621-231

Optimization of 6-Bar Watt Linkages Adjustable for the Coupler Motion*

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In such cases as conveying bodies by use of motions of coupler links, a kinematic synthesis has to be made considering the coupler directions. In this paper, dimensional analysis and synthesis of a six-link Watt mechanism are carried out. The parameters of the mechanism are split into the ones which determine the shape of the coupler curve or the direction of the coupler and the ones which determine the position and magnitude of the curve. Moreover they are qualitatively classified into three kinds. By use of these parameters, an optimum synthesis of a Watt mechanism is carried out such as to satisfy every planar path with two directions of a body, without any trial and error.

1. Introduction

In dimensional synthesis of path generating mechanisms, the direction of coupler is an important condition to be taken into consideration. Planar six-link path-generator has been analyzed⁽¹⁾, and kinematic syntheses⁽²⁾⁻⁽⁴⁾ and some practical applications^{(5),(6)} have been suggested, but the direction of coupler has never been taken into consideration. In such kinematic synthesis, the generated curve obtained by solving a coupler equation which satisfies the precision points very often becomes discontinuous between the precision points.

In this paper, the coupler motion of a six-link Watt mechanism is analyzed, and the kinematic parameters are classified qualitatively. Then by using such parameters a synthesis of a Watt mechanism satisfying two positions of coupler and the locus between them is carried out.

2. Analysis of coupler motion

Six-link Watt mechanism is shown in Fig.1, in which a is the fixed link, b is the driving link, c, d, and e are the couplers, and f is the driven link. The vector $\vec{EE'}$ on the coupler e determines the direction of body, and the locus of the point E determines the path.

2.1 Parameters for coupler motion

The position of the vector $\vec{EE'}$ is determined by 16 parameters $b, g_{ef}, g_{fd}, d,$

$e, e', f_{ca}, f_{ce}, \mu_{cf}, \mu_{ef}, \mu_e, \mu_c,$ and $J_{ab}(X_{ab}, Y_{ab}), J_{fa}(X_{fa}, Y_{fa})$. The coupler curve described by $E(X, Y)$ is expressed as an algebraic equation of degree 16 in X and Y ,⁽⁷⁾ each coefficient of which is a function of 15 parameters except μ_e , and it is very hard to solve such higher order simultaneous equations by giving the precision points. Then the authors suggest a new method for optimum synthesis of the mechanism.

As shown in the Figure, let Φ be the angle of the link f_{ca} from the axis X , ϕ be the angle of the link e from f_{ce} , and such angles at the coupler positions 1 and 2 be Φ_1, ϕ_1 , and Φ_2, ϕ_2 , then the variation of the coupler direction between two positions is $(\Phi + \phi)_2 - (\Phi + \phi)_1 = (\Phi_2 + \phi_2) - (\Phi_1 + \phi_1)$, which is unconcerned with μ_e . Further, let Θ_1 and Θ_2 be the input angles, then the above variation is determined by $\Theta_2 - \Theta_1$, and the shape of the coupler curve by Θ_1 (or Θ_2).

Therefore the number of parameters which determine the motion of $\vec{EE'}$ becomes 17, in which Θ_1 and Θ_2 are included, and μ_e

Fig.1

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excluded.

2.2 Classification of parameters

Let us call the four-link mechanisms $abc_f f_{ca}$ and $c_{ef} def_{ce}$ whose frames are f_{ca} and f_{ce} , respectively, the mech.I and mech.II. Then, an arbitrary revolution or similar transformation of the locus of $\overline{EE'}$ with respect to J_{fa} is possible by changing the position and dimension of the mech.II, while the mech.I is fixed, as shown in Fig. 2(a) and (b). Considering such characteristics of the Watt mechanism, the above 17 parameters are classified as follows:

(i) The parameters which determine the shape of the coupler curve and the direction change of the coupler; the shape of the Watt chain (a/f_{ca} , b/f_{ca} , c_{ef}/f_{ca} , c_{ef}/f_{ce} , d/f_{ce} , e/f_{ce} , $\mu_{cf} + \nu_{cf}$), input angles (θ_1 , θ_2), and the shape of the coupler (e'/f_{ca} , μ_{ef}).

