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On Ideal Shapes of the Relieved Surface of Hob Tooth*

By Kenichi TERASHIMA** and Taku UENO***

The gear hob should generate exactly an originally specified tooth form even after re-sharpening. According to the manufacturing method now in use, the solid hob is relieved by feeding a grinding wheel to the direction of hob radius, and the built hob is relieved by inclining the blade made as a segment of thread with continuous surface.

Even if re-ground precisely in a specified way, these hobs always have their peculiar tooth profile errors on the profile of re-sharpened cutting edge.

In this paper, the ideal shapes of relieved hob tooth and the profile of the grinding wheel to grind such a tooth are made numerically clear, and the new manufacturing methods of both solid hob and built hob are introduced.

1. Introduction

The gear hob has to generate correctly the involute tooth profile after resharpening as a new one does. However, the side face of solid hob teeth has been relieved by feeding a grinding wheel in the direction of hob radius, and the built hob has been relieved by inclining the hob blades (F in Fig. 11) made as a segment of screw thread (Fig. 10).

Even if such hobs are resharpened precisely in a specified way, the profiles of reground cutting edge always have peculiar tooth form errors depending upon the relieving method. Such errors have been pointed out by many reports^{(1)~(5)}, but it seems that hob makers have only made their efforts of sharpening correctly the edges of new articles, and have been not only unconcerned about, but even have kept away, from this relieving problem.

Figure 1 shows the errors produced by resharpening the hobs defined by Japanese Industrial Standard, and accuracy grades of gear generated with the reground hob. It also can explain error increase with an increase in regrinding times, module figures and especially thread numbers.

Figure 2 indicates the edge profile errors produced 10 degrees before and behind the position of the nominal diameter of 130 mm, where the edge profile is correctly made. The module of hob is 10 mm, and the number of threads is triple. It is not desirable from the viewpoint of the generated gear accuracy

to use the ordinary hob resharpened many times.

Recently the accurate gears are required, while the multi-thread hobs which are undesirable in accuracy are used to increase gear productivity, so that it is not allowable to neglect such errors of the re-sharpened cutting edge.

On a grinding method of the tooth side face in which such errors do not appear by resharpening, A. Budnick has reported his method by which a grinding wheel for relieving is fed in the direction of hob axis, but a sectional profile of grinding wheel is reproduced

Resharpening the usual hob by 15 degrees, the JIS accuracy of generated gear turns respectively worse about 6, 4, and 1 grade in module 4 mm; and about 8, 5, and 3 grade in module 8 mm for triple, double, and single thread. It is ideal that the reground hob keeps the original accuracy in new articles.

Fig. 1 Tooth profile errors of reground hob, and accuracy of generated gear

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** Professor, Mechanical Engineering Department, Sasebo Technical College.

*** Professor, Faculty of Engineering, Kyushu University, Fukuoka.

on the tooth side surface.

However the shapes of relieved tooth face are delicately deformed by rake angle, helical angle of gash and thread number, and the wheel profile cannot be reproduced on the tooth flank face because of interference*. Therefore the hob, which generates exact gears even after regrinding, cannot be made without analyzing mathematically the tooth flank form and without considering theoretically the grinding wheel profile.

In this paper, the ideal shapes of relieved tooth were theoretically analyzed including the hobs with rake angle and/or helical gash, and the sectional profiles of grinding wheel which can grind the ideal shapes were mathematically made clear. New manufacturing methods were respectively investigated for solid hobs and built hobs having ideal flank teeth.

Symbols used in this paper are illustrated in Table 1.

2. Ideal shapes of relieved tooth flank

Twelve conditions shown in Table 2 ① are necessary to design a hob considering the relieved tooth face. The calculating formulas of ideal tooth face are numerically derived by the use of all con-

ditions as follows; The helical hands of tooth thread, the direction of hob axis, the relations between plus minus sign $\pm$ and suffixes 1 & 2 in the equation, and the used terms are illustrated in Fig. 3.

2.1 Profiles of cutting edges

Cutting edges should exist on an involute helicoidal surface in order to act the same as the rack-typed cutter. An equation of involute helicoid described by the cutting edge at nominal diameter position can be shown by means of co-ordinates of any point P [Fig. 3(b) and (d)] on the helicoidal surface, as follows;

$$\begin{aligned} \bar{X} &= R \cos \omega, \quad \bar{Y} = R \sin \omega, \quad \bar{Z}_{12} = \pm F_{(R)} + l\omega / (2\pi) \\ F_{(R)} &= 0.5W_{ta} + l[(\sqrt{R_a^2 - R_g^2} - \sqrt{R^2 - R_g^2})/R_g \\ &\quad - \{\cos^{-1}(R_g/R_a) - \cos^{-1}(R_g/R)\}]/(2\pi) \end{aligned} \quad \dots\dots\dots (1)$$

In this paper, suffix 12 stands for suffix 1 or 2 corresponding to upper or lower sign of either $\pm$ or $\mp$. This relation between suffix 1, 2 and sign $\pm$ is explained in Table 1 and in Fig. 3(a).

