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Study on Machine Tool Slideway* (Analysis of Sliding Motion by Simulation)

By Katuhiro NAKASHIMA**, Masaru TAKATA***,
and Yasufusa MORITA****

A kinetic equation of machine tool slideway motion is obtained considering slideway conditions, and the coefficients of the equation are expressed as a state function which is related to driving velocity and film thickness in a steady state. The transient response of the state function in such a nonsteady state as stick-slip motion can be approximated by the time lag of first order.

The analysis of the kinetic equation by local linearization supports the following experimental phenomena. Namely, stable sliding occurs even in the negative slope range of characteristic curve of friction, and the larger the rigidity or oil viscosity, the wider the stable range is. Since there are many factors which influence the coefficients of the equation, it is difficult to solve the equation analytically. The simulation on an analogue computer, therefore, has been attempted. The results thereof show that the obtained equation represents the actual behavior of sliding motion well.

1. Introduction

In machine tools, it is important from the viewpoint of accuracy and efficiency to assure the geometrically correct and smooth movements of the tool and workpiece. The frictional state of a lubricated slideway, however, depends on the load, sliding velocity and properties of lubricant. A wide variation of frictional state from solid contact to fluid friction causes various problems^{(1) (2) (3)}. While in the range of mixed lubrication, the problems of high friction force, stick-slip motion, wear, etc. are encountered, in the range of fluid film lubrication, there arises the problem of a slider floating due to the oil film. In the present paper, the motion on a slideway, such as stick-slip which takes place at a low sliding velocity is discussed. There have been many papers hitherto written on stick-slip motion. In one of them an analysis is made neglecting the time dependence through the line segment approximation of characteristic curve of friction⁽⁴⁾, while in another the static friction

is discussed assuming that it increases with the stick time and that the kinetic friction changes by the fracture and recovery of the oil film when slipping⁽⁵⁾. While the change of static friction with time is analyzed in connection with the relative distance of two sliding surfaces, namely, the film thickness, the effect of load and sliding velocity is not clarified satisfactorily.

Okamura et al., showed that the behavior of the oil film has a close relation to stick-slip motion, and analyzed, moreover, the frictional characteristics considering the film thickness on the slideway^{(6) (7)}.

It is, however, necessary in analyzing the phenomena of stick-slip motion to know the variation of the friction in motion. So at first a mathematical model of slideway motion is derived here and then the formation of friction, the relation between friction and film thickness and the variation of the oil film in motion are considered. It is difficult to solve analytically the mathematical model because of its nonlinearity. Therefore two methods to check the mathematical model are done. The first method solves a linear model by local linearization, and the second carries out the simulation using an analogue computer.

Since the results of simulation have proved

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** Researcher, Government Industrial Research Institute, Tosu.

*** Professor, Faculty of Engineering, Kyushu University.

**** Professor, Kurume Technical College.

that the mathematical model simulates the actual motion well, the simulation is extended to obtain better conditions which prevent stick-slip motion while varying the coefficients of the mathematical model.

2. Nomenclature

- P = normal load of slider
 p_i = normal load at each contact point
 P_e = normal load of elastic contact part
 P_f = normal load of fluid film lubricated part
 $\alpha = P_e/P$
 F = friction force of slider
 F_e = friction force due to elastic contact
 F_f = friction force due to fluid film lubrication
 F_0 = sticking friction force
 μ = friction coefficient
 $\mu_e = F_e/P$, $\mu_f = F_f/P$, $\mu_0 = F_0/P$
 μ_{e0} = friction coefficient when only the elastic contact appears
 μ_{f0} = friction coefficient under full fluid film lubrication condition
 x = displacement of slider
 $\dot{x}$ = sliding velocity of slider
 $\ddot{x}$ = acceleration of slider
 y = spring deflection
 k = spring constant
 t = time
 g = acceleration of gravity
 v = driving velocity
 h = film thickness
 r_i, r_i' = radius of each contact point
 $R_i = r_i \cdot r_i' / (r_i + r_i')$, $R = \sum_{i=1}^n R_i/n$
 n = number of contact points
 E = modulus of elasticity
 S = shearing strength per unit area at elastic contact point
 a_i = half width of elastic contact point
 d = elastic deformation