(ii) The parameters which determine the position and dimension of the coupler curve; fixed pivot J_{fa} (X_{fa} , Y_{fa}), the angle between the link a and axis X, the shape of the driven link (f_{ca} , f_{ce} , μ_{cf}), and μ_{ef} .

Now put the origin on J_{fa} , the axis X along a, $f_{ca}=f_{ce}=1.00$, and $\mu_{cf}=180^\circ$, then the parameters which determine the motion of the coupler are the shape of the six-link chain, input angles, and the shape of the coupler.

2.2.1 Classification of the angular relation

(a)

(b)

Fig. 2

Put $\angle J_{fa} J_{cf} J_{bc} = \Psi$, then the angular relation $\Phi \sim \phi$ is split into two input-output relations $\Phi \sim \Psi$ and $\Psi \sim \phi$ of the mech.I and II. Such two angular relations are classified under the following conditions.

(1) The link b turns 2π rad. around a, and a swings about f_{ca} .

(2) The allowable value of the maximum pressure angles of the mech.I and II are $\alpha_p (=60^\circ)$.

(3) The allowable value of the maximum ratio of link lengths is 5.

Since the links a and c_{bf} swing about f without stopping except at the edge points from the condition (1), the relation $\Phi \sim \Psi$ is illustrated in Fig. 3. Now, put $\Delta\Phi = \Phi_{max} - \Phi_{min}$, $\Delta\Psi = \Psi_{max} - \Psi_{min}$, and $\Delta\Phi' = \Phi(\Psi_{min}) - \Phi(\Psi_{max})$, then $0 < \Delta\Psi \leq 120^\circ$ is obtained considering $|\Psi - 90^\circ| \leq \alpha_p$ in the

Fig. 3

(a)

(b)

Fig. 4

condition (2). From the condition (1) the pressure angle α of the mech. I should satisfy $\alpha \leq \Delta\Phi/2$, then $0 < \Delta\Phi < 120^\circ$ is obtained. From the above results, we may classify $\Delta\Phi$ and $\Delta\Psi$ as follows:

$$\begin{aligned} \Delta\Phi &= 30^\circ \times n_1 \quad (n_1 = 1, 2, 3) \\ \Delta\Psi &= 30^\circ \times n_2 \quad (n_2 = 1, 2, 3, 4) \end{aligned} \quad \text{----- (1)}$$

Now we obtain $(\Delta\Phi/\Delta\Psi)_{\min}$ when $\Delta\Phi < 0$ by use of the above values of $\Delta\Phi$ and $\Delta\Psi$. Let the links f_{ca} of the mechanisms $J_{ab} J_{bc} J_{cf} J_{fa}$

Fig. 5

(a)

(b)
Fig. 6

and $J_{ab} J_{bc} J_{cf} J_{fa}$ at the positions $\Psi = \Psi_{\max}$ and $\Psi_{\min}$, respectively, coincide with each other, as shown in Fig. 4(a). In the figure, the middle point M of $J_{bc} J_{bc}'$ should exist in the inside of the circle whose diameter is $J_{cf} J_{fa}$, and in order minimize $\Delta\Phi$, $\angle J_{fa} J_{cf} J_{bc}' = 90^\circ - \alpha = 30^\circ$ is the necessary condition. Considering the above conditions, $(\Delta\Phi/\Delta\Psi)_{\min}$ is illustrated in Fig. (b), and $\Delta\Phi/\Delta\Psi$ may be classified as follows:

$$\begin{aligned} \Delta\Phi/\Delta\Psi &= 0.1 \times (10 - n_3) \\ n_3 &= 1, 2, \dots, 9 \quad (\Delta\Psi = 30^\circ, 120^\circ) \\ &1, 2, \dots, 11 \quad (\Delta\Psi = 60^\circ) \\ &1, 2, \dots, 10 \quad (\Delta\Psi = 90^\circ) \end{aligned} \quad \text{---- (2)}$$

Since the mech. II is a double rocker mechanism, the angular relation $\Psi \sim \Phi$ is shown in Fig. 5. This angular displacement curve has one extremal value, which is evident after a synthesis of the mech. II giving the angular ranges $\Delta\Psi$ and $\Delta\Phi$.