Supposing that the cutting face of hob having rake angle and helical gash is a spiral-stairs-like ruled

Table 1 Nomenclature

In this paper, suffix 12 stands for suffix 1 or 2 corresponding to upper or lower sign of either $\pm$ or $\mp$ in equation, and corresponding to right or left hand of thread in accordance with rules illustrated in Fig. 3(a). These 1 and 2 are omitted in the obvious cases.

A :	center distance between hob and wheel axis
α_a :	axial pressure angle (at pitch point)
α_c :	cutter pressure angle
$\alpha_{(R)}$:	axial pressure angle (at any radius R of hob)
β_s :	setting angle of grinding wheel
γ :	lead angle of hob thread (on the pitch cylinder)
γ_{12} :	lead angle of tooth flank in both sides
γ_g :	lead angle of hob thread (on the base cylinder)
Γ :	setting angle of hob
δ :	rake angle of tooth
e :	peripheral relief amount
ϵ :	side relief angle of tooth
$R_{(R)}$:	axially sectional profile of involute helicoid
ϕ :	lead angle of hob gash
$G_{(R)}$:	axially sectional profile of relieved tooth flank
h :	cam lift mm/cam revolution
l :	lead of tooth thread (on pitch cylinder)
Δl :	side relief amount mm/hob revolution
L :	lead of helical gash; straight gash $L = \infty$
m_n, m_a :	normal, axial module
N_g :	number of gashes
$\Pi_{(R, \delta)}$:	see Fig. 3(d)
R, R_a, R_p, R_d, R_g :	any, outside, pitch circle, root circle, and base circle radius of hob
r_p :	setting circle radius of grinding wheel (see Fig. 6)
S_p :	axial thickness of relieved tooth (the pitch cylinder)
θ :	positioning angle of tooth section
W_{ta} :	axial thickness of thread described by top edge
ψ :	conical base angle of dish shaped grinding wheel
ω :	position angle from cutting face to any point, Fig. 3(d)
Z_n :	thread number of hob
ζ :	peripheral relief angle of hob

(a) Errors in the solid hob relieved by feeding radially the grinding wheel

(b) Errors in the built hob relieved by inclining the comblike blades
Module 10mm, nominal outside diam. 130mm, triple thread, straight gash

Fig. 2

* This "interference" does not mean that the grinding wheel touches once again the finished tooth surface, but that the wheel grinds off excessively the addendum and dedendum portion because of the lead angle different on the tooth flanks.

surface drawn by a generating line n [Fig. 3(c)], which rotates in lead L , touching a base cylinder n' and at the same time being parallel with $\bar{X}\bar{Y}$ -plane; the equation of the cutting face is shown with co-ordinates of any point Q [Fig. 3(b)] on the face.

$$\left. \begin{aligned} \bar{X} &= R \cos \omega, \quad \bar{Y} = R \sin \omega, \\ \bar{Z} &= -L\{\omega - \Pi_{(R, \delta)}\} / (2\pi) \\ \Pi_{(R, \delta)} &= 0.5\pi - \delta \\ &\quad - \cos^{-1}\{R_a/R\} \sin \delta \end{aligned} \right\} \dots\dots (2) \quad \text{Fig. 3(d)}$$

Table 2 Necessary specifications for designing

①	Specifi- cation	$m_n(m_a), R_a, R_p, \alpha_c, \epsilon_{12} (h \text{ or } \zeta),$ $N_G, \delta, Z_n, \phi, \Gamma, \theta, \text{Helix hand}$
②	Transfor- mation $m_a \rightarrow m_n$ $h \rightarrow \epsilon_{12}$ $\zeta \rightarrow \epsilon_{12}$	When specifying values of above parentheses; $m_n = m_a \cos \gamma$ $h \rightarrow h_{12} N_G = \Delta l_{12} - \gamma_{12} - \epsilon_{12}$ $\zeta \rightarrow \Delta l$ of Eq. (15) $\rightarrow \epsilon_{12}$
	Lead angle Lead Gash lead Lead angle of tooth flank Side relief Radius of base circle Lead angle ③ Peripheral relief Tooth top thickness Coefficient	$\sin \gamma = 0.5 m_n Z_n / R_p, \tan \gamma = 0.5 m_a Z_n / R_p$ $l = 2\pi R_p \tan \gamma, \quad l = \pi m_a Z_n$ $L = 2\pi R_p \tan \phi, \quad L = \infty$: Straight gash $\gamma_{12} = \gamma \mp \epsilon_{12} = \tan^{-1}\{(l \mp \Delta l_{12}) / (2\pi R_p)\}$ $\Delta l_{12} = \pm l \mp 2\pi R_p \tan \gamma_{12}, \quad \Delta l_{12} = h_{12} N_G$ $R_b = l \cos \Gamma / (2\pi \sqrt{\tan^2 \alpha_c + \sin^2 \Gamma})$ $\gamma_\theta = \tan^{-1}\{l / (2\pi R_\theta)\}$: On base cylinder $e = \text{Eq. (14)}$ $W_{1a} = \text{Eq. (11)}$ $W = l \sqrt{R_p^2 - R_\theta^2} / (2\pi R_p R_\theta)$ $I_{12} = (L + l \mp \Delta l_{12}) / (L + l)$ $V = L / (L + l), \quad J_{12} = \Delta l_{12} / (2\pi)$ $K_{12} = \Delta l_{12} R_a \sin \delta$ $\quad \quad \quad / (2\pi R_p \sqrt{R_p^2 - R_a^2} \sin^2 \delta)$ $M_{12} = (l \mp \Delta l_{12}) / (2\pi)$