3. Equation of motion on slideway

According to the simplified model of slideway depicted in Fig. 1, the following equation of motion is derived.

$$(P/g)\ddot{x} = -F + k(vt - x) \quad \dots\dots\dots (1)$$

Putting

$$y = vt - x, \quad \ddot{y} = -\ddot{x} \quad \dots\dots\dots (2)$$

Fig. 1 A schematic model of the slideway

Equation (1) is rewritten as

$$\ddot{y} = (g/P)(F - ky) \quad \dots\dots\dots (3)$$

$$\left. \begin{aligned} \text{here } F &= ky & \text{when } \dot{x} = 0 \text{ and } P\mu \geq |ky| \\ F &= \pm P\mu & \text{when } \dot{x} \neq 0 \text{ or } P\mu < |ky| \end{aligned} \right\} \quad \dots\dots (4)$$

The relation between friction coefficient and sliding velocity is generally shown as in Fig. 2. If the characteristic curve of friction of Fig. 2 can be applied to a transient state where the sliding velocity varies suddenly, as in the case of stick-slip motion, then the slider motion must always be in an unstable state in the range of the negative slope of this curve due to the negative resistance force. A stable motion is, however, often observed actually in the range of negative slope. This contradiction can be explained by considering the transient characteristics of friction force due to the variation of sliding velocity. The fact that friction force changes differently from its steady characteristics when the sliding velocity changes suddenly is mentioned in the past experimental data^{(5) (8)}. But the theoretical analysis of this phenomenon will be a future task.

In this paper, the mathematical formulation of the mechanism for friction force to be generated is tried in relation to the film thickness based on some assumptions.

4. Consideration on generation of friction force and its formulation

In sliding between just finished surfaces, the running-in may proceed in such a way that higher asperities of each surface contact and slide each other, being accompanied with plastic deformation. When the process of running-in has been completed, it may be considered that the contact occurs at many pairs of projected points having certain curvatures, and the same height which exists uniformly on the contacting surfaces as shown in Fig. 3, and that an elastic contact (Hertzian contact) occurs because the contact pressure is relatively low.

Without lubricant the friction coefficient is

Fig. 2 Characteristic curve of friction

usually smaller than 0.3. This value is smaller than the one obtained with surfaces cleaned by acetone and the like. This seems to suggest that ordinary contact or slide occurs between the contaminated layers. Here the shearing strength of the layer is assumed to be constant. Under a lubricated condition, the elastic deformation decreases due to the formation of oil film. Hence, a friction force to shear the oil film is generated. When surfaces stick to each other, another friction force is produced with a higher value.

From the facts described above the friction force of slider, whether in motion or not, is given by

$$F = F_c + F_f + F_0 \quad (5)$$

In the next section, the mathematical model of F_c , F_f and F_0 are derived.

4.1 Kinetic friction force (F_c , F_f)

Under a uniform load without lubrication, a_i and d in Fig 3 are given by $a_i = k_1(p_i R_i)^{1/3}/E^{1/3}$, $d = k_2 p_i^{2/3}/(E^2 R_i)^{1/3}$, from Hertzian theory, where $\sum_{i=1}^n p_i = P$, k_1 and k_2 are constants. The kinetic friction force without lubricant is expressed as $F_c = \sum_{i=1}^n \pi a_i^2 S$. If the difference between R_i and R is sufficiently small, and the number of contact points is small, then $\sum_{i=1}^n \pi a_i^2$ and $\sum_{i=1}^n \pi a_i^2 S$ will be nearly equal to the values in the cases where all radii of contacting points are equal to R . Thus, for simplicity all radii are approximated to be R . So d is rewritten as $d = k_2 P^{2/3}/(E^2 n^2 R)^{1/3}$.

Under a lubricated condition, the recovery from elastic deformation through oil film formation reduces the load at elastic contact points. Under the lubricated condition F_c is, therefore, obtained by calculating the value P_c as a function of the film thickness h . It is assumed that in an elastic contact with a fluid film between surfaces the slider is lifted uniformly over the whole contact area. Hence $d - h = k_2 (P_c^2/E^2 n^2 R)^{1/3} (h \leq d)$ is obtained.

