621-231:621.833:621-755

Fundamental Researches on a Perfectly Balanced
Rotation-Reciprocation Mechanism*

(Report 2, Vibration on an Eccentric Geared
Device of a Crankshaft Planetary
Motion System and a Dynamic
Balancing Machine)

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This report relates to an eccentric geared device of crankshaft planetary motion system designed to satisfy two theories in the 1st report and a dynamic balancing machine with a vibrating table designed for this study.

The construction of its device and constructional comparison between this device and known slider crank device, theoretical analysis on its motion and perfect balance, are described. From these results, it is made clear that the inertia force of reciprocating mass is perfectly balanced, and a moment due to its inertia force does not occur in this device. Further, vibration analysis on the vibrating table with a testing device in the dynamic balancing machine and its characteristics are described.

1. Introduction

This report relates to a perfectly-balanced eccentric geared rotation-reciprocation device of crankshaft planetary motion system designed to satisfy two theories in the 1st report^{a)}, and a dynamic balancing machine for this study.

The construction of this device and constructional comparison between this device and known slider crank device, theoretical analyses on its motion and perfect balance, are described. The principle and the characteristics of the vibrating table in a dynamic balancing machine manufactured to measure the amplitude of vibration caused by unbalanced forces and torques exerted on the support of testing device are presented. Then, experiments on vibration of this device by the foresaid vibrating table, are carried out.

2. Eccentric geared device of crankshaft planetary motion system

* Received 19th July, 1971.

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2.1 Construction, design and comparison

Fig.1 shows a longitudinal sectional

1 cylinder	2 piston
3 connecting rod	4 needle bearing
5 crankpin	6 crankshaft
7 needle bearing	
8 revolving eccentric disc	
9 ball bearing	10 rotating gear
11 revolving gear	12 eccentric gear
13 reversing gear	14 eccentric gear shaft
15 reduction gear	16 reduction shaft
17 cylinder head	18 compression chamber

Fig.1 Sectional view of this device

view of this device and Fig.2 its appearance. And Table 1 shows its specifications. In Fig.1, rotating gear meshes with eccentric gear, reversing gear with reduction gear and its reduction gear with revolving gear. The number of teeth in rotating gear is equal to that in eccentric gear, and the number of teeth in revolving gear equal to that in reversing gear. And, as shown in Fig.1, distance between axis O_1 of crankpin and axis O_2 of crankshaft is equal to that between axis O_2 and axis O_1 of revolving eccentric discs. They are 1 respectively.

Crankshaft is provided with rotation balance weights W_4 and W_5 diametrically opposite to the big end of rod. Revolving eccentric discs in the left and right sides are respectively provided with revolution balance weights W_6 and W_7 diametrically opposite to crankpin. These balance weights are designed such as to satisfy Eqs.(4) and (5), which will be mentioned later.

Next, constructional comparison between this device and the conventional slider crank device is made. It can be considered that this device is superior to the known slider crank device in the following respects.

(1) As rod performs linear reciprocating motion, rod receives only compressive force and tensile force axially and no bending moments act on rod.

(2) No side forces of slider are applied to cylinder wall.

(3) The maximum slider speed is smaller than that of the known slider crank device having the same slider stroke as this device.

(4) The ratio of rod length to crank radius is not restricted on account of no side forces.

The above-mentioned merits are common to all of the perfectly balanced rotation-

reciprocation devices.

On the contrary, this device has complex construction and large size in comparison with the known slider crank device. And also, as shown in Fig.1, revolving eccentric discs in the left and right sides can't be united in one body. So, rotating motion of crankshaft and revolving motion of eccentric discs are apt to go wrong.

2.2 Analysis on linear reciprocating motion

It is assumed throughout that rotation and revolution members rotate at constant angular velocity, and that frictional forces among members and the effects of clearance between shaft and bearing, gear tooth errors, backlashes of gears and various errors can be neglected.