Fig. 3 Illustration of technical terms, co-ordinate systems, etc. used in this paper

If the rake angle δ is negative in Eq. (2), add the negative sign to the calculated result of the right side of equation $\Pi_{(R, \delta)}$ with positive δ . When the hand of gash helix is the same as that of the tooth thread, the sign of L becomes negative.

Let ω_0 represent ω which is obtained from the equation $\bar{Z}_{12} = \bar{Z}$ using Eqs. (1) and (2), then Eq. (3) obtained by substituting ω_0 into Eq. (1) expresses the cutting edge profile at the nominal position.

$$\left. \begin{aligned} \bar{X}_{12} &= R \cos \omega_0, \quad \bar{Y}_{12} = R \sin \omega_0 \\ \bar{Z}_{12} &= L\{\pm F_{(R)} + l \Pi_{(R, \delta)} / (2\pi)\} / (L + l) \\ \omega_0 &= \mp \{2\pi F_{(R)} \mp L \Pi_{(R, \delta)}\} / (L + l) \end{aligned} \right\} \dots (3)$$

2.2 Equations for ideal shapes of relieved tooth surface

Resharpener the cutting face to revolve the original face about hob axis, the tooth flank always has to give a new cutting edge which describes the initial rack profile. Accordingly both the cutting edge profiles before and after regrinding must individually exist on two involute helicoidal surfaces having the same shape and parallel to each other.

Therefore ideal tooth flank must be a thread surface described by the cutting edge profile [Eq. (3)] which moves spirally along the thread flank [Eq. (1)], parallel to hob axis, and in a direction decreasing the tooth thickness. Designating the revolving angle of cutting edge profile as ω and a displacement of profile as $\Delta l_{12}\omega$, the equation of ideal tooth flank is shown with the co-ordinates of arbitrary point P' [Fig. 3(b)] on the flank as follows;

$$\left. \begin{aligned} \bar{X} &= R \cos \omega, \quad \bar{Y} = R \sin \omega, \\ \bar{Z}_{12} &= \bar{Z}_{12} \text{ of Eq. (3)} + (\omega - \omega_0)(l \mp \Delta l_{12}) / (2\pi) \\ &\quad = \pm I_{12} F_{(R)} + M_{12} \omega \pm J_{12} V \Pi_{(R, \delta)} \\ \omega_0 &: \text{from Eq. (3)} \\ I_{12} &= (L + l \mp \Delta l_{12}) / (L + l), \quad V = L / (L + l) \\ M_{12} &= (l \mp \Delta l_{12}) / (2\pi), \quad J_{12} = \Delta l_{12} / (2\pi) \end{aligned} \right\} \dots\dots (4)$$

After resharpener the cutting face by angle θ [Fig. 3(b)], and performing the transformation from the co-ordinate $\bar{X}\bar{Y}\bar{Z}$ to a new coordinate XYZ which is moved spirally from $\bar{X}\bar{Y}$ plane by $l\theta/(2\pi)$, (three thick solid lines in Fig. 3(c)), X and Z coordinates in Eq. (2) and (4) become;

$$\left. \begin{aligned} Z &= -L\{\omega - \theta - \Pi_{(R, \delta)}\} / (2\pi) \\ X &= R \cos(\omega - \theta) \\ Z_{12} &= \pm I_{12} F_{(R)} + M_{12}(\omega - \theta) \pm J_{12} V \Pi_{(R, \delta)} \end{aligned} \right\} \dots\dots (5)$$

ω_0 represent ω which is obtained from equation $Z = Z_{12}$, and the equation for the cutting edge profile produced in the relieved tooth flank becomes Eq. (6).

$$\left. \begin{aligned} X_{12} &= R \cos(\omega_0' - \theta), & Y_{12} &= R \sin(\omega_0' - \theta) \\ Z_{12} &= V \{ \pm F_{(R)} + l \Pi_{(R, \delta)} / (2\pi) \} \\ &\quad \mp \{ J_{12} L / (L + l \mp J_{12} l) \} \theta \\ \omega_0' &= \omega_0 + \theta / I_{12} \end{aligned} \right\} \dots\dots\dots (6)$$

An equation for thread surface described by cutting edge of Eq. (6) with co-ordinates of a point P'' on the surface, is obtained as follows;

$$\left. \begin{aligned} X &= R \cos(\omega - \theta), & Y &= R \sin(\omega - \theta) \\ Z_{12} &= Z_{12} \text{ of Eq. (6)} + (\omega - \theta - \omega_0') l / (2\pi) \\ &= \pm F_{(R)} + l(\omega - \theta) / (2\pi) \mp (J_{12} / I_{12}) \theta \end{aligned} \right\} \dots\dots\dots (7)$$

Equation (7) has an entirely identical shape to involute helicoidal surface of Eq. (1), except a shift of $(J_{12} / I_{12}) \theta$ in the direction of hob axis.