Fig. 3 Schematic model representing the contact state of the slideway

Therefore, P_c is expressed as follows:

$$P_c^{2/3} = P^{2/3} - \{(E^2 n^2 R)^{1/3}/k_2\} h \quad (6)$$

Then F_c is given by relations $F_c = \sum_{i=1}^n \pi a_i^2 S$ and $P_i = P_c/n$ as follows:

$$F_c = \pi k_1^2 (n R^2/E^2)^{1/3} \{P^{2/3} - h(E^2 n^2 R)^{1/3}/k_2\} S \quad (7)$$

By solving Reynolds' equation, P_f and F_f due to the film thickness h are given by

$$P_f = C_{p1}(h)\dot{x} - C_{p2}(h)dh/dt \quad (8)$$

$$F_f = C_{f1}(h)\dot{x} - C_{f2}(h)dh/dt \quad (9)$$

where C_{p1} , C_{p2} , C_{f1} and C_{f2} are the coefficients determined by the slider shape, which are all functions of h . Although the film thickness is not uniform over the whole contact area, as shown in Fig. 3, the magnitude of elastic recovery has been assumed to be a representative value of the film thickness.

Equation (8) is a first nonlinear differential equation which gives the response of h for $\dot{x}$ and is rewritten as

$$dh/dt = \{\dot{x} - P_f(h)/C_{p1}(h)\} C_{p1}(h)/C_{p2}(h) \quad (10)$$

Here let us define the following function, and name it the "state function".

$$\delta = P_f(h)/C_{p1}(h) \quad (11)$$

This takes a value $\dot{x}$ under the steady state $\dot{x} = v$, and both δ and h are assumed as uniquely corresponding to each other. From the relation $d\delta/dt = (d\delta/dh)(dh/dt)$ and Eq. (11), Eq. (10) is rewritten as

$$d\delta/dt = (\dot{x} - \delta)\gamma \quad (12)$$

where

$$\gamma = F_\gamma(\delta) = \{C_{p1}(h)/C_{p2}(h)\}(d\delta/dh) \quad (13)$$

Equation (12) indicates that δ takes the same value of $\dot{x}$ under the steady state, and that it behaves as the time lag of first order under a transient state.

Also Eqs. (7), (8) and (9) will be expressed as follows.

$$F_c = F_c(\delta) = P\mu_c \quad (14)$$

$$P_f = P_f(\delta) = k_{p1}(\delta)\dot{x} - k_{p2}(\delta)dh/dt \quad (15)$$

$$F_f = P\mu_f = k_{f1}(\delta)\dot{x} - k_{f2}(\delta)dh/dt \quad (16)$$

From Eqs. (15) and (16), μ_f is given by

$$\mu_f = F_a(\delta) + F_\tau(\delta)\dot{x} \quad (17)$$

where

$$F_a(\delta) = k_{f2}(\delta)P_f(\delta)/\{Pk_{p2}(\delta)\} \quad (18)$$

$$F_\tau(\delta) = \tau = k_{f1}(\delta)/P - k_{f2}(\delta)k_{p1}(\delta)/\{Pk_{p2}(\delta)\} \quad (19)$$

The term $F_a(\delta)$ in Eq. (17) which is not proportional to $\dot{x}$, is the term attributed to the change of film thickness.

Now let us write μ_1 and μ_2 as follows.

$$\mu_1 = \mu_c + F_a(\delta) = M_1(\delta) \dots\dots\dots(20)$$

$$\mu_2 = F_r(\delta)\dot{x} = \tau\dot{x} \dots\dots\dots(21)$$

Then it will be found that μ_1 represents the part of the friction coefficient determined explicitly by δ , and μ_2 is that of the friction coefficient which depends on $\dot{x}$. Thus, the total friction force in motion is given by

$$F_c + F_f = P(\mu_1 + \mu_2) \dots\dots\dots(22)$$

4.2 Variation of sticking friction force (F_0)

The sticking friction force F_0 is written as follows:

$$F_0 = P\mu_0 \dots\dots\dots(23)$$

There have been various studies on F_0 , in which it is pointed out that the friction coefficient increases with an increase of sticking time. In this study the same trend has been observed as shown in Fig. 14, and the relation between the increase of μ_0 and the sticking time can be approximated by an exponential function in the same manner as described by Rabinowicz⁽⁹⁾. Since the variable part of μ_0 is considered as a function of δ , the maximum of μ_0 can be written as $\mu_s = M_s(\delta)$.

