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Behavior of Fatigue Crack Propagation*

(In the Case of Repeated Torque)

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There are many problems on behavior of a fatigue crack that are not still revealed by many studies with an electron microscope. Macroscopic behavior of the fatigue crack cannot be completely explained by its electron microscopic one. Medium scopic observation between the macroscopic and the electron microscopic one should be made to fill the gap between them. In this investigation, the vicinity of the fatigue crack tip is observed every several cycles of stress by an optical microscope of 280 magnification and the fracture surface is observed by the electron microscope. The relation between an optical microscopic observation and an electron microscopic one is discussed and the following conclusions are obtained:

(1) It is observed that a fatigue crack intermittently propagates. So, there are dormant terms in a fatigue crack propagation process.

(2) The dormant terms ΔN and a crack length Δl which intermittently propagates after a dormant term depend on material and applied stress. The former is determined by alternating stress and the latter determined by maximum stress.

(3) Striations are found on the fracture surface on which a fatigue crack intermittently propagates.

1. Introduction

Many studies on the behavior of a fatigue crack propagation have been made recently. In most of them, macroscopic and microscopic rates of the fatigue crack propagation are related to a stress intensity factor⁽¹⁾. However, it seems that the relation between the rate of the fatigue crack propagation and the stress intensity factor fails to state the physical properties of the crack propagation.

The microscopic behavior of the fatigue crack is presumed on the basis of an observation of the fracture surface using an electron microscope. It does not always explain the macroscopic behavior.

The dislocation, energy, nucleation model and so on have been presented as the mechanism of the fatigue crack propagation. In many models, it is assumed that the fatigue crack propagates by a minute length at every one cycle of stress. On the basis of this assumption, the microscopic rate obtained from striations on the fatigue fracture

surface does not coincide with the macroscopic one at lower and higher levels of the stress⁽²⁾. Therefore, this assumption is not always true.

It has been shown that there is a dormant period in the fatigue crack propagation^{(3) (4)}. However, its properties and the relation between it and the striations are not known. This is an important problem to clarify the mechanism of the fatigue crack propagation.

In this paper, the tip of a fatigue crack propagating from a slit subjected to a repeated torque is observed at every few stress cycles. It is shown that there is a dormant period in the fatigue crack propagation, and that the dormant period and the length of the propagation after the dormant period are respectively related to the stress. In addition, the observation of the fatigue fracture surface makes it possible to discuss the relation between the dormant phenomenon and the striations on the fracture surface.

2. Experimental procedure

A carbon steel S35C for structure use and aluminium alloy 56S are used in this experiment. The mechanical properties and the chemical composition of these materials are shown in Figs. 1

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and 2. As shown in Fig. 2, the specimen is a thin hollow cylinder of outer and inner diameters 30 mm and 28 mm respectively. When the torque is applied to this specimen, the plane stress state is expected to occur. A slit of 2 mm long, of which the curvature of the edge is 0.25 mm and the major axis makes an angle of 45° to the axis of the specimen, is machined by electro spark in the center of the specimen. A crack is initiated from the slit and it propagates along its major axis. The surface of the specimen of the carbon steel S35C for structure use is burnished with sand papers up to # 1000, and buffed with powder of CrO_2 to eliminate cutting marks. Then, the specimen is annealed in a vacuum for 40 minutes at 830°C . The specimen of Al alloy 56S is also annealed in a vacuum for 40 minutes at 350°C , and then is electrolytic-polished in a solution of phosphoric acid.

The specimen treated as above is attached to a Schenk type fatigue testing machine and it is subjected to a repeated torque. In our previous experiment⁽⁵⁾, a crack, which is initiated from the slit as shown in Fig. 1 under the repeated torque, propagates at a constant rate until it becomes 1 mm in length. Therefore, as it could be assumed that the crack propagates in the same manner until it becomes 1 mm long, the observation was made at tips of the cracks of 0.4 mm to 0.6 mm long without making the crack length uniform. A copy of the appearance in the vicinity of the crack tip on the surface of the specimen is taken

Fig. 1 Shape of the specimen

Table 1 Mechanical properties and chemical composition of the carbon steel S35C

Yield stress kg/mm ²	Tensile strength kg/mm ²	Elongation %	Reduction of area %
32.9	60.1	28.9	47.0

(%)

C	Si	Mn	P	S	Cu
0.39	0.24	0.74	0.016	0.014	0.09

Table 2 Mechanical properties and chemical composition of Al-alloy 56S (%)

Cu	Si	Fe	Mn	Mg	Cr	Zn
tr	0.63	0.78	0.09	6.13	0.054	tr

0.2 % proof stress 10.8 kg/mm²

(a) At σ_a of 16.0 kg/mm²

Fig. 2 Change of the appearance in the vicinity of a crack tip in carbon steel S35C under repeated stress

on a 0.8 mm thick foil of acetylene-cellulose by using a replica method. The area containing the crack tip on the copy is photographed through the optical microscope of 280 magnification and finally magnified to 840 diameters on a print.