The gear train in this device is shown in Fig.3. Nomenclature in Fig.3 is as follows. O_1, O_2, O_4, O_5 : axis of revolving eccentric disc/crankshaft/eccentric gear shaft/reduction gear shaft, O_6 : center of eccentric gear, r_1, r_2, r_3 : pitch radius of revolving gear and reversing gear/rotating gear and eccentric gear/reduction gear, l : eccentricity, θ : angle of revolution, A, B, C : pitch point between rotating gear and eccentric gear/revolving gear and reduction gear/reversing gear and reduction gear, $\omega_{rot}, \omega_{rev}, \omega_{red}$: absolute angular velocity vector of rotating gear by rotation/revolving gear/reduction gear, $\beta = \angle AO_4O_1$, V_{Arot}, V_{Are}, V_A : velocity vector of point A on rotating gear by ω_{rot} /rotating gear by ω_{rev} /eccentric gear, $R_{AA} (|R_{AA}| = R_{AA}) R_{2A}, R_{12}$: position vectors, $O-xy$: cartesian coordinate.

As shown in Figs. 1 and 3, the rotation of rotating gear on O_2 is transmitted to eccentric gear having eccentricity 1. The rotation of eccentric gear is transmitted, through reversing gear, to reduction gear. Reduction gear, in turn, transmits the rotation to revolving eccentric discs on O_1 through revolving gear.

Thus, rotating gear (crankshaft) rotates on O_2 at ω_{red} and revolves on O_1 at ω_{rev} in the reverse direction.

Fig.2 Appearance of this device

Table 1 Specification of an eccentric geared device

Cylinder bore × Slider stroke (mm)	40 × 40
Length of connecting rod (mm)	151.2
Weight of connecting rod (g)	580
Length of crank arm (mm)	10
Eccentricity of eccentric disc (mm)	10
Reciprocating weight (g)	745
Rotating weight (g)	3175
Revolving weight (g)	8755
Total weight (g)	20000
Type of crankshaft	one body

Fig.3 Gear train

Then the following equations are obtained by referring to Fig.3.

$$\left. \begin{aligned} V_{Arev} &= \omega_{rev} \times R_{12} = \omega_{rev} \times (i l \cos \theta - j l \sin \theta) \\ V_{Arot} &= \omega_{rot} \times R_{2A} = \omega_{rot} \times (-i l_2) \\ V_A &= \omega_{rev} \times R_{4A} = \omega_{rev} \times (i R_{4A} \cos \beta - j R_{4A} \sin \beta) \\ R_{4A} &= \sqrt{l_1^2 + 2 l_2 l \cos \theta + l^2} \\ \sin \beta &= \frac{l \sin \theta}{\sqrt{l_1^2 + 2 l_2 l \cos \theta + l^2}}, \cos \beta = \frac{l \cos \theta + l_2}{\sqrt{l_1^2 + 2 l_2 l \cos \theta + l^2}} \end{aligned} \right\} \text{---(1)}$$

Where i, j = unit vectors in x and y direction.

The condition for pure rolling on rollong circles of gears is

$$V_A = V_{Arot} + V_{Arev} \text{---(2)}$$

From Eqs. (1) and (2)

$$\omega_{rev} = -\omega_{rot} \text{---(3)}$$

As clear from Eq. (3), the rotating gear rotates on O_2 and revolves on O_1 at the same angular velocity in the reverse direction to each other.

Consequently, according to the theories in the 1st report⁽¹⁾, axis O_3 in Fig.1 performs a linear reciprocating motion having stroke $4l$ on the longitudinal axis of cylinder in Fig.1.

(a) Construction of slider crank portion

(b) For rotation

(c) For revolution

Fig.4 Condition of dynamic balance

2.3 Analysis on perfect balance

Fig.4 (a), (b) and (c) show the construction of slider crank portion, and dynamic balances for rotation and revolution in the top dead point of this device respectively. Nomenclature in Fig.4 is as follows. W_r : reciprocating weight, W_4, W_5 : balance weights for rotation, W_6, W_7 : balance weights for revolution, W_2 : rotating weight except W_r, W_4 and W_5 , W_{rot} : total rotating weight ($=W_r + W_4 + W_5 + W_2$), $l, R_4, R_5, a_4, a_5, D_1, R_6, R_7, b_6, b_7, D_2$: distances. Other nomenclature is the same as those in Fig.3.