When the slider starts sliding again, the stick friction disappears, although the mechanism has not been clarified yet. Therefore, it has been assumed here that μ_0 disappears exponentially as in the case where it increases, both exponential characteristics having different time constants. Based upon the considerations made above, the model of μ_0 is given by

$$d\mu_0/dt = (\mu_s - \mu_0)\beta \dots\dots\dots(24)$$

where $\mu_s = M_s(\delta)$, $\beta = \beta_s$ when $\dot{x} = 0$

and $\mu_s = 0$, $\beta = \beta_0$ when $\dot{x} \neq 0$

here, both β_s and β_0 are related to the rate of increase and disappearance of μ_0 , where β_s is assumed to be sufficiently small compared with β_0 considering the rapid disappearance of μ_0 .

From Eqs. (22) and (23), Eq. (15) is finally expressed as

$$F = P(\mu_0 + \mu_1 + \mu_2) \dots\dots\dots(25)$$

Equation (25) is the aimed mathematical model of friction force, which is represented by a function

Fig. 4 Increase of stick friction coefficient versus stick time

of the state function.

The validity of the mathematical model must be checked by investigating whether the actual results can be explained well or not by this model. The functional formulae $F_r(\delta)$, $M_1(\delta)$, $F_c(\delta)$ and $M_s(\delta)$, the coefficients β_s and β_0 in the mathematical model will be determined by the physical factors of an actual apparatus. The main factors may be properties of the lubricant, characteristics of surfaces and weight of the slider. However, it is difficult to obtain explicitly the relations between these physical factors and the functional formulae or the coefficients by experiment. Admitting the relations thus derived the mathematical model can hardly be solved analytically because of its nonlinearity.

This model is, therefore, simulated by an analogue computer, and the results are compared with the experimental ones to examine the validity of the model. In carrying out the simulation, however, all the factors of the functional formulae must be estimated by the experimental results. Furthermore, in order to see the effect of each coefficient, every varied value of the coefficient must be compared one by one. The computation of the stable or unstable range of slider motion becomes tedious because of the numerous combinations of the coefficient. As a result, firstly the linearized model by means of local linearization is solved since it enables us to check the validity of the mathematical model under less assumptions than in the case of simulation, and to investigate the qualitative effect of each coefficient on the slider motion.

5. Analysis by local linearization

Linearizing the mathematical model with regard to an arbitrary minute increment in the vicinity of the steady state excluding $\dot{x} = 0$, the local stability of the slider motion is examined and the results are compared with experimental ones.

With respect to the infinitesimally small deviation from the steady state $\dot{x} = \delta = v_0$, the following equation is derived from Eq. (1).

$$(P/g)\Delta\ddot{x} = -\Delta F + k\Delta y, \quad \Delta\dot{y} = \Delta v - \Delta\dot{x} \dots\dots\dots(26)$$

Where Δ represents the infinitesimally small deviation of each variable. At the constant driving velocity, Δv is zero. From Eqs. (20), (21) and (22) the expressions $\Delta F = P(\Delta\mu_1 + \Delta\mu_2)$, $\Delta\mu_1 = [d\mu_1/d\delta]_{v_0}\Delta\delta$ and $\Delta\mu_2 = [d\tau/d\delta]_{v_0}v_0\Delta\delta + \tau_0\Delta\dot{x}$ are obtained. Rewriting ΔF by introducing η ,

$$\eta = [d\mu_1/d\delta]_{v_0} + [d\tau/d\delta]_{v_0}v_0 \dots\dots\dots(27)$$

$$\Delta F = P(\eta\Delta\delta + \tau_0\Delta\dot{x}) \dots\dots\dots(28)$$

From Eq. (12) the following equation is obtained.