3. Experimental results

In this experiment, it is found that the fatigue crack intermittently propagates after a short dormant period and that the dormant period and the length of the intermittent propagation of the fatigue crack after this period depend on the applied stress. It will be shown below for two materials of S35C and Al alloy 56S how the appearance of the crack tip changes as the stress cycle is repeated.

3.1 Carbon steel S35C for structure use

Figure 2 shows the change of the appearance in the vicinity of the crack tip as the stress cycle is repeated in the case of a zero mean stress. (a) shows results in the case of a stress amplitude of 16.0 kg/mm². The upper left in Fig. 2(a) shows

the crack tip at N cycles of the repeated stress. A black and thick line is a main crack. Slip lines incline at an angle of about 45° to the direction of the main crack propagation, and those which look like microcracks exist in the slip lines. The main crack in parallel and vertically propagates in a zig-zag way to the slip line. A step of the zig-zag way is about 10 microns long. By repeating 8 cycles of the stress in addition to N cycles, the main crack tip opens a little but does not propagate. However, microcracks along the slip line increase in size and number. After repeating the stress of $N+22$ cycles, the main crack tip opens wider. However, its position does not change. A few of the microcracks existing along the slip lines will combine with each other. To repeat the stress of $N+34$ cycles increases this tendency. At $N+38$ cycles, two microcracks quite grown are observed ahead of the main crack tip. One of them completely combines with the main crack. At $N+46$ cycles, these two microcracks combine with each other, and the main crack propagates by 20 microns.

(b) shows the result of the stress amplitude of 9.0 kg/mm². A crack opening displacement is much smaller than that at the stress amplitude of 16.0 kg/mm², and the plastic zone, namely, the slip line ahead of the crack tip can not be observed through the optical microscope. The main crack propagates not in a zig-zag way, but straight. In the case of the stress amplitude of 16.0 kg/mm², the plastic zone ahead of the crack tip is large, therefore it is considered to be closer to the plane stress state. In the case of the stress amplitude of 9.0 kg/mm², the plastic zone ahead of the crack tip is small, therefore it is considered to be closer to the plane strain state. The position of the main crack tip, the crack opening displacement and the appearance in the vicinity of the crack tip at $N+2000$ cycles are similar to those at N cycles. This state is kept up to $N+7000$ cycles. At $N+10000$ cycles, the main crack propagates by about 4 microns. An electron microscope of 2000 magnification makes the plastic zone ahead of the crack tip visible, which is not observed under an optical micro-

(b) At σ_a of 9.0 kg/mm²
Fig. 2 Change of the appearance in the vicinity of a crack tip in carbon steel S35C under repeated stress

scope ($\times 280$).

3.2 Anticorrosion aluminium alloy 56S

Figure 3 shows results on Al alloy 56S. (a) is the result at the stress amplitude of 10.5 kg/mm^2 , which causes the specimen to fracture at about 10000 cycles. Difference in material does not essentially change the appearance in the vicinity of the crack tip. An area deformed non-linearly and coarse slip lines exist ahead of the crack tip. A zig-zag like a microcrack along and vertical to the slip lines exists ahead of the crack tip. The main crack vertically crosses the slip lines. At the stress cycles of $N+16$, a zig-zag microcrack hardly grows and a new one is initiated ahead of the main crack tip. This new microcrack combines with the main crack and the main crack propagates a little. At $N+26$ cycles, the zig-zag microcrack almost combines with the main crack and at $N+36$ cycles, they completely combine

with each other. The main crack propagates by about 20 microns.

(b) illustrates the results at the stress amplitude of 4.0 kg/mm^2 . At a low level of the stress, the slip line can not be observed ahead of the crack tip just as in the case of the carbon steel S35C. By repeating the stress from N cycles to $N+400$ cycles, a fine line appears ahead of the main crack tip and it becomes clear at $N+600$ cycles. This fine line branches along the boundary of the grain of which the center is almost in the direction of crack propagation. At $N+800$ cycles, this line grows a little. However, it is not a crack which has a free surface separating the grain from the adjacent one (this must be something that takes place due to a grain boundary slip). After $N+1000$ cycles, this line becomes a crack which has a free surface, and the main crack propagates by about 8 microns.

As shown above, in the carbon steel S35C and anticorrosion aluminium alloy 56S subjected to the repeated torque, it is observed that the fatigue crack intermittently propagates after the dormant period ΔN , and it is expected that the dormant period ΔN and the length of the intermittent propagation of the fatigue crack depend on the applied stress and the material.

