	98.534 98.447	
$\cos\varphi_3$		97.898 97.815	98.832 98.769	99.497 99.452				99.409 99.452	98.717 98.769	97.733 97.815	
η	-0.180 -0.343	0.943 0.935	1.616 1.621	0.608 0.699	-2.099 -1.945	-3.689 -3.815	-1.334 -1.672	0.438 0.382	0.956 1.039	0.070 0.145	-4.031 -3.807
η'				0.982	1.333					2.856	2.259

Table 3 The calculation results of ρ coordinate of point Q_2 and the energy ratio η (when $M12 \times 1.25$)
upper: 3-dimensional analysis, lower: 2-dimensional analysis unit: mm for ρ , % for η

2α	65°	64°	63°	62°	61°	60°	59°	58°	57°	56°	55°
${}_IQ_2$	④	④	③	③	②	②	⑦	⑦	⑧	⑧	⑨
${}_{II}Q_2$	⑤	④	④	③	③	②	②	⑦	⑦	⑧	⑧
${}_I\rho_2$	5.719 5.745	5.150 5.166	5.457 5.442	5.906 5.886	5.824 5.841	5.305 5.323	5.314 5.272	5.857 5.839	5.915 5.924	5.437 5.448	5.010 4.999
${}_{II}\rho_2$	5.958 5.939	5.700 5.722	5.149 5.173	5.449 5.426	5.908 5.886	5.842 5.865	5.329 5.369	5.206 5.190	5.799 5.782	5.966 5.981	5.476 5.523
η	-1.016 -0.908	-1.595 -1.661	0.530 0.504	1.567 1.570	-1.090 -0.973	-3.735 -3.837	-0.044 -0.391	1.108 1.115	-0.532 -0.737	-5.841 -5.821	-2.061 -2.476
η'	0.887				2.605				3.458		

Notes: $R_{m1}=5.3234$, $R_{e1}=5.5941$

Fig. 5 2nd incidence of the stress waves (2-dimension analysis)

decreases as it approaches 60° . It means that the magnitude of fastening action is principally decided by that of $\epsilon_{2c}\epsilon_{3p}$. ρ_{2c} at $2\alpha=64^\circ$ and ρ_{2c} at $2\alpha=58^\circ$ are equal, and ρ_{2c} at $2\alpha=62^\circ$ and ρ_{2c} at $2\alpha=56^\circ$ are equal as seen from Table 1. By comparison of each $\epsilon_{2c}\epsilon_{3p}$ and each η at these 2α , it is found that the smaller 2α is, the weaker the fastening action becomes acceleratively when each ϵ_{2c} or each ρ_{2c} is equal at these 2α .

The error of η in two-dimensional analysis is 0.224% at most and it is small in general. The relative error is also small in general except in the case of η being very small at 65° or 56° . And it can be said that the error is small in two-dimensional analysis because this report discusses the problem when η is large.

Table 3 shows partial results of analysis for M12 fine thread. It is clear now from η in the table that (1) a strong loosening action occurs when $2\alpha=60^\circ$ and $55^\circ\sim 56^\circ$, (2) the fastening action occurs when $2\alpha=62^\circ\sim 63^\circ$ and 58° , and that the strongest action occurs when it is 62° . Therefore, the recommendable angle is 62° in this case.

The effect of thread top can be negligible when it is a rolled thread which is generally used and it is difficult to process the top perfectly even. Supposing the top is perfectly even, it is clear from Tables 2 and 3 that a thread angle which is 1° smaller than the above mentioned recommendable angles and also thread angles $57^\circ\sim 58^\circ$ cause strong fastening actions, because $\eta+\eta'$ becomes positive. The amount is almost equal to η at the recommendable angle in the former case (1) and it is fairly large in the latter (2).

4.3 Estimation of loosening and fastening

Figure 5 shows the change of position Q_2 with

2α which is obtained in two-dimensional analysis. Line R and point R with suffixes show the related position of thread dimension as shown in Fig. 1. The thick line shows the range of 2nd incidences, the upper end being ${}_H Q_2$, the lower end being ${}_I Q_2$; the right oblique thick line shows the pressure flank and the left oblique thick line shows the clearance flank. The thick line which goes out of line R_{ma} to the right side shows the waves which impinge the top, and the thick line which goes out of line R_{mi} to the left side shows the waves which impinge the bottom. $\bigcirc$ is the flank number; which is shown in Fig. 2.

For example, ${}_H Q_2$ is on (4) and ${}_I Q_2$ is on (3) when $M8 \times 1.25$, $2\alpha=65^\circ$, and also it means that both ${}_H Q_2$ and ${}_I Q_2$ are on (3) when it is 64° .

We can say from the above consideration that the loosening action occurs when the right oblique thick line between line R_{ma} and line R_{mi} is long, and the fastening action occurs when the left oblique thick line between them is long. The magnitudes of loosening and fastening actions are accurately assumed and conclusion (1) is derived by checking the thick line of other screw threads and comparing (1) the notices mentioned in paragraph 4.1, (2) the general tendency of change η mentioned in paragraph 4.2, and (3) the thick line of M12 coarse and fine thread and η in Tables 2 and 3.

Next, it is indicated that the effect of combination of major diameter and pitch of screw thread is very big.

The changes of thick lines of each screw thread with by 2α are classified into (1) M8, M10, M12, M14, (2) M16, $M8 \times 1$, $M10 \times 1.25$ and (3) $M12 \times 1.25$, $M14 \times 1.5$, $M16 \times 1.5$. Pitch-diameter ratio in each class is (1) 0.15, (2) 0.125, and (3) about 0.1. Therefore, the recommendable angle is 63° just as M12 when pitch-diameter ratio is 0.15, it is 62° just as $M12 \times 1.25$ when the ratio is about 0.1, and it is 62.5° when the ratio is 0.125 since it is between classes (1) and (3).

The screw threads which have these recommendable angles are insensitive to angle error since no loosening action occurs even when the angle changes at $\pm 1^\circ$.

5. Conclusions

The following sums up the study results:

(1) A strong loosening action occurs when thread angles are 60° and 55° . The fastening action

occurs when they are $62^{\circ}\sim 63^{\circ}$ and $57^{\circ}\sim 58^{\circ}$, but it is stronger in the former than in the latter.

(2) The recommendable angle which causes the strongest fastening action is 63° when the pitch-diameter ratio is 0.15; it is 62.5° for 0.125; and it is 62° for almost 0.1. Moreover, these screw threads are insensitive to the angle error.

(3) The interference of "reaction loosening", which was based on the experiment in the last report, has been theoretically verified in this report.

(4) A two-dimensional analysis cannot make clear the effect of the lead angle of thread; but, its analytic error is small in general, thus it is useful enough for obtaining thread angles which make loosening and fastening actions bigger.

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