In the same way as paragraph 5.2 in the 1st report⁽¹⁾, dynamic balanced conditions of this device are given as follows.

The balanced conditions for rotation are

$$\left. \begin{aligned} W_r l &= W_4 R_4 + W_5 R_5, \quad R_4 W_4 a_4 = R_5 W_5 a_5 \\ (D_1 W_5 R_5 &= a_4 W_r l, \quad D_1 W_4 R_4 = a_5 W_r l) \end{aligned} \right\} \text{---(4)}$$

The balanced conditions for revolution are

$$\left. \begin{aligned} W_{rot} l &= W_6 R_6 + W_7 R_7, \quad R_6 W_6 b_6 = R_7 W_7 b_7 \\ (D_2 W_7 R_7 &= b_6 W_{rot} l, \quad D_2 W_6 R_6 = b_7 W_{rot} l) \end{aligned} \right\} \text{---(5)}$$

As this device is designed such as to satisfy the balanced conditions, that is, Eqs. (4) and (5), inertia forces and torques are perfectly balanced in this device theoretically.

3. Testing apparatus and measuring methods

3.1 Dynamic balancing method

Dynamic balancing test for a single rotor is performed by well-known methods. Table 2 shows the specifications of the dynamic balancer.

In Table 2, unbalance is the distance (μ) between the center of the gravity and axis of the rotation of testing rotor. In this study, rotational speed of testing rotor is 1085 rpm and the unbalances on the correction surfaces of testing rotor are made smaller than 5μ .

Then, some rotors in this device are balanced by using the vibrating table (which is mentioned later) as a balancing machine.

3.2 Measurement on the amplitude of vibration

Mean amplitude of vibration, the amplitude of vibration and its waveform, the amplitude of vibration having the same or twice period as rotational speed of crankshaft, rotational speed, phase, etc. can be measured by vibrating table in the

Table 2 Specification of dynamic balancer

Weight of testing device	0.1~10 kg
Rotational speed	1085, 2160 rpm (60 Hz)
Accuracy of unbalance	$\pm 5\%$ when unbalance, 10μ
Accuracy of phase	$\pm 5\%$ when unbalance, 10μ

Fig.5 Block diagram of vibration measurement

dynamic balancing machine. Fig.5 shows a block diagram of vibration measurement and Table 3 the specifications of the testing apparatus. In Fig.5, rotational speed detector is composed of a photo transistor and a miniature bulb, and is used for measurement of rotational speed, phase and the amplitude of vibration having the same or twice period as rotational speed of crankshaft. Besides, trigger marker is mounted so as to make the waveform of vibration amplitude start at the top dead point in the testing device. As the dynamic balancing machine is a very essential apparatus, the construction, principle and characteristics in this machine are presented in next chapter.

4. Dynamic balancing machine

4.1 General descriptions

This dynamic balancing machine is specially designed and manufactured for this study in order to measure the amplitude of vibration caused by unbalanced forces and torques exerted on the support of testing device. Fig.6 shows appearance of vibration measurement by means of this dynamic balancing machine. A testing device is mounted rigidly on the vibrating table, and the crankshaft is driven by a motor via universal joint. The amplitude of the vibrating table is detected by a displacement pick up detector and its output is observed by means of a cathode ray oscilloscope and a meter. Table 4 shows the specification of this machine.