4. Discussion

4.1 Relation between the applied stress and the dormant period ΔN

Figure 4 illustrates the relation between the applied stress and the dormant period ΔN for carbon steel S35C and Al alloy 56S. The stress amplitude σ_a is taken on the abscissa and the dormant period ΔN on a log scale, on the ordinate in this diagram. The results on S35C are shown in Fig. 4 (a). The experimental results are scattered widely. However, the averaged results signified by large $\bigcirc$ can be linearly related to the stress amplitude. The results under the mean stress of 3.6 kg/mm^2 are signified by $\oplus$ marks in this diagram and they are scattered as widely as in the case of the mean stress of

(a) At σ_a of 10.5 kg/mm^2

Fig. 3 Change of the appearance of the vicinity of a crack tip in Al-alloy 56S under repeated stress

zero. Therefore, the mean stress affects the dormant period ΔN in the fatigue crack propagation so little that the effect is hardly recognized. This is similar to the effect of the mean stress on the fatigue crack initiation. It seems very interesting.

The stress is increased very much at the crack tip and the material in the vicinity of the crack tip is deformed non-elastically. Therefore, the dormant period ΔN can be considered as the cycles required to fracture a very small area at the crack tip due to the plastic fatigue.

The relation shown in Fig. 4 is very similar to that between the stress amplitude and the life to the fatigue crack initiation at the notch root of a low-carbon steel subjected to the plastic fatigue load.

In Fig. 4(b), the results on Al alloy 56S are shown. Just as S35C, the mean stress hardly

affects the dormant period ΔN .

From these figures, the relations between the dormant period and stress amplitude are obtained as follows;

for carbon steel S35C,

$$\log \Delta N = -0.323\sigma_a + 6.680 \quad \dots\dots\dots(1)$$

and for Al alloy 56S,

$$\log \Delta N = -0.328\sigma_a + 4.230 \quad \dots\dots\dots(2)$$

4.2 Relation between the applied stress and the length Δl of the intermittent propagation of the fatigue crack

The length Δl of the intermittent propagation of a fatigue crack is related to the maximum applied stress in Fig. 5. The results on carbon steel S35C in Fig. 5 are scattered. This is considered to be due to the inhomogeneity of the grain (size, orientation, kind and so on) at the crack tip in the direction of the crack propagation just as in the case of the dormant period ΔN . Large

○ marks signify the averages of the results at each maximum stress level and fall on a straight line.

It is reported that the rate of the fatigue crack propagation is proportional to the size of the plastic zone ahead of the crack tip⁽⁶⁾. This suggests that the plastic zone is an important factor for the fatigue crack propagation.

When an infinite plane or a half plane containing a crack is subjected to tension, the elastic stress distribution in the vicinity of the crack tip is obtained exactly. On the other hand, a few stress distributions due to the crack originating from a notch as a circular or an elliptical hole are theoretically analysed, but the numerical calculation of the stress is not made.

Regarding a state where a crack is initiated from a slit as a superposed notch, the stress distribution in the vicinity of the crack tip is approximately obtained by using the approximate formula for the stress concentration factor of the superposed notch as follows; $\alpha = \alpha_I \times \alpha_{II}$ *

* In reference (7), $\alpha = \alpha_I + \alpha_{II} - 1$ is used because of its easy calculation. Here, we use $\alpha = \alpha_I \cdot \alpha_{II}$ which is considered to be more accurate.

(b) At σ_a of 4.0 kg/mm²

Fig. 3 Change of the appearance of the vicinity of a crack tip in Al-alloy 56S under repeated stress

Where α , α_I and α_{II} are stress concentration factors of respectively ⁽⁷⁾ ⁽⁸⁾, the superposed notch, the slit and the crack.

The plastic zone is assumed as a region where the applied stress ahead of the crack tip is greater than the yield stress of the material. The relation at various levels of the maximum nominal stress applied to the specimen between the size λ of the plastic zone ahead of the crack tip and the crack length indicates that the size λ is almost uniform for the crack length up to about that of the semi-major axis of the slit. The ratio c of Δl to λ is independent of the maximum nominal stress applied to the specimen and it is almost constant. The value of c is 13.3 for the carbon steel S35C. The relation between the maximum applied stress and λ/c is shown by a solid line in Fig. 5. Namely, the relationship of $\Delta l = \lambda/c$ is established. The solid line in Fig. 5 (a) may be approximated with a straight line, a broken one in this figure. This relation is expressed by

$$\log \Delta l = 0.065 \sigma_{\max} - 3.072 \dots \dots \dots (3)$$

where

$$\sigma_{\max} = \sigma_a + \sigma_m$$

σ_a is a stress amplitude and σ_m , a mean stress.