$\Delta\delta = (\Delta\dot{x} - \Delta\dot{\delta})\tau_0$ (29)
 considering Eqs. (26), (27), (28) and (29), a block diagram is shown in Fig. 5. The characteristic equation of this block diagram is given by

$$s^3 + (\gamma_0 + g\tau_0)s^2 + \{g\tau_0\gamma_0 + g\eta\tau_0 + (k/P)g\}s + (kg/P)\gamma_0 = 0 \quad \text{.....(30)}$$

Replacing gk/P by ω^2 , and putting $l\omega/g$, $m\omega/g$ and $n\omega$ as η , τ_0 and γ_0 , respectively, l , m and n

Fig. 5 Block diagram

Fig. 6 Range of stability by parameters l , m and n Fig. 7 Range of stability by parameters a , b and c

can be nondimensional variables in place of η , τ_0 and γ_0 , respectively. Hence, Eq. (30) is rewritten as follows:

$$s^3 + (m+n)\omega s^2 + (mn+ln+1)\omega^2 s + n\omega^3 = 0, \quad m, n > 0 \quad \text{.....(31)}$$

By Routh's stability criterion, the stability condition of Eq. (31) is

$$l > 1/(m+n) - 1/n - m \quad \text{.....(32)}$$

Putting $a\gamma_0^2 = gk/P$, $b\gamma_0/g = \eta$ and $c\gamma_0/g = \tau_0$, k/P , η and τ_0 are normalized with respect to γ_0 as a , b and c , respectively. Hence, Eq. (30) is also rewritten as follows:

$$s^3 + (1+c)\gamma_0 s^2 + (a+b+c)\gamma_0^2 s + a\gamma_0^3 = 0, \quad a, c > 0 \quad \text{.....(33)}$$

The stability condition of Eq. (33) is

$$b > a/(1+c) - a - c \quad \text{.....(34)}$$

The two stability conditions, Eqs. (32) and (34) are shown in Figs. 6 and 7, respectively, where the stable range is denoted by the oblique lines. A noticeable point in Figs. 6 and 7 is that there exists a stable range, even if l or b representing the slope of the characteristic curve of friction is negative. As mentioned in Chapter 3, a stick-slip motion occurs evidently in the negative slope range if the transient character of friction is ignored. However, if the transient character obtained in Chapter 4 is taken into account, a stable sliding can possibly occur in the negative slope range.

Figures 6 and 7 show that the smaller the value of n and the larger the values of a and c , the wider the stable range for the infinitesimally small deviation is. Figure 6 indicates that, when the infinitesimally small deviation from the steady state exists, it becomes easier for the slider motion to shift to the stable state, provided that the time required for the shifted state to return to the steady state is more shortened. The value a corresponds to the spring constant. The value c is related to fluid film friction, and is proportional to the viscosity of lubricant. Since the results described above are valid near the steady state, they can not be applied to the range deviating from the steady state. Nevertheless, by utilizing the present model the same actual behaviors such that the larger the spring constant, the wider the stable range is⁽¹⁰⁾, and a stable slide can occur even in the negative slope range of the characteristic curve of friction can be quantitatively analyzed.

6. Simulation on an analogue computer

This chapter deals with the simulation by setting the functional formulae and coefficients in the mathematical model through an analogue computer, and the results are compared with the experimental

ones. Besides, some effects of the coefficients in the mathematical model on the slider motion are investigated.

6.1 Setting of δ , μ_0 , μ_1 and μ_2

6.1.1 Setting of δ

By Eq. (13), the state function δ is programmed as in Fig. 8. As δ is equal to the driving velocity v in a steady state, the relation between v and F_r can be adopted in place of the relation between δ and F_r . From the experimental result that a larger value of v shortens the transient period, it is presumable that F_r increases in proportion to δ , although the functional relation between δ and F_r is not clear. The functional relation has been assumed as shown in Fig. 9 for convenience of simulation, so that the functional relation can be varied by altering the value of Γ in the simulation.

6.1.2 Setting of μ_0

By Eq. (24), μ_0 is programmed as in Fig. 10. By assuming that $M_s(\delta)$ takes the maximum increase of stick friction coefficient at $\delta=0$, and it becomes zero when δ exceeds a certain value δ_c , $M_s(0)$ is connected to δ_c with a straight line, as shown in Fig. 11.