When a hob, which has the cutting edge to describe the thread surface shown in Eq. (7), is mounted slantwise by setting angle Γ , an equation for imaginary profile, which is an envelope represented on gear blank by cutting edge, is derived by Morozumi's theory⁽⁷⁾ for hobbing mechanism.

In accordance with his theory, Eq. (7) is transformed in a new co-ordinate $X'Y'Z'$ revolved by angle Γ about X -axis, then a plane curve is obtained by projecting the transformed Eq. (7) on $X'Z'$ -plane, and finally the envelope of curves drawn by each point on the cutting edge profile can be determined as Eq. (8).

$$\begin{aligned} Z_{12}' &= \mp \cos \Gamma \sqrt{\tan^2 \gamma_g - \tan^2 \Gamma} X' \\ &\quad \pm \tan \gamma_g \cos \Gamma \sqrt{R_a^2 - R_g^2} \mp \{ l \cos \Gamma / (2\pi) \} \\ &\quad \times \{ \cos^{-1}(R_g / R_a) - \cos^{-1}(\tan \Gamma / \tan \gamma_g) \} \\ &\quad \pm 0.5 W_{ta} \cos \Gamma \mp J_{12} \theta \cos \Gamma / I_{12} \dots\dots\dots (8) \end{aligned}$$

Imaginary edges of Eq. (8), linear function of X' , act as rack type cutter; and radius R_g of hob base circle is deduced as Eq. (9), because the coefficient of X' should be equal to cutter pressure angle α_c .

$$R_g = l \cos \Gamma / (2\pi \sqrt{\tan^2 \alpha_c + \sin^2 \Gamma}) \dots\dots\dots (9)$$

The last term $(J_{12} / I_{12}) \theta \cos \Gamma$ in Eq. (8) shows an amount of the tooth thickness to be decreased by relieving. In order to make a thin rack (Fig. 4) have the same profile as the rack at nominal diameter position, a thin top portion (a shadowed trapezoid in

a : Imaginary rack drawn by edge at nominal hob diameter
b : Imaginary rack thinned by relieving the tooth

Fig. 4 Peripheral relief e

Fig. 4) should be ground off as peripheral relieving. From Fig. 4, the relieving amount e takes the form

$$e = 0.5 \cos \Gamma (J_1 / I_1 + J_2 / I_2) \theta / \tan \alpha_c \dots\dots\dots (10)$$

The sum $Z_1' + (-Z_2')$ obtained by substituting R_p into X' of Eq. (8) must be equal to the tooth thickness $\pi m_n / 2$ on rack pitch line, so that the width W_{ta} , which is axial length of thread described by top edge, is shown as Eq. (11).

$$\begin{aligned} W_{ta} &= 2[0.25 \pi m_n + \tan \gamma_g \cos \Gamma \\ &\quad \times \{ (R_p \sqrt{1 - \tan^2 \Gamma / \tan^2 \gamma_g} - \sqrt{R_a^2 - R_g^2}) \\ &\quad + R_g \{ \cos^{-1}(R_g / R_a) \\ &\quad - \cos^{-1}(\tan \Gamma / \tan \gamma_g) \} \}] / \cos \Gamma \dots\dots\dots (11) \end{aligned}$$

In the above equation, setting angle Γ should be calculated by hob lead angle γ for the ordinary hob, and Γ should be zero for the zero setting angle hob (rectangular hob).

2.3 Axially sectional profile of tooth flank, and pressure angle

Putting $\omega = 0$ in the formulas X and Z_{12} of Eq. (5), an equation for axially sectional profile of hob tooth can be derived at angular position θ . Substituting $\delta = 0$ and/or $L = \infty$ in the above formulas, the equations of hob without rake angle and/or with straight gash are respectively obtained as Eq. (12) in Table 3. Differentiating Eq. (12) by R , the axial pressure angle α_a at the pitch point is found by putting $R = R_p$ as Eq. (13) in Table 3. The feeding depth e for peripheral clearance becomes Eq. (14) depending upon helix of gash.

The last term $J_{12} \theta$ of Eq. (12) expresses an axial displacement of the sectional profile of the relieved tooth, and this displacement is proportional to positioning angle θ of the sectional profile.