4.2 Construction of vibrating table and principle on its table

Fig.7 shows the construction of vibrating table. As shown in Fig.7, it is constituted by vibrating body, support, rod springs to receive shaking forces on x-y plane and crossed plate springs to receive

Apparatus	Specification
Displacement pick up	Frequency range 20~10000 rpm
Dynamic balancing machine	Rotational speed range 1500~10000 Hz
Trigger marker transducer	Allowable rotational speed Max. 10000rpm
Trigger marker Control unit	Rise time of pulse 1μs
Oscilloscope	Frequency range 0~500kHz

shaking forces in z direction.

The principle of this vibrating table with a testing device is illustrated in Fig.8. Nomenclature in Fig.8 is as follows. O: center of elasticity in vibrating body, G: center of gravity in vibrating body with testing device, a, b: distances as shown in Fig.8.

The vibrating body is connected to a support by springs and the support is fixed on the ground. In this investigation, the weight, the dimension and the mounting of testing device are restricted such as to satisfy the conditions in which axes x, y and z with the origin at O are nearly coincident with the principal axes of inertia of the vibrating body with a

Table 4 Specification of dynamic balancing machine

Testing device	weight	10~60 (kg)
	Max.length	Within 600 (mm)
	Max.width	Within 500 (mm)
Rotational speed range	Velt drive	1500~10000 (rpm)
Motor	2.2 KW Induction motor (variable speed)	
Unbalance measurement	Detected direction	Horizontal and vertical
	Balancing limit	Unbalance 1~2 (μ)
Vibration measurement	Object	Amplitude
	Range	0~250 (μ)
	Accuracy	±25%, Unbalance, 1~2 (μ) ±5%, Unbalance, 10 (μ)
Others	Terminal for waveform and frequency analysis

Fig.6 Appearance of vibration measurement

testing device. As clear from the above-mentioned explanation, this vibrating body with a testing device can perform linear displacements in directions x, y, z and angular displacements on axes x, y, z and these six displacements are so independent of each other that a coupled vibration is not produced. We decide to put the longitudinal axis of cylinder in z - x plane and crankshaft axis O_1 in y - z plane, in order that unbalanced forces and torques may mainly occur in z - x plane-parallel and distance between this z - x plane-parallel and z - y plane may be fairly small in comparison with distance a .

Consequently, as shown in Fig.9, the amplitude of vibration of the vibrating body with a testing device is completely detected, in the direction shown by arrows, by Pick ups V.F., V.B. and H. In this study, testing device and pick ups are to be mounted on the vibrating body in the above-mentioned manner.

As clear from the above-mentioned characteristics of this machine, a dynamic balancing machine with a vibrating table

can be used for not only vibration measurement but also dynamic balancing test.

4.3 Characteristics of vibrating table

As shaking forces and torques having higher harmonics occur in this device, frequency characteristics of the vibrating table over a wide range are obtained. Vibrating body with a testing device whose weight is about 20(kg), is given an impact by hammer through its center of gravity in displacement direction of each pick up, and then it is made to vibrate freely. Their frequency analyses on the displacements detected by the pick ups are shown in Fig.10. As shown in Fig.10, resonances are observed within the frequencies ranging from 1000 to 1500 rpm and from 6000 to 9000 rpm.

Fig.11 shows an apparatus of unbalanced rotor. This rotor is provided with many tapped holes and can be used as an unbalanced rotor having various unbalances by putting some bolts in their holes. Characteristic tests of vibrating table are carried out by means of this apparatus.

Fig.12 shows the waveform of vibration amplitude of vibrating body in the case of rotor speed 1800 rpm and unbalance 1627(gmm). As clear from Fig.12, all of the waveforms are sine curves having the same period as the rotor speed.

Fig.13 shows the relation between rotor speed and vibration amplitude of vibrating body in the case of constant unbalance 210(gmm). Resonances are observed at 7000 rpm in Pick up H. and at 6000 rpm in Pick up V.F., when extraordinary noise occurs and metrescale indication becomes unstable. Fig.14 shows the relation between rotor speed and vibration amplitude of vibrating body within rotor speed ranging from 2000 to 5000 rpm chosen for this investigation. In Fig.14, five kinds of unbalances (at the intervals of about 500gmm from 500 to 2500 gmm) are taken as

parameters. It is the effect of resonances that amplitudes detected in Pick up H. decrease as rotor speed increases, and that those in Pick up V.F. increase at both ends in rotor speed range.