6.1.3 Setting of μ_1 and μ_2

By Eqs. (20) and (21), μ_1 and μ_2 are pro-

Fig. 8 Generation of δ

Fig. 9 Assumed relation between $\delta(v)$ and $F_r(\delta)/\Gamma$

Fig. 10 Generation of μ_0

grammed, as shown in Fig. 12. The functional formulae of $M_1(\delta)$ and $F_r(\delta)$ are determined in the following way. If μ_{c0} is the friction coefficient at the purely elastic contact, μ_{f0} is the friction coefficient when lubricated by a full fluid film and α is the ratio of P_c and P , then the friction coefficient μ in motion is expressed as follows:

$$\mu = \alpha\mu_{c0} + (1-\alpha)\mu_{f0} \quad (35)$$

The terms $\alpha\mu_{c0}$ and $(1-\alpha)\mu_{f0}$ in Eq. (35) correspond to the terms μ_1 and μ_2 in Eq. (22), respectively. Consequently, if the value of $\alpha\mu_{c0}$ and $(1-\alpha)\mu_{f0}$ are obtained, the functional formulae of $M_1(\delta)$ and $F_r(\delta)$ which correspond to μ_1 and μ_2 , respectively, can be set on an analogue computer. Both $\alpha\mu_{c0}$ and $(1-\alpha)\mu_{f0}$ are obtained in the following way, using the characteristic curve of friction (solid line) in Fig. 13.

There is a region of complete fluid lubrication above the critical sliding velocity v_c in Fig. 13, and in that region, according to the fluid lubrication

Fig. 11 Relation between $\delta(v)$ and $M_s(\delta)$

Fig. 12 Generation of μ_1 and μ_2

Fig. 13 Decomposition of the characteristic curve of friction

theory $\mu = \mu_{f0} = c_f \sqrt{v}$ holds, where c_f is a constant. Extrapolating that equation to the region below the critical velocity, as shown by a broken line in Fig. 13, the broken line will represent the characteristic curve of friction in the region of com-

plete fluid lubrication over the whole region of driving velocity. On the other hand, the value of μ_{c0} is usually constant, regardless of the driving velocity as shown by two-dot-dash line in Fig. 13. The value of α for any driving velocity is, there-

fore, obtained from the lines of μ , μ_{c0} and μ_{f0} in Fig. 13 and from Eq. (35). As a result $M_1(\delta)$ and $F_r(\delta)$ can be obtained.

6.2 Setting of slider motion and whole program

By Eq. (3), the slider motion is programmed as represented by a part A in Fig. 14. Since there never occurs a negative value of $\dot{x}$ when the slider is driven unidirectionally, $\dot{x}$ is so programmed as not to have a negative value. The value of Δ is set as small as possible within the stable region considering the motion of the relay elements to be used. The whole program of slider motion for

Fig. 14 Program of the model

Fig. 15 Oscillations of spring deflection (Simulation)

Fig. 16 Oscillations of spring deflection (Experiment)

Fig. 17 The change of μ with $\dot{x}$

Fig. 18 Period of stick-slip

simulation is shown in Fig. 14.

6.3 Result of simulation

6.3.1 Simulation of the mathematical model

In this paragraph the validity of the mathematical model is examined according to 6.1 and 6.2. The simulation is carried out under the following conditions: The load per unit area is 2.0 kg/cm², and the lubricant is a machine oil No.1.

In carrying out the simulation, firstly β_0 , β_s , and Γ are varied so that the amplitude of spring deflection obtained by the simulation may be approximated to that obtained by the experiment at each prescribed driving velocity. However, a set of β_0 , β_s , and Γ thus obtained at a certain driving

Fig. 19 Effect of β_s and Γ on the number of times of stick-slip Fig. 20 Effect of β_s and Γ on amplitude of spring force and frequency of stick-slip (Solid sign: Frequency of stick-slip)

velocity, does not always meet the above requirement at the other driving velocities. Therefore, an adequate set of β_0 , β_s and Γ which meets the requirement over a wide range of driving velocities is searched.

The oscillations of the spring obtained by utilizing the set of β_0 , β_s and Γ thus obtained are shown in Fig. 15, and the actual oscillations used as the model of simulation are shown in Fig. 16, the two indicating a good simulation. The simulation is carried out in the negative slope range of the characteristic curve of friction.

