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Dynamic Characteristic of the Noncircular
Planetary Gear Mechanisms with Nonuniform Motion*

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The authors have already synthesized the noncircular planetary gear mechanisms with nonuniform motion by using the noncircular gears for the planetary gear mechanisms.

In this paper, the dynamic characteristics of those mechanisms are made clear by analysis and experiment. First, the torque fluctuation occurring in the driving axle of the mechanism in operating the mechanism is analysed and its equation is induced. Next, the mechanism is modeled to a vibration system and the equation of motion is induced and the dynamic displacement curve is calculated. Moreover, considering the mechanism as an equivalent moment of inertia, the equation of motion is induced and the value of the torque fluctuation is calculated. And then, by comparing and discussing the theoretical results with experimental results, the basis of design of the mechanisms is made clear.

1. Introduction

The authors have already classified the types of the noncircular planetary gear mechanisms with nonuniform motion by using the noncircular gears for the planetary gear mechanisms, and shown examples of calculation on the swing, swing and rotate, and intermittent and rotate mechanisms. Then, the mechanisms were made after inducing and expression to design the noncircular gears. Moreover, the static behavior of the mechanisms has been measured and, as the result, it has been shown that the mechanisms have good character as function-generators.⁽¹⁾

In this paper, to make clear, about each mechanism with nonuniform motion, how the vibration or fluctuation of torque which is caused when the mechanisms are operated influences the accuracy of rotation, analysis and experiment on the characteristics of the mechanisms are made, and the results are discussed.

2. Nomenclature

b : length of planet carrier (center distance)
 F : force vector arising on the center of gravity of planetary gears
 F_i : force vector added to planetary gears from planetary gear axle
 F_d : force vector of planetary gear c added by sun gear d

F_a : force vector of planetary gear e added by sun gear a
 G : center of gravity of planetary gears
 I_p : moment of inertia of planetary gears about planet axle
 I_d : moment of inertia of planetary gear d and output axle about output axis
 m_p : mass of planetary gear
 n, n' : gear ratio of spur gears
 O : axis of sun gear axle
 O_p : axis of planetary gear axle
 P : intermeshing point of sun gear d and planetary gear c
 Q : intermeshing point of sun gear a and planetary gear e
 r : varying radius of pitch curve of noncircular gears
 S : peripheral length of pitch curve of noncircular gears
 W : vector of gravity arising at the center of gravity of planetary gears
 α : angle between planet carrier at reference place and vertical line
 θ : rotating angle of input axle
 θ_H : period of rotating angle of input axle
 θ : displacement angle of pitch curve of noncircular gears
 φ : rotating angle of output axle
 φ_H : advanced angle after one period of input

3. Analysis of dynamic character

A planetary gear mechanism with one pair of spur gears a, e (or d, c) and another with one pair of noncircular gears d, c (or a, e) are shown in Fig.1, and the former is named I-1 type and the latter I-2 type mechanism. Moreover, the mechanisms are classified as II and III types, in the former, gear a , in the lat-

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ter, a and d being the internal gears, respectively. Each value of mechanism is shown in Table 1 and experimental apparatus is shown in Fig.2.

3.1 Analysis of fluctuation of input torque

In the case of rotating the input axle of the mechanism with constant angular velocity, the equation of the fluctuation of the input torque is decided with the static balancing of the force and moment acting on the planetary gears, with the frictional force neglected.

3.1.1 I type mechanism

In Fig.3, the reference place is set as in the case when the center of gravity G of the planetary gear is on the extension of the line connecting the center O

Fig.1 I type mechanism

Fig.2 Experimental apparatus of I-1 type mechanism

of the sun gear and the center O_p of the planetary gear. At first, the force vector F arising on the center of gravity is obtained, considering the mass of the planetary gear with unbalance WR to be concentrated at the center of gravity. For the center of gravity

$$R_G = b + R = be^{i\theta} + re^{i\psi} = r_G e^{i\theta_G} \quad (1)$$

and, for the acceleration

$$\ddot{R}_G = (-\ddot{\theta}^2 + i\ddot{\theta})be^{i\theta} + (-\ddot{\psi}^2 + i\ddot{\psi})re^{i\psi} \quad (2)$$

on the other hand, we obtain

$$\dot{\psi} = \dot{\theta} + \dot{\theta}_p, \quad \ddot{\psi} = \ddot{\theta} + \ddot{\theta}_p \quad (3)$$

Substituting Eq.(2) in Eq.(3) and setting $\dot{\theta}$ equal to zero as the input axle rotates with constant angular velocity, we have

$$\ddot{R}_G = -\dot{\theta}^2 R_G - (2\dot{\theta}\dot{\theta}_p + \dot{\theta}_p^2)R + i\ddot{\theta}_p R \quad (4)$$

$$F = -m_p \ddot{R}_G = m_p \dot{\theta}^2 R_G + m_p (2\dot{\theta}\dot{\theta}_p + \dot{\theta}_p^2)R - i m_p \ddot{\theta}_p R \quad (5)$$

Then, the moment of inertia of the planet gear is considered in the following three types, assuming the gear as a rigid body.

i) When the gear is considered a particle about axle : I_{p0}

$$I_{p0} = m_p r^2 \quad (6)$$

Fig.3 Relation Between intermeshing and force (I type mechanism)

Table 1 Each value of mechanism

	Intermittent and rotate (I-1 type)	Swing and rotate (I-2 type)	Swing (I-1 type)
Period of input	$\frac{9}{4}\pi$	2π	2π
Advanced angle	$\frac{1}{4}\pi$	$\frac{1}{12}\pi$	0
Displacement curve	$0 \leq \theta \leq \frac{9}{8}\pi$ $\bar{x} = \frac{2}{9}(\theta - \frac{9}{16}\sin \frac{16}{9}\theta)$ $\frac{9}{8}\pi \leq \theta \leq \frac{9}{4}\pi$ $\bar{x} = \frac{1}{4}\pi$	$0 \leq \theta \leq \pi$ $\bar{x} = \frac{\theta}{6} - \frac{1}{12}\sin 2\theta$ $\pi \leq \theta \leq 2\pi$ $\bar{x} = \frac{\pi}{4} - \frac{\theta}{12} + \frac{1}{24}\sin 2\theta$	$\bar{x} = \frac{\pi}{6}