Accordingly, it is necessary for grinding wheel

Table 3 Sectional profile of relieved tooth flank, and pressure angle at pitch point

Mark*, **: Zero, positive or negative rake angle

Profile	Gash	Rake angle	$F_{(R)}$: Eq.(1), W_{ta} : Eq.(11), $\Pi_{(R, \delta)}$: Eq.(2) Coefficients V, W, I, J, K : See Table 2-③
Profile of tooth flank	Straight	*	$Z_{12} = \pm F_{(R)} \mp J_{12} \theta$
		**	$Z_{12} = \pm F_{(R)} \pm J_{12} \Pi_{(R, \delta)} \mp J_{12} \theta$
	Helix	*	$Z_{12} = \pm I_{12} F_{(R)} \mp J_{12} \theta$
		**	$Z_{12} = \pm I_{12} F_{(R)} \mp V J_{12} \Pi_{(R, \delta)} \mp J_{12} \theta$
Pressure angle	Straight	*	$\tan \alpha_{a12} = \mp W$
		**	$\tan \alpha_{a12} = \mp (W + K_{12})$
	Helix	*	$\tan \alpha_{a12} = \mp I_{12} W$
		**	$\tan \alpha_{a12} = \mp (I_{12} W + V K_{12})$
Peripheral relief	Straight gash		$e = (0.5 \cos \Gamma / \tan \alpha_c) (J_1 + J_2) \theta$
	Helical gash		$e = (0.5 \cos \Gamma / \tan \alpha_c) (J_1 / I_1 + J_2 / I_2) \theta$

to be fed in the direction of hob axis, and for tooth flank to be relieved in proportion to $J_{12}\theta$.

The relief angle is determined by blank materials, cutting conditions and other factors, but in the hob with ideally relieved tooth flank, peripheral relief angle ζ and side relief angle ε are related by Eq. (15).

Relieving the both side flanks with the same cam, the axial relief depth Δl_{12} per hob revolution can be put equal to $\Delta l_1 = \Delta l_2$, and then $\tan \zeta$ is equivalent to $(de/d\theta)/R_a$ obtained from Eq. (14), therefore ε_1 and ε_2 are related to ζ by the following equation.

$$\left. \begin{array}{l} \text{Straight gash :} \\ \Delta l = B/2, \quad \Delta l = \Delta l_1 = \Delta l_2 \\ \text{Helical gash :} \\ \Delta l = (L+I)^2 (\sqrt{1+B^2/(L+I)^2} - 1) / B \\ \varepsilon_{12} = \pm \gamma \mp \tan^{-1} \{ (I \mp \Delta l) / (2\pi R_p) \} \end{array} \right\} \dots (15)$$

here $B = 4\pi R_a \tan \alpha_c \tan \zeta / \cos \Gamma$

2.4 Calculating procedure and computed examples

The calculation sequence is shown in Table 4, for

Table 4 Flow chart for profile calculation

both the hob dimensions and the tooth sectional profile.

Figure 5 illustrates the results of calculated by varying thread number N_G , rake angle δ , hand of gash helix, and position angle θ , in the Japanese Industrial Standard hob (m_n : 10 mm, R_a : 65 mm, R_p : 52.5 mm, and α_c : 20 degrees), which is given side relieving angle $\varepsilon = 3^\circ 30'$.

Figure 5(a) shows that the edge profile of single threaded hob has been approximated by straight line and yet this linear edge should be properly modified in a large module hob. Figure 5(b) and (c) show that the amount to be modified increases rapidly as the thread number increases, and a hob with helical gash and/or rake angle is asymmetric in the sectional profile of tooth flank.

3. Grinding wheel for relieving

The ideal shapes of relieved tooth flank are the loci described by the sectional profile of Eq. (12), when the profile is moved spirally by lead $l + \Delta l_2$ or $l - \Delta l_1$ [Δl_{12} in Table 2 (3)] along the hob axis.

The profile of grinding wheel is not reproduced

Table 5 Specifications for designing the built hob

Module	$m_n = 5$
Outside radius	$R_a = 55$
Pitch circle radius	$R_p = 48.75$ ($R_d = 42.5$)
Gash number	$N_G = 10$
Rake angle	$\delta = 8^\circ$
Thread number	$Z_n = 3$
Gash type	$\phi = 0.5\pi$ Straight gash
Setting angle	$\Gamma = 7^\circ$
Position angle	$\theta = 10^\circ$
Peripheral relief angle	$\zeta = 8^\circ$
Calculated	* $\zeta = 7^\circ 07' 06''$
Side relief angle (Left)	* $\varepsilon_1 = 2^\circ 55' 26''$
(Right side)	* $\varepsilon_2 = 2^\circ 52' 41''$
Helix hand	Right

Specification of grinding wheel

Type	Dish-shape
Radius of pitch circle	$r_p = 50$
Inclined angle	$\beta_{s12} = 7_{12}$

Module 10 mm, Cutter pressure angle 20 degrees, Outside radius 65 mm, Side relief angle $3^\circ 30'$

Fig. 5 Axially sectional profiles of tooth with ideal flank

exactly on the tooth flank due to the interference* of wheel, and a slight difference exists between wheel profiles for axially relieving and radially relieving; for these reasons, it is necessary to compute the exact wheel profile.