From these results, it becomes clear that the relation between unbalance and vibration amplitude is satisfactory within rotor speeds ranging from 2000 to 5000 rpm. Thus, the experimental results of vibration measurement is to be appreciated from Figs. 11~14. But these frequency characteristics are not satisfactory concerning vibrations having higher harmonics, so those vibrations should be appreciated carefully.

5. Experimental results and discussions

5.1 Dynamic balancing

Dynamic balancing for eccentric gear shaft and reduction gear shaft with all parts to be secured on each shaft as shown in Fig.1, and for rotation and revolution are carried out according to paragraph 3.1.

Figs.15 and 16 show the balancing methods and the results of balances for rotation and revolution respectively. In these figures, L and R show left and right correction surfaces respectively and all parts except equivalent mass are the same as in Fig.1. As shown in Fig.15, the hollow cylinder balanced on its axis,

Fig.10 Frequency characteristics of vibrating table

Fig.11 Apparatus of unbalanced rotor

having the same weight as reciprocating weight, is concentrically secured on the crankpin and they are dynamically balanced on axis O_2 . And then, for revolution, as shown in Fig.16, the balanced rotating parts in Fig.15 are mounted in revolving eccentric discs and this device without eccentric gear shaft is secured on the vibrating body. The quality of revolution balance is not good, because revolving eccentric discs in the left and right sides are not united in one body.

5.2 Vibration measurement

Fig.17 shows the results of vibration measurement. Fig.18 shows the waveforms of

Fig.12 Waveform of vibration amplitude

Fig.13 Vibration amplitude of vibrating table in wide range

vibration amplitude at crank speed 1800 rpm and their Fourier analyses. In these figures, "load" means compressive force in compression chamber. Fourier analyses are obtained till 15th harmonics and ten terms are shown here in order to its magnitude.

In Fig.17, total amplitudes of vibration are larger than those having the same period as crank speed. It is sure that the former is affected by the friction among members, the clearance between shaft and bearing, etc. at no load, and in addition to them, crank torque by compressive force at load. And also, vibration amplitudes at load are larger than those at no load, for the former are affected by crank torque by compressive force. As the abovementioned matters are common to the results of vibration tests in this study, we are not to explain on these problems hereinafter.

The large amplitudes of vibration detected by Pick up V.F. at low crank speed are caused by the characteristics of the vibrating table in Fig.14. Judging from Figs.13,17 and 18, the large total-amplitude of vibration at crank speed 2000 rpm, results from the coincidence of natural frequency of the vibrating table with 3rd or 4th harmonics in crank torque and friction loss torque. The amplitudes detected by Pick up V.B. are almost the same as those by Pick up V.F. As shown in Fig.17 (b), there is little difference between vibration amplitudes detected by

Fig.14 Vibration amplitude of vibrating table in testing range

Pick up H. at no load and those at load. It is because the vibration caused by torques is not detected by Pick up H.

From the above-mentioned results, referring to Fig.14, the vibration amplitude in this device corresponds to that in rotor having unbalance, 500 gmm. However, unbalance in this device calculated by the equation, $2 \times [\text{reciprocating weight}] \times [\text{eccentricity (one fourth of slider stroke)}]$, is about 14900 gmm, when unbalanced inertia force due to reciprocating mass is not balanced.

The amplitudes shown in Fig.17, are affected by the clearance between shaft and bearing, friction force and torque, accuracy of machining and assembling.

Total weight (g)	3175
Unbalance in L (gmm)	4
Unbalance in R (gmm)	5
JIS-Balance grade, L	2
JIS-Balance grade, R	2
Rotational speed (rpm)	1085

Fig.15 Balancing method and the result of balance for rotation

Total weight (g)	8755
Unbalance in L (gmm)	60
Unbalance in R (gmm)	65
JIS-Balance grade, L	4
JIS-Balance grade, R	4
Rotational speed (rpm)	3000

Fig.16 Balancing method and the result of balance for revolution

Therefore, it is ascertained from the vibration tests that inertia force and torque due to reciprocating mass in this device are perfectly balanced theoretically.