Just as the carbon steel S35C, the results on Al alloy 56S are shown in Fig. 5 (b). The results

(a) Relation between the stress amplitude and ΔN for carbon steel S35C

(b) Relation between the stress amplitude and ΔN for Al-alloy 56S

Fig. 4

for the maximum stress of 8.0 kg/mm² include those for the mean stresses of 0.0, 2.0 and 4.0 kg/mm². The values of Δl at the mean stresses of 2.0 and 4.0 kg/mm² are almost same and they are slightly greater than those at the mean stress of zero. The maximum stress can be regarded as the most important factor to determine the length Δl of the intermittent propagation of the fatigue crack as well as in the case of the carbon steel S35C. The relation between the maximum stress and the quotient of the size λ of the plastic zone divided by the constant c of 50.0 is shown by a solid line in Fig. 5 (b). A straight broken line which fits the experimental results is steeper than this solid line. It is due to the simple assumption used for the calculation of the size of the

(a) Relation between the maximum applied stress and the length of the intermittent propagation of a fatigue crack for carbon steel S35C

(b) Relation between the maximum applied stress and the length of the intermittent propagation of a fatigue crack for Al-alloy 56S

Fig. 5

plastic zone, however, that a large plastic zone takes place ahead of the crack tip because the applied stress is close to the yield stress of the material. However, the solid line well expresses the tendency of the experimental results. The broken line is expressed by

$$\log \Delta l = 0.149\sigma_{\max} - 3.291 \dots\dots\dots (4)$$

4.3 The macroscopic rate of the fatigue crack propagation and the averaged rate of the intermittent propagation

Under this experimental condition, it is observed that the fatigue crack intermittently propagates by the uniform length Δl after the uniform dormant period ΔN till it becomes the length (1 mm) of the semi-major axis of the slit. The averaged rate of the crack which propagates stepwise is expected to agree with the macroscopic rate obtained from the crack propagation curve. The averaged rate of the intermittent propagation is obtained from Eqs. (1) and (3) as follows;

$$\log \Delta l - \log \Delta N = 0.065(\sigma_a + \sigma_m) - 3.072 \\ - (-0.323\sigma_a + 6.680)$$

$$\log \frac{\Delta l}{\Delta N} = 0.388\sigma_a + 0.065\sigma_m - 9.752 \dots\dots\dots (5)$$

The macroscopic rate obtained from the crack propagation curve for the similar specimen is expressed by ⁽⁸⁾

$$\log \frac{dl}{dN} = 0.366\sigma_a + 0.054\sigma_m - 9.256 \dots\dots\dots (6)$$

Although the coefficients of σ_a and σ_m in Eq. (5) is a little larger than those in Eq. (6), both equations well agree with each other. These equations are interpreted graphically in Fig. 6. The intermittent rate is smaller than the macroscopic rate under the effect of the mean stress of about 3.0 kg/mm². It is mainly due to the difference in the constant terms of both equations. However, Eqs. (5) and (6) comparatively agree with each other.

From the above, it can be concluded that in this experiment, the macroscopic propagation

Fig. 6 Comparison between macroscopic rate and averaged one of intermittent propagation

behavior of the fatigue crack (crack propagation curve) can be sufficiently explained by the propagation behavior of the crack observed through the medium visual field (optical microscope).

This paper does not deal with the case where the fatigue crack propagates at every one cycle independently of the repeated stress. It is not clarified in this paper how the effect of the stress amplitude and the mean stress on the rate of the fatigue crack propagation can be explained by the equation such as Eq. (5) obtained from Eqs. (1) and (3). This has to be clarified by a new experiment.

4.4 The dormant phenomenon and the striations

If a striation on the fatigue fracture surface

$\sigma_a = 4.0 \text{ kg/mm}^2$ $\times 5100$
 $8.0 \times 10^{-7} \text{ mm/c}$ from crack propagation diagram
 $1.67 \times 10^{-3} \text{ mm/c}$ from striation pitch

$\sigma_a = 10.5 \text{ kg/mm}^2$ $\times 3580 \times 3/4$
 $9.5 \times 10^{-4} \text{ mm/c}$ from crack propagation diagram
 $4.0 \sim 5.0 \times 10^{-4} \text{ mm/c}$ from striation pitch

Fig. 7 Striations on the fatigue fracture surface of Al-alloy 56S

is formed when a fatigue crack propagates at every one cycle as considered in general, the striations should not be formed on the fracture surface under the condition of the experiment that the fatigue crack is dormant.

Electron micrographs of the fracture surface of the Al alloy 56S in this experiment are shown in Fig. 7. A transgranular fracture and the plane strain state are included in the conditions for the striations being formed. In this experiment, the stress in the vicinity of the crack tip is close to the plane stress state. However, at both high and low levels of the stress amplitudes of 10.5 and 4.0 kg/mm², the striations are observed. As the striations are observed not on the whole of the fracture surface, but on a patch area, the following are conceivable for the formation of the striations;

(1) the striations are formed by a certain mechanism while the crack is dormant, and

(2) they are formed on the area where the crack is active and it propagates at every one stress cycle.