3.1 Axial profile of grinding wheel

By making a grinding wheel contact with a ground tooth flank at the machining position, a touching curve is obtained between two surfaces.

The wheel profile should be the same as that of a rotating body obtained by revolving the touching curve about the wheel axis.

Figure 6 shows a schema in which a circle (radius r_p) on the wheel surface is attached to pitch point Q (radius R_p & tooth thickness S_p) of hob, and also the wheel axis is inclined by angle β_s around axis x after paralleling the hob axis Z and the wheel axis z at distance $R_p + r_p$. The X -axis passes through the midpoint of tooth top.

An equation of the touching curve is derived from the fact that the line mn which is normal to the tooth flank and which passes through an arbitrary point m on the touching curve, intersects with the wheel axis z .

The co-ordinates (X, Y, Z) and (x, y, z) of any point on the tooth flank are derived as Eqs. (16) and (17).

$$\left. \begin{aligned} X &= R \cos \theta, \quad Y = R \sin \theta, \quad \text{Suffix 12: See Table 1} \\ Z_{12} &= G_{(R)} + M_{12} \theta \\ G_{(R)} &= \pm I_{12} F_{(R)} \pm V J_{12} III_{(R, \delta)} \end{aligned} \right\} \dots\dots\dots (16)$$

$$\left. \begin{aligned} x &= X - A, \quad y_{12} = Y \cos \beta_s + (Z_{12} - S_p) \sin \beta_s \\ z_{12} &= (Z_{12} - S_p) \cos \beta_s - Y \sin \beta_s, \quad A = R_p + r_p \end{aligned} \right\} \dots\dots\dots (17)$$

Add a suffix m to symbols, when the point P is on the touching curve, then the equation of the normal at point P_m is;

Fig. 6 Co-ordinate systems, and setting of grinding wheel

$$\left. \begin{aligned} (x-x_m)/D_{yzm} &= (y-y_m)/D_{zxm} = (z-z_m)/D_{zym} \\ D_{yzm} &= \begin{vmatrix} \partial y / \partial R & \partial z / \partial R \\ \partial y / \partial \theta & \partial z / \partial \theta \end{vmatrix}_{R=R_m, \theta=\theta_m} \end{aligned} \right\} \dots\dots\dots (18)$$

where the symbol D_{yzm} is the above Jacobian. Projecting the normal of Eq. (18) on xy -plane,

$$y = (D_{zxm}/D_{yzm})x + (D_{yzm}y_m - D_{zxm}x_m)/D_{yzm} \dots\dots (19)$$

The above linear equation should pass through the origin of xy -plane, as the normal intersects with z -axis.

Thus, this condition becomes,

$$D_{yzm}y_m - D_{zxm}x_m = 0, \quad D_{yzm} \neq 0$$

Expanding the Jacobian D and ignoring suffix m , the above equation takes the form;

$$\left. \begin{aligned} E[R \sin \theta \cos \beta_s + \{G_{(R)} + M_{12} \theta - S_p\} \sin \beta_s] \\ - [-\cos \beta_s \{R \alpha_{(R)} \sin \theta + M_{12} \cos \theta\} + R \sin \beta_s] \\ \times (R \cos \theta - A) &= 0 \\ E = M_{12} \sin \theta - R \alpha_{(R)} \cos \theta &\neq 0 \\ \alpha_{(R)} &= \mp \{I_{12} l \sqrt{R^2 - R_p^2} / (2\pi R_p R) \\ &\quad + V J_{12} R_s \sin \delta / (R \sqrt{R^2 - R_s^2} \sin^2 \delta)\} \end{aligned} \right\} \dots\dots\dots (20)$$

After giving a suitable value to radius R and finding the θ which satisfies Eq. (20), from Eq. (16) and (17), the co-ordinates z_{12} from origin O and wheel radius r are obtained as Eq. (21).

$$\left. \begin{aligned} z_{12} &= (Z_{12} - S_p) \cos \beta_s - Y \sin \beta_s \\ r_{12} &= \sqrt{x^2 + y_{12}^2} \end{aligned} \right\} \dots\dots\dots (21)$$

The whole profile of wheel can be found by varying R between R_a and R_d .

3.2 Base cone of grinding wheel

The wheel surface usually is concave as shown in Fig. 7. Accordingly, in the wheel, supposing a cone which has the generating line $b\bar{x}$ and axis z , and knowing the difference between curved wheel profile and straight generating line, the wheel surface is easily modified.

Finding a generating line $dn\bar{x}$, which intersects with wheel axis z and touches the tooth flank, the shape of base cone is determined by both wheel radius

Fig. 7 Expression of wheel sectional profile by base cone

up as a hob having center O, they are relieved peripherally and laterally by inclining them by angle j on the hob pitch cylinder.

In such a way, even if the cutting edge has exact profile at position A, the edge profile at B or C deviates from theoretical profile as shown in Fig. 2(b). Such errors can not be avoided as far as this method is used, accordingly the advantage of the long life has no meaning for accuracy.