Then we may classify the displacement curve $\Psi \sim \Phi$ into type (1) to type (8) in Fig. 5, in which $r (< 1)$ means the ratio of the ranges of Φ between the extremal value and the edge values.

In order to classify the range $\Delta\Phi$ qualitatively, let us obtain $|\Delta\Phi|_{\max}$ giving α and $\Delta\Psi$. When $\Delta\Phi > 0$, the pressure angles at the edge points should coincide with the allowable value. Put a point J_{de} , in Fig. 6(a), on the arc which has a chord $J_{cd} J_{cf}$ whose angle of circumference is $90^\circ + \alpha$, and a point J_{cd}' satisfying $J_{de} J_{cd}' = J_{de} J_{cd}$ and $\angle J_{ef} J_{de} J_{cd}' = 90^\circ - \alpha$. Moreover, put J_{cf}' which satisfies $J_{cf} J_{cd}' - J_{cf} J_{ef} = J_{ef} J_{cd}$ and exists on the arc whose angle of circumference is $\Delta\Psi$, and J_{cd}' on $J_{cf} J_{ef}$ satisfying $J_{cf} J_{cd}' = J_{cf} J_{cd}$ and $J_{ef} J_{cd}' = J_{ef} J_{cd}$. Then the maximum swing angle of $e(J_{de} J_{ef})$ about f_{ce} ($J_{cf} J_{ef}$) is $\Delta\Phi_{\max} = 180^\circ - \angle J_{cf} J_{ef} J_{cd}$. When $\Delta\Phi < 0$, in Fig. (b), the pressure angle should coincide with the allowable value at an edge point, and c_{df} and d should be collinear at the other point. Similarly to the above case, the maximum swing angle is obtained by $|\Delta\Phi|_{\max} = \angle J_{de} J_{ef} J_{cd}'$. Given $1/5 \leq e/d \leq 5$, $\alpha = 60^\circ$, and $\Delta\Psi = 30^\circ \times n_2$ ($n_2 = 1, 2, 3, 4$), the required angle $|\Delta\Phi|_{\max}$ is obtained as shown in Fig. 7, and may be classified as follows:

Fig. 7

a mech. II. In this process, putting J_{cd3} as a mirror image of J_{cd1} with respect to $J_{cf}J_{de1}$, the dimensions c_{df} and d are determined such that the pressure angle of the mechanism at the point of J_{cd3} or of intersection of $J_{cd1}J_{cd3}$ with $f_{ce}(J_{cf}J_{ef})$ may be less than or equal to α_p . The type (4) curve is generated by the mechanism obtained by inverting the above mechanism about $J_{cf}J_{de1}$. The type (5) curve is generated as follows. In Fig. (b), J_{dei} ($i = 1, 2, 3$) exists on a circle J_{ef} whose radius is e , and $\angle J_{de1}J_{ef}J_{de2} = r\Delta\phi$, $\angle J_{de2}J_{ef}J_{de3} = -\Delta\phi$. Moreover, J_{cdi} ($i = 1, 2$) exists on circles J_{de1} whose radii are d , and J_{cf} exists on $J_{de2}J_{cd2}$ and satisfies $J_{cf}J_{cd1} = J_{cf}J_{cd2}$ ($=c_{df}$) and $\angle J_{cd1}J_{cf}J_{cd2} = \Delta\psi/4$. When $d = \text{const}$, J_{cf} describes a solid curve. Obtain the point of intersection of this curve with the circle J_{de3} whose radius is d , and we have a mech. II shown by bold lines. In this process, the dimension d is determined such that the pressure angles at the edge points of swinging of c_{df} and at the position when c_{df} and $f_{ce}(J_{cf}J_{ef})$ are collinear may be less than or equal to α_p . The type (6) curve is generated by the