Although the exact distance between two striations is not determined because of the uncertainty of the striation at the stress amplitude of 10.5 kg/mm², it is 4.0×10^{-4} to 5.0×10^{-4} mm. If these striations are formed when the crack propagates at every one stress cycle as described in (2), the rate of the crack propagation is 4.0×10^{-4} to 5.0×10^{-4} mm/cycle. The rate obtained from a crack propagation curve is 9.5×10^{-4} mm/cycle and it is about twice as large as that obtained from the striations. This difference is not so large as to deny (2) if it is considered that the observation of the striations is limited to within a small area and that the rate of the crack propagation in the material is scattered.

On the other hand, the rate of the crack pro-

Fig. 8 Fractograph of the fracture surface of Al-alloy specimen which is statically fractured immediately after the surface crack completes the dormant period

pagation obtained from the distance of the striations at the stress amplitude of 4.0 kg/mm² is 8.7×10^{-7} mm/cycle and about one-tenth as large as that obtained from the crack propagation curve, 1.67×10^{-5} mm/cycle. As the similar result is obtained⁽⁹⁾, for an Al alloy 2024-T3 it can be considered that the striation is not always formed at every one stress cycle in whatever condition. Figure 8 shows an electron fractograph of the fracture surface of the Al alloy specimen which is statically fractured immediately after the surface crack of a certain length completes the dormant period and propagates by about 2.5×10^{-2} mm. The static fracture surface is seen on the right side of the figure and the fatigue one, on the left where the striations are observed. The boundary between two fracture surfaces is almost at the center of the figure. The interval divided by directed lines corresponds to the length Δl of the intermittent propagation of the crack of 2.5×10^{-2} mm at the stress amplitude of 10.5 kg/mm², and shows the area where the crack propagates intermittently. In this region, the striations are formed. If the

Fig. 9 Model of formation of striations in one intermittent propagation of the fatigue crack

crack behavior observed on the surface exactly coincides with that on the inside, a certain mechanism of forming the striations in the dormant period of the crack should exist as described in (1). Many striations must be formed in one intermittent propagation of the crack due to this mechanism. This mechanism can be explained from the model⁽¹⁰⁾ of the fatigue crack propagation proposed by one of the authors which combines the mechanism of the crack propagation by J. Holden with the observation of the striations by P. J. E. Forsyth.

A plastic zone takes place in the vicinity of the crack tip due to the large stress concentration. A high level of the stress repeated in the dormant period largely deforms the grains in the plastic zone and it forms subgrains in them. The movement of the dislocations by repetition of the stress forms atomic vacancies. These vacancies gather on the subgrain boundary to form voids as shown in Fig. 9(a). The voids increase in number and size by combining with each other under the additional repetition of the stress as shown in Fig. 9(b). They decrease the sectional area of the part where the crack is going to propagate. When the number of the voids reaches a certain value, the material between the voids is statically fractured by the maximum stress of the cycle and the crack propagates through the voids as shown in Fig. 9(c). A high applied stress is sufficient to fracture a large bulk of the material even between a small number of the voids. On the other hand, a low applied stress needs so many voids that the distance between the voids should be sufficiently short. If the repetitions of the brittle fracture and the ductile one are observed as the striations⁽¹¹⁾, the striations are formed on the fracture surface since the void corresponds to the brittle fracture portion and the fracture of the material between the voids, to the ductile one. Wide striations are formed at a high level of the stress and narrow ones, at a low level of the stress. The difference, as indicated in Fig. 8, between the macroscopic rate of the crack propagation and the distance of the striations can be explained by this mechanism. It is considered that the area without striations is fractured by shearing along the slip lines or by the intergranular means.

In the case where the crack behavior on the surface is different from its inner behavior, it is considered that the inner crack propagates at every one stress cycle during the dormant period of the

surface crack and the striations shown in Fig. 8 are formed at every one stress cycle. The several mechanisms of the formation of the striation in this case were described previously.

It can be sufficiently explained by both relations, as described above, between the surface crack and the inner one why the striations are formed on the corresponding fracture surface where the surface crack intermittently propagates after the dormant period.

5. Conclusions

The areas at the crack tips of the carbon steel S35C and the anticorrosion Al alloy 56S are observed and the following conclusions are obtained:

(1) In both materials S35C and 56S, a fatigue crack propagates intermittently. Namely, the dormancy of the crack is observed.

(2) The dormant period depends not on the mean stress but only on the stress amplitude.

(3) The length Δl in the intermittent propagation of the crack after the dormant period depends on the maximum stress and is proportional to the size of the plastic zone ahead of the crack tip.

(4) Striations are observed on the fracture surface where the surface crack intermittently propagates. This observation result is explained by indicating the fact that the striations can be formed whether the surface crack behavior coincides with the inner one or not.