The author proposes a new manufacturing method without such a defect.

5.1 Tooth flank of built hob, and grinding wheel

In the built hob, the profile of blade tooth may be also made to have profiles calculated by the equation in Table 3. For that purpose, the wheel is modified on its surface as described in paragraphs 3.1 and 3.2, and it is set in the way of Fig. 6, and then it may be fed either by lead $l - \Delta l_1$ in the right side flank or by $l + \Delta l_2$ in the left.

The hub of the built hob generally is grooved as flute A in Fig. 11, and blades F are fixed in their own flutes. The teeth of a complete hob have to be arranged along hob thread D. In order to make a continuous thread surface C with lead $l + \Delta l_2$, the comblike blades are shifted in turn by ΔS_2 in the grooves; thus the right flank C can be continuously ground as screw grinding.

The left flank is ground after arranging blades

along surface B. After finished on both side flanks, blades are fixed in each flute along the original helix D. Thus, an ideal tooth flank is given to this built hob. The shifts ΔS_1 and ΔS_2 can be calculated from the next.

$$\left. \begin{aligned} \Delta S_1 &= \Delta l_1 / N_G, \quad \Delta S_2 = \Delta l_2 / N_G \\ N_G &: \text{Gash number} \end{aligned} \right\} \dots\dots\dots (25)$$

The peripheral relieving and the tip rounding should be done in the same way as in manufacturing solid hobs. If the gash number is even, then remove alternately the blades inserted in hub flutes, and the large diameter wheel for peripheral relieving can be used effectively.

5.2 Relation between hob diameter and relief angle

To make a continuous flank, the relieving lead $\Delta S_{12} N_G = \Delta l_{12}$ must be equal to an integer multiple of axial pitch πm_a , otherwise the tooth flank will not be continuous like E in Fig. 11. For this reason, a relation shown by Eq. (26) should be established among the relief angle ζ and ϵ , the outside diameter R_a of hob and the thread number Z_n .

$$\left. \begin{aligned} \epsilon_{12} &= \pm \gamma \mp \sin^{-1} [0.5(Z_n \mp n) / \{(R_a / m_n) - 1.25\}] \\ \text{or } \Delta l_{12} &= \pm l \mp \pi (Z_n \mp n) m_n / \cos \gamma, \quad n: 1, 2, 3, \dots\dots\dots \\ &1.25: \text{Addendum coefficient.} \end{aligned} \right\} \dots\dots\dots (26)$$

The built hobs generally are made with a straight gash and with the setting angle equal to lead angle, and they are relieved on the right and the left flanks with the same cam. Putting $\Delta l = \Delta l_1 = \Delta l_2$ and $\Gamma = \gamma$ both in the equation of straight gash among Eq. (14) and in equation Δl_{12} of Eq. (26), the following is obtained;

$$\zeta = \tan^{-1} \{ (0.5n / \tan \alpha_c) / (R_a / m_n) \} \dots\dots\dots (27)$$

The above is the necessary condition to make a continuous ideal flank.

The calculating procedure is as follows; first of all, the outside diameter R_a is determined, which gives strict limits to the hob design; next, the peripheral relief angle ζ is chosen, which is related to the cutting quality of tooth edge and with the edge wear; and then, n is determined as an integer closest to the value computed by Eq. (27), which expresses blade shift as a multiplying factor of axial pitch πm_a .

By the reverse calculation using Eq. (27), a new ζ is computed from the chosen integer n . If the new ζ differs within about 3° from the original ζ , the profile calculation has to be carried forward using the new ζ , but the machining may be done using the original ζ .

In such a way, tooth depths become different from the required ones except at the nominal diameter position, but, this only gives an effect on the tooth depth of cut gear and does not influence the con-

Fig. 10 Hob blade and screw segment

Fig. 11 Continuous thread surface made by shifting the blades

jugating accuracy of gears. If the new ζ is different more than 3° from the original data, the hob design should be started again by varying radius R_a .

5.3 Design example

Table 5 shows the design condition of a triple threaded built hob in detail with module 5 mm, outside diameter 100 mm and rake angle 8° , where *-marked ζ and ϵ are angles in the case of one pitch shift $n=1$, and are computed by Eqs. (26) and (27).

The reversely calculated ζ is $7^\circ 07'$ and the initial ζ is 8° . Since the difference between the two angles is small, the profile calculation is carried out using $7^\circ 07'$ and the grinding for peripheral relieving is performed using 8° . In this case, the tooth depths at 10° before or behind the nominal position become different by 0.15 mm from theoretical ones.