same process with the type (5) curve by substituting $3\Delta\psi/4$, $\Delta\psi/4$, $\Delta\phi$, and $r\Delta\phi$ for $\Delta\psi/4$, $3\Delta\psi/4$, $r\Delta\phi$ and $\Delta\phi$, respectively. Moreover, the types (7) and (8) curves are generated by the mechanisms obtained by inverting the ones which generate the types (6) and (5) curves, respectively, about $J_{de1}J_{de2}$.

3.3 Number of mechanisms synthesized

Now given 117 kinds of the angular relations $\Phi \sim \Psi$ and 220 kinds of $\Psi \sim \phi$ classified in paragraph 2.2.1, the mech. I and II are synthesized by the processes in sections 3.1 and 3.2; as the result, the shapes of the displacement curves which can be generated, the number of the mech. I and II synthesized, and that of the Watt chains constructed as combinations of the mech. I and II are summarized in Table 1. In the table, $[n_2]$ ($n_2 = 1, 2, 3, 4$) means the types of mechanisms which satisfy $\Delta\psi = 30^\circ \times n_2$, and (m) ($m = 1, 2, \dots, 8$) means the types of the mech. II which generate the type (m) curves. In the column of mech. I, the ratios $\Delta\phi/\Delta\Phi$ are determined at intervals of $0.3 \sim 0.5$, and in the column

Table 1 Shape of displacement curves and number of mechanisms

Types of mech. I and II [n ₂]	Mech. I				Mech. II								No. of 6-link chains
	Shape of disp. curve			No. of mecha- nisms	Type (m) and shape of disp. curve							No. of mecha- nisms	
	ΔΨ*	ΔΦ*	ΔΦ'/ΔΦ		(1) (2) (3),(4) (5),(8)** (6)*,(7)								
					Δφ*	-Δφ*	±Δφ*	±Δφ*	r	±Δφ*	r		
[1]	30	30	0.2	3	30 ⋮ 180	30	30	30	0.2	30	0.2	27	81
			0.5			60	—	60	0.1	60	0.2		
			0.9			90	—	90	0.1	90	0.1		
						120	—	—	—	120	0.1		
						150	—	—	—	—	—		
[2]**	60**	30**	-0.1**	5	30 ⋮ 210	30	30	30	0.2	30	0.2	32	160
			0.4			60	—	60	0.2	60	0.3		
			0.9			90	—	90	0.2	90	0.1		
	60		0.5			120	—	120**	0.2**	120	0.1		
			0.9			150	—	—	—	150	0.1		
[3]	90	30	0.0	5	30 ⋮ 210	30	30	30	0.2	30	0.3	33	165
			0.3			60	60	60	0.2	60	0.3		
	60		0.4			90	—	90	0.2	90	0.1		
			0.8			120	—	120	0.2	120	0.1		
			0.9			150	—	—	—	150	0.1		
[4]*	120*	30	0.2	3	30 ⋮ 240	30	30*	30	0.3	30	0.3	34	102
		60*	0.4*			60	60	60	0.2	60	0.3		
						90	—	90	0.2	90	0.3		
		90	0.7			120	—	120	0.2	120*	0.1*		
						150	—	—	—	150	0.1*		
	16				28	18	12	30	38	126	508		

- (Notes) 1. When the types of disp. curve of mech. II are (3), (5), (6), $\Delta\phi > 0$, and when (4), (7), (8), $\Delta\phi < 0$.
 2. "*" means the type and shape of the mechanism and curve obtained in Ex.1, and "**" means the ones obtained in Ex.2.

of mech. II, the ratios mean their own maximum values among 0.1, 0.2, and 0.3. As is evident from the table, the numbers of the mech. I and II are 16 and 126, and the number of the Watt chains obtained is 508.