6. Conclusion

By the theories and tests on this device, it is ascertained that an eccentric geared device of a crankshaft planetary motion system conforming to the two basic theories¹⁰ is perfectly balanced.

However, this device has complex construction and large size in comparison with known slider crank device. So, main purpose of study is to make the construction of this device simpler.

A dynamic balancing machine with a vibrating table specially designed for vibration measurement is analyzed. From the analysis, it is made clear that the vibrating table with a testing device which satisfies some conditions is a vibrating system without coupled vibration, and that vibration in this device can be detected by Pick ups H., V.F. and V.B.

From the characteristic tests on vibrating table, the relation between vibration amplitude and rotor speed in rotational speeds ranging 1500~5000 rpm and 1500~7000 rpm, and its natural frequency are obtained.

Reference

- (1) K.Ishida: Bulletin of the JSME, Vol.17, No.103 (1974), P.

Fig.17 Amplitude of vibration

Rotational speed of crankshaft 1800 rpm				
In the case of load			In the case of no load	
	Wave form	Analysis	Wave form	Analysis
Pick up H. Vibration amplitude μ		6.05 sin(θ + 350°) 0.60 sin(2θ + 212°) 0.35 sin(3θ + 86°) 1.00 sin(4θ + 37°) 0.15 sin(5θ + 260°) 0.10 sin(6θ + 83°) 0.30 sin(8θ + 126°) 0.20 sin(9θ + 92°) 0.20 sin(10θ + 291°) 0.15 sin(12θ + 234°)		5.45 sin(θ + 74°) 0.35 sin(2θ + 343°) 0.15 sin(3θ + 56°) 0.20 sin(4θ + 62°) 0.15 sin(5θ + 222°) 0.15 sin(8θ + 293°) 0.10 sin(10θ + 18°) 0.10 sin(11θ + 108°) 0.15 sin(12θ + 246°) 0.10 sin(15θ + 270°)
		5.75 sin(θ + 186°) 1.30 sin(2θ + 154°) 1.10 sin(3θ + 36°) 1.35 sin(4θ + 227°) 0.70 sin(5θ + 316°) 0.15 sin(6θ + 295°) 0.10 sin(7θ + 287°) 0.10 sin(8θ + 319°) 0.10 sin(9θ + 170°) 0.15 sin(10θ + 31°)		6.45 sin(θ + 171°) 0.20 sin(2θ + 157°) 0.75 sin(3θ + 227°) 0.85 sin(4θ + 124°) 0.10 sin(5θ + 165°) 0.15 sin(6θ + 236°) 0.10 sin(7θ + 194°) 0.25 sin(10θ + 3°) 0.20 sin(11θ + 336°) 0.20 sin(12θ + 168°)
		5.05 sin(θ + 345°) 0.65 sin(2θ + 294°) 2.35 sin(3θ + 247°) 1.05 sin(4θ + 158°) 0.20 sin(5θ + 86°) 0.60 sin(6θ + 157°) 0.20 sin(7θ + 159°) 0.40 sin(8θ + 114°) 0.25 sin(11θ + 341°) 0.20 sin(13θ + 266°)		5.10 sin(θ + 53°) 0.40 sin(2θ + 203°) 0.30 sin(3θ + 335°) 0.20 sin(5θ + 291°) 0.20 sin(9θ + 10°) 0.15 sin(10θ + 211°) 0.20 sin(11θ + 93°) 0.15 sin(12θ + 335°) 0.15 sin(13θ + 222°) 0.15 sin(14θ + 91°)
	Crank angle deg		Crank angle deg	

Fig.18 Waveform of total vibration amplitude and its Fourier analysis