Since the blade shifting is not necessary for making a continuous flank, a solid hob can be designed easily without such a limitation between R_a and ζ . Table 6 illustrates various dimensions and the coefficients computed by ② and ③ of Table 2 for profile calculation, hob design and machining. Table 7

Table 6 Specifications for manufacturing the built hob

Lead angle of hob thread	$\gamma = 8^\circ 51' 00''$
Axial pitch	$\pi m_a = 15.8972$
lead	$l = 47.6917$
Gash lead	$L = \infty$
Lead angle of tooth flank	$\gamma_1 = 5^\circ 55' 34''$ $\gamma_2 = 11^\circ 43' 41''$
Side relief (in lead)	$\Delta l_1 = 15.8972$ $\Delta l_2 = 15.8972$
Radius of base circle	$R_0 = 18.9802$
Tooth thickness on pitch line	$S_{p1} + S_{p2} = 8.0628$
Tooth top thickness	$W_{ta} = 3.3179$
Lead of tooth flank (Left side)	$l - \Delta l_1 = 31.7944$
(Right side)	$l + \Delta l_2 = 63.5889$
Cam lift for peripheral relief	$h_3 = 4.80$
When solid hob is made by above data;	
Cam lift for side relief	$h_{12} = 1.5897$

Table 7 Axial profile of relieved tooth flank, and grinding wheel for relieving

Radius of hob R	Profile of tooth Z_{12}	Sectional profile of grinding wheel			
		For left flank		For right flank	
		$\bar{x}_1$ mm	$\bar{y}_1 \mu$	$\bar{x}_2$ mm	$\bar{y}_2 \mu$
55.000	1.659	6.667	13.4	6.614	5.7
53.438	2.255	5.001	7.7	4.967	2.9
51.875	2.849	3.335	3.5	3.317	1.1
50.313	3.441	1.668	0.9	1.662	0.2
48.750	4.031	0	0	0	0
47.188	4.619	1.669	0.9	1.670	0.2
45.625	5.204	3.341	3.7	3.349	0.4
44.063	5.786	5.014	8.6	5.041	0.5
42.500	6.364	6.691	15.7	6.747	-0.1
$\alpha_{a12} = 20^\circ 38' 56''$		$\psi_1 = 20^\circ 32' 52''$		$\psi_2 = 20^\circ 15' 11''$

Dimensions written in the above developed views are those of triple thread hob

Fig. 12 Grinding method of side tooth flank

Fig. 13 Setting and feeding of wheel

Fig. 14 Grinding for peripheral relief, and hob outline

a : Sectional profile at the nominal diameter
b and c : Profile at the 10 degrees before and behind the nominal position a

Fig. 15 Axially sectional profiles of tooth flank and grinding wheel

shows individually the sectional profiles of the tooth flank and the wheel, calculated by following the procedure of Table 4. In Fig. 12 through Fig. 15, the machining methods are given to help understanding.

6. Conclusions

The solid hob relieved by feeding a grinding wheel radially to the hob radius, and the built hob relieved by inclining the hob blades, always have their own peculiar tooth profile errors on the resharpened tooth face. It is desirable for the shapes of tooth flank not to have such errors at the reground edge.

The essentials of this paper may be summarized as follows;

(1) The ideal shapes of hob tooth flank were quantitatively determined, which had been qualitatively considered.

(2) The strict calculating method for the profile of a grinding wheel has been worked out, by which an ideal flank can be formed considering the interference effect.

(3) By analyzing these ideal flanks, it was made numerically clear that such flanks can not be made without axial relieving.

(4) New manufacturing methods for the solid hobs and built hobs with the ideal flank have been proposed.

Finally, to make ideal hobs which do not have the tooth profile error after resharpening, special machines and difficult techniques are not required. A minor modification of ordinary hob grinding machines is sufficient for this purpose. The authors expect that their simple method will soon be practically applied.

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Discussion

M. MOROZUMI (Shinshu University):

The profile errors produced at the resharpened edge of radially relieved solid hob should relate to the diameter of the used grinding wheel. However, this effect is not clear in Figs. 1 and 2(a).

A. Fieseler and K. Braun have described it theoretically and practically only in reference (5) among (1) through (5) regarding the tooth flank. Even in their theory, the effect of wheel diameter is not considered.

In what way is such an effect considered in Fig. 1 and Fig. 2(a)? By what theoretical grounds are the computed results supported?

Authors' closure

It is pointed out that such an error is influenced by the effect of wheel diameter. But, in order to estimate roughly, this effect was neglected as follows.

In the Append.-Fig. 1, supposing that ab is

a sectional profile of involute helicoid, the part ac of ab should be used as cutting edge in the straight gash hob, and this ab should act as rack profile cutter for gear blank. The sectional profile de of a tooth thinned by relieving must be equal to a profile made

Append.-Fig. 1 Cutting edge profile of straight gash hob

by moving ab parallel to hob axis, because this edge also must act as the same profile as the rack before re-grinding.

The part $a''c''$ is a profile made by moving ac radially by peripheral relief $\angle R$, and then the axial differences $a''d$, $p''p$, and ec'' between profiles de and $a''c''$ are shown as the errors in Figs. 1 and 2(a).

In the pencil type wheel having an approximately linear touching line, the profile errors approach almost nearly the values shown in the figure, but they are slightly different in the dish type wheel having a somewhat curved touching line.

In this paper, however, the wheel diameter and all the other conditions are considered in the ideal profile calculation.