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Discussion

J. HOSHINO (Nippon Kaiji Kyokai):

I read with interest the method of an analysis on the mechanism of the fatigue crack propagation. Although my questions seem to concern fundamental problems, I want to ask about the following points:

(1) The mechanism of the crack propagation shown in Fig. 9 is considered as one in an infinite body without the edge plane as a boundary. On the other hand, an observation in the vicinity of the crack tip in this experiment is made on the surface where the stress and displacement states must be essentially different from those on the inside of the infinite body. In authors' experiment, the thickness of the specimen is 1 mm for the stress in the specimen which is close to the plane stress state. The questioner, however, considers that the discontinuity due to the free surface is of higher order than that due to the difference of the orientations of the crystals in the polycrystalline. Therefore, a discussion of the fracture mechanism based on the observation result of the free surface should naturally be limited. Namely, it is allowed from the industrial point of view that the rate of the crack propagation is defined as $\Delta l / \Delta N$ from the observation on the surface which gives the length Δl of the crack propagation after ΔN stress cycles. However, it is a mistake that the rate of discontinuous crack propagations is expressed by dl/dN . Additionally, the behavior of the crack incubation obtained from the observations on the surface will be fairly different from that of the crack through the whole thickness of the specimen considering that a crystal having a free surface is deformed easily. Therefore, the behavior is not universal. If the purpose of a study of the fatigue crack propagation is a clarification of the fracture mechanism, the investigation from the standpoint of the physical properties, for example, a detailed observation of the difference in appearance between the surface layer and the inner portion of the fracture surface will be necessary.

(2) In Figs. 4(a) and 5(a) showing the relations between ΔN and σ_a , Δl and σ_{max} respectively, do the marks $\bigcirc$ and $\oplus$ for the average values indicate the averages of the experimental results? Specifically speaking, they are the results at $\sigma_a = 16.0, 10.0$ and 8.0 kg/mm^2 in Fig. 4(a) and at $\sigma_{max} = 19.6, 14.6$ and 12.6 kg/mm^2 in Fig. 5(a). Needless to say, I think that the average value of $\log \Delta N$ is the gravity of points plotted on the

figure.

(3) How is the distinction between the slip line and the crack initiation? I think that the slip line is the one which does not satisfy the conforming requirement of displacement but satisfies the equivalent requirement of the force in the continuum, the crack, the one which does not satisfy both requirements. Regarding the material having the boundary (free surface), it is possible that the slip line on the surface is a crack inside the material. I wish to know the means of their distinction by the observation of the surface.

Authors' closure

(1) We consider that the model of the fatigue crack propagation shown in Fig. 9 may be applied not only to the crack propagation in the infinite body but also to that in the plate. In this experiment, the observation in the vicinity of the crack tip through the optical microscope could not find any voids, but in the fatigue test using the foil which can be observed with a transmission electron microscope, many voids were found out in the vicinity of the fatigue crack^(*1). This result confirms that the model shown in Fig. 9 is valid for the plate. Namely, there may not be any essential difference between the action of a free surface and that of a grain boundary to the void as a boundary.

In general, the fatigue crack propagation curve obtained by macroscopic observation is expressed by a smooth line on assumption that a fatigue crack always propagates continuously during even one stress cycle. However, the fatigue crack propagates discontinuously during one stress cycle in fact. Therefore, the rate obtained from the smooth curve of the crack propagation can be regarded as the average value of the rate of the crack propagation within a certain term and it is considered to be essentially similar to the averaged rate $\Delta l / \Delta N$ of the crack which propagates intermittently.

The dormancy in the surface crack propagation is observed. However, as pointed out by questioner, it is sufficiently possible that the crack propagation behavior on the surface is different from that in the inside of the material. Such an experimental result was obtained^(*2).

It is not clarified in this experiment whether

(*1) Karashima, S., *The 16th Japan National Symposium on Strength, Fracture and Fatigue* (in Japanese), (1971), p. 62.

(*2) Mori, M. and Koterazawa, R., *Preprint of Japan Soc. Mech. Engrs.* (in Japanese), No. 714-2 (1971), p. 99.

the inner crack incubates or not. It can not be considered that the behavior of the surface crack has no relation to that of the internal crack. For example, it is impossible that the internal crack continues to propagate while the surface crack lies dormant. In the other words, it is natural to consider that the rate obtained from the observation of the surface crack propagation coincides with the averaged rate of the internal crack. In this experiment, the crack tip is almost linear through the thickness of the specimen, and this fact endorses the matter described above. However, we are going to make further study on the relation between the macroscopic rate of the internal crack and that of the surface crack using the solid bar and the thick plate specimens.

(2) The large marks of $\bigcirc$ and $\textcircled{1}$ in Figs. 4(a) and 5(a) denote the values of $\log \overline{\Delta l}$ and $\log \overline{\Delta N}$, respectively, where $\overline{\Delta l} = \frac{1}{n} \sum_{i=1}^n \Delta l_i$ and $\overline{\Delta N} = \frac{1}{n} \sum_{i=1}^n \Delta N_i$. Therefore, they do not coincide with the gravity of the groups of the experimental results on the figures.