4. Synthesis of six-link Watt mechanism

As shown in Fig. 11, choose a six-link chain from among the ones satisfying the ideal function of $\Phi \sim \phi$, and determine the input angles θ_1 , θ_2 , and the coupler angle ϕ_1 in order to satisfy the given variation of the coupler direction $(\Phi + \phi)_2$, then a six-link mechanism is obtained such as to satisfy the variation of direction between the positions 1 and 2. Given e' and μ_{ef} , the coupler curve is determined. Modify this curve as shown in Fig. 2, and let the coupler points corresponding to $\theta = \theta_1$ and $\theta_1 + \theta_2$ coincide with the given edge points in Fig. 12. The error between the actual

Fig. 11

Fig. 12

curve drawn by the bold solid line and the ideal curve drawn by the thin solid line is defined as a distance from a coupler point to the ideal curve. Now, let δ be the maximum error in the range 1~2, and σ_1, σ_2 be the meet angles of both curves at the edge points. From among the Watt mechanisms in Table 1, choose ones which have the errors $\delta, \sigma_1, \sigma_2$ less than their own allowable values δ_p and σ_p , and moreover, determine one whose error δ is minimum. In such process, an optimum synthesis of a six-link mechanism may be realized

5. Example

5.1 Example 1

The ideal path is shown in Fig. 13 by the dotted lines and arrows, i.e., $(\Phi + \phi)_2 = 90^\circ$. The result for $\delta_p = 0.5$, $\sigma_p = \pm 45^\circ$ is shown in Table 2 and Fig. 13; the type of the mech. I is [4], and that of the mech. II is [4]-(6), whose displacement curve has a shape bearing the symbol "*" in Table 1. The obtained mechanism satisfies the direction of body precisely at the edge points, and the maximum error of the coupler curve is $\delta = 0.050$ for the unit length 1.00 of the path line segment, and the maximum pressure angle is 57.5° .

5.2 Example 2

The ideal path is shown in Fig. 14, i.e., $(\Phi + \phi)_2 = 45^\circ$, which cannot be obtained by a four-link mechanism. The results for the same allowable value as section 5.1 are shown in Table 2 and Fig. 14; the type of the mech. I is [2], and that of the mech. II is [2]-(8), whose displacement curve has a shape bearing the symbol "***". The maximum error is $\delta = 0.072$, and the maximum pressure angle is 58.0° .

In the above examples, the optimum syntheses are carried out automatically by use of a digital computer (HITAC 5020), and the computing times are about 20 minutes.

Fig. 13

6. Conclusions

The results obtained in this paper are summarized as follows:

(1) 17 parameters which determine the coupler motion of a Watt mechanism are classified into (i) three parameters which determine the shape of the coupler curve and the variation of the coupler direction, and (ii) four parameters which determine the position and dimension of the curve.

Moreover, considering the characteristics of the mechanism, these parameters are classified into three, that is, the shape of the Watt chain, the input angles, and the shape of the coupler.

(2) The above three kinds of parameters are classified qualitatively by giving the allowable value of the maximum pressure angle and of the maximum ratio of link lengths, and the rotatability conditions of the driving link.

(3) In the classification of the input angles, they are determined such as to satisfy the variation of the coupler direction, as the result, a precise synthesis about the direction of body has become possible.

(4) By use of the above parameters, an optimum synthesis of a six-link mechanism is carried out such as to satisfy every planar path with the direction of body at two positions.

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Table 2 Parameters of mechanism

parameters	Example 1	Example 2
a	1.074 98	0.882 18
b	0.499 74	0.227 23
c_{df}	0.620 64	0.456 65
μ_{cf}	67.800°	118.983°
c_{df}	2.519 07	0.775 18
d	1.884 08	0.260 07
e	0.774 67	0.274 57
μ_{ef}	-128.600°	171.813°
e'	3.000 00	1.000 00
$f_{ca}(=f_{ce})$	1.000 00	1.000 00
Input angles θ_1	-140.200°	-43.184°
angles θ_{12}	180.001°	141.475°

Fig. 14