The variation of the friction coefficient with regard to the sliding velocity in stick-slip or in transient motion is shown in Fig. 17. The behaviors shown in Fig. 17 seem to simulate very well the previous experimental results^{(8) (11)}, that is, when the slip begins, μ decreases rapidly, but the rate of decrease diminishes as the sliding velocity approaches the maximum value and it makes a gradual decrease until the next stick. The period of stick-slip motion at each driving velocity which has been obtained by the simulation is compared with the experimental value in Fig. 18. From the results mentioned above, it is confirmed that the mathematical model well simulates the experimental results.

6.3.2 Effects of β_s and Γ

Since the effects of the coefficients in the mathematical model are investigated easily with an analogue computer, the values of β_s and Γ are varied near the values used in 6.3.1 in order to know their influences on the slider motion.

The number of sticking times counted from

Fig. 21 Effect of Γ on transient time

the beginning of driving to continuous slide is given in Fig. 19. This number serves as the ground for the judgement whether stick-slip motion occurs easily or not. Figure 19 shows how the larger value of β_s tends to cause stick-slip motion and how little Γ influences stick-slip motion over the range of variations in this experiment. The amplitude of spring force and the period of stick-slip motion are also shown in Fig. 20. The values β_s and Γ have influence on the amplitude and the period of stick-slip near the boundary between stable and unstable ranges. They, however, have little influence on them at a low driving velocity far from the boundary region. The transient time from the beginning of drive to the steady state sliding is shown in Fig. 21, which represents the effect of Γ in a high driving velocity region where stick-slip motion does not occur at a steady state. Figure 21 shows that the larger the value of Γ , the smaller the transient time is. On the other hand, it is confirmed that the magnitude of β_s gives little influence on the transient time.

From the results obtained above, it is clear that a smaller value of β_s and a larger value of Γ are desirable in slideway. The small value of β_s causes a slow growth of sticking friction, and the large value of Γ develops a quick formation of oil film.

7. Conclusion

A mathematical model of lubricated slideway motion has been presented based on the assumption of Hertzian contact between two sliding surfaces. The validity of the presented model is confirmed in two ways, namely, by solving the linear model by local linearization, and by introducing the method of simulation with an analogue computer. The results thus obtained agree satisfactorily with the actual results. To examine the presented model, however, many values of functional formulae and coefficients of the model must be assumed. The relation between those assumed values and the physical factors of the slideway has not been clear yet and it is the subject for a future study.

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References

- (1) Matsuzaki, A., *Jour. Japan Soc. Lub. Engrs.* (in Japanese), Vol. 12, No. 5 (1967-5), p. 183.

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- (2) Opitz, H., *Schmieretechnik*, 15. Jahrg. März/April Nr. 2 (1968), S. 77.
 - (3) Mitsui, M., *Jour. Mechanical Laboratory* (in Japanese), Vol. 19, No. 1 (1965), p. 1.
 - (4) Takano, E., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol. 33, No. 253 (1969-9), p. 1138.
 - (5) Kato, S., et al., *Trans. American Soc. Mech. Engrs.*, Vol. 94, Ser. F, No. 4 (1972), p. 234.
 - (6) Okamura, K., et al., *Jour. Japan Soc. Precision Engrs.* (in Japanese), Vol. 35, No. 4 (1969-4), p. 215.
 - (7) Okamura, k., et al., *Jour. Japan Soc. Precision Engrs.* (in Japanese), Vol. 35, No. 8 (1969-8), p. 490.
 - (8) Okamura, K., et al., *Jour. Japan Soc. Precision Engrs.* (in Japanese), Vol. 34, No. 12 (1968-12), p. 774.
 - (9) Rabinowicz, E., *Friction and Wear of Materials*, (1965), p. 60, John Wiley & Sons.
 - (10) Rabinowicz, E., *Friction and Wear of Materials*, (1965), p. 100, John Wiley & Sons.
 - (11) Lyons, W. J. and Scheier, S. C., *Jour. Appl. Phys.*, Vol. 36, No. 6 (1965), p. 2020.
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