(3) The distinction between the slip line and the crack is made as follows: The crack is the one which opens by application of the tensile stress and the slip line is the one which does not open by that. Even if the slip line which does not open on the surface by the tensile stress is a crack in the inside, it should be regarded as the slip line by observation of the surface.

J. HOSHINO (Nippon Kaiji Kyokai):

I wish to ask again about (2) in the previous questions. The authors say that the averages in Figs. 4(a) and 5(a) are respectively $\frac{1}{n} \sum_{i=1}^n \Delta N_i$ and $\frac{1}{n} \sum_{i=1}^n \Delta l_i$, but I consider that to approximately derive Eqs. (1) and (3) from these figures is to obtain a regression line for the experimental results plotted on the coordinates of σ_a and $\log \Delta N_i$ or $\sigma_{\max}$ and $\log \Delta l_i$. $\frac{1}{n} \sum_{i=1}^n \log \Delta N_i$ and $\frac{1}{n} \sum_{i=1}^n \log \Delta l_i$ should be the averages of the experimental results. If an arithmetic average of ΔN_i or Δl_i is used as the authors' opinion, the abscissa of Fig. 4(a) or 5(a) should be marked on a real scale and Eqs. (1) and (3) should be expressed by

$$\Delta N = -A\sigma_a + B \quad \dots\dots\dots(1)$$

$$\Delta l = C\sigma_{\max} - D \quad \dots\dots\dots(3)$$

According to such a consideration, a reexamination must be made after Eq. (5).

Authors' closure

As pointed out by the questioner, when a regression line for the experimental results expressed in logarithm, $\log \Delta l_i$ or $\log \Delta N_i$, is obtained to clarify the relation between the stress and $\log \Delta l$ or $\log \Delta N$, the average for each level of the stress should be shown by

$$\overline{\log \Delta N} = \frac{1}{n} \sum_{i=1}^n \log \Delta N_i, \quad \overline{\log \Delta l} = \frac{1}{n} \sum_{i=1}^n \log \Delta l_i$$

However, in this paper, Eqs. (1) and (3) are obtained from the regression lines not for the experimental results, $\log \Delta N_i$ and $\log \Delta l_i$ in Figs. 4(a) and 5(a), but for $\log \overline{\Delta N}$ and $\log \overline{\Delta l}$, where $\overline{\Delta N}$ is $\frac{1}{n} \Delta N_i$ and $\overline{\Delta l}$ is $\frac{1}{n} \sum_{i=1}^n \Delta l_i$.

A regression line between the real values of ΔN_i or Δl_i and the stress becomes a curve. In order to obtain a relation between ΔN or Δl and the stress, the fact is utilized that this curve can be approximated by a straight line in a semi-logarithmic diagram. That is illustrated in Figs. 4(a) and 5(a). Therefore, the straight lines in these figures express the relations between $\log \overline{\Delta N}$ and $\log \overline{\Delta l}$, and the stress, $\log \overline{\Delta N}$ and $\log \overline{\Delta l}$ should be shown as the averaged values.

In regard to the latter point of the questions, since the relations between $\overline{\Delta N}$ and $\overline{\Delta l}$, and the stress are expressed by curves as described above and can be expressed not by

$$\overline{\Delta N} = -A\sigma_a + B$$

but by the Eq. (1)

$$\log \overline{\Delta N} = -A\sigma_a + B$$

the reexamination after Eq. (5) is considered to be unnecessary. This way of arrangement of the results is significant to obtain the rate of the crack propagation of Eq. (5). The rate of the fatigue crack propagation is derived using $\log \overline{\Delta N}$ and $\log \overline{\Delta l}$ pointed out by the questioner as follows;

$$\overline{\log \Delta l} - \overline{\log \Delta N} = \log \frac{\overline{\Delta l}}{\overline{\Delta N}} = f_1(\sigma_a, \sigma_m) \dots\dots(i)$$

This equation expresses the relation between the average of the logarithms of the propagation rates and the stress. On the other hand, according to our manner of the arrangement in Figs. 4(a) and 5(a),

$$\log \overline{\Delta l} - \log \overline{\Delta N} = \log \left(\frac{\overline{\Delta l}}{\overline{\Delta N}} \right) = f(\sigma_a, \sigma_m) \dots(ii)$$

is obtained. This equation expresses the relation between the logarithm of the averaged rate and the stress. Equation (ii), which is deformed into the expression;

$$\left(\overline{\Delta l} / \overline{\Delta N} \right) = \exp \{ f(\sigma_a, \sigma_m) \}$$

can be directly compared with Eq. (6) which expresses the relation between the stress and the

averaged rate obtained from the crack propagation curve. However, the direct comparison of Eq. (i) and Eq. (6) can not be made in the strict sense.

Considering the above points, the arrangement on the basis not of $\overline{\log \Delta N}$ and $\overline{\log \Delta l}$, but of $\log \overline{\Delta N}$ and $\log \overline{\Delta l}$ is made in this paper. Such an arrangement is made for the Al alloy too. As the number of the experimental results is small, the averages of ΔN and Δl are not shown in Figs. 4 (b) and 5 (b). However, Eqs. (2) and (4) express the relations between the stress and $\log \overline{\Delta N}$, the stress and $\log \overline{\Delta l}$, respectively.

J. HOSHINO (Nippon Kaiji Kyokai):

On the diagram of real coordinates, the experimental results should be arranged by using the arithmetic average and on the diagram of log-arithmetic coordinates, by using the geometrical average. The regression line which is obtained for the arithmetic averages plotted on the semi-logarithmic diagram as described in the authors' closure, does not satisfy the requirement to minimize difference from the experimental data and can not represent the group of the experimental data. Therefore, derivation of the equation in this paper seems to have no grounds.

Since $(\log \overline{\Delta N_i}) / (\log \overline{\Delta l_i})$ in Eq. (i) in the authors' second closure is equal to $\log (\sqrt[n]{\Delta N_1 \Delta N_2 \cdots \Delta N_n}) / \log (\sqrt[n]{\Delta l_1 \Delta l_2 \cdots \Delta l_n})$, it is not unreasonable that $(\overline{\Delta l} / \overline{\Delta N})$ geometrical average $= \exp\{f(\sigma_a, \sigma_m)\}$.

Authors' closure

As explained previously, the measured results of ΔN and Δl at certain values of σ_a and $\sigma_{\max}$ are scattered at random. Therefore, their arithmetic averages $\overline{\Delta N}$ and $\overline{\Delta l}$ are regarded as the dormant period and the length of the intermittent propagation of the fatigue crack for certain values of σ_a and $\sigma_{\max}$ respectively. In order to obtain the relations between σ_a and $\overline{\Delta N}$, $\sigma_{\max}$ and $\overline{\Delta l}$, the lines which are most fitting to the results of $\overline{\Delta N}$ and $\overline{\Delta l}$ for different values of σ_a and $\sigma_{\max}$ are examined. Therefore, one value of $\overline{\Delta N}$ or $\overline{\Delta l}$ corresponds to the result for each value of σ_a or $\sigma_{\max}$. Then, the relations between σ_a and $\overline{\Delta N}$, $\sigma_{\max}$ and $\overline{\Delta l}$ are regarded as linear on a semi-logarithmic diagram. In this case, the regression line should be obtained not for the individual experimental result of ΔN or Δl , but for $\overline{\Delta N}$ or $\overline{\Delta l}$. It is considered that there is such misunderstanding because the individual experimental results of ΔN and Δl were plotted in Figs. 4(a) and 5(a) with $\overline{\Delta N}$ and $\overline{\Delta l}$ respectively. However, we believe that the drift of

$\frac{dl}{dN} = \frac{l_2 - l_1}{N_2 - N_1}$: macroscopic propagation rate of a crack
(a) Macroscopic crack propagation curve

(b) Small intermittent propagation of a crack obtained from the detailed observation of crack propagation

$$\text{from (a), (b), } \frac{dl}{dN} = \frac{\sum_{i=1}^n \Delta l_i}{\sum_{i=1}^n \Delta N_i} = \frac{\frac{1}{n} \sum_{i=1}^n \Delta l_i}{\frac{1}{n} \sum_{i=1}^n \Delta N_i} = \frac{\overline{\Delta l}}{\overline{\Delta N}}$$

Append.-Fig. 1

our argument is correct.

Regarding the latter question, as pointed out by questioner, the relation between the stress and the geometrical average of $\Delta l / \Delta N$ is surely obtained from $\overline{\log \Delta l}$ and $\overline{\log \Delta N}$. However, the previous explanation means the following: The macroscopic crack propagation curve, which is used to obtain the crack propagation rate dl/dN in Eq. (6), is considered as resulting from continuity of the intermittent propagation process of the fatigue crack illustrated in this paper. (see Append.-Fig. 1) The crack propagation curve is linear for a long period after the initiation of the crack. The rate in this period is nearly equal to the rate $\overline{\Delta l} / \overline{\Delta N}$ obtained from the arithmetic averages of Δl and ΔN in the intermittent crack propagation over a small length as shown in Append.-Fig. 1. Therefore, the averaged rate $\overline{\Delta l} / \overline{\Delta N}$ of the intermittent propagation obtained in this paper can be directly compared with the macroscopic rate of the crack propagation.

It is therefore significant to obtain the relations between the stress and the arithmetic average $\overline{\Delta N}$ of ΔN , the stress and the arithmetic average $\overline{\Delta l}$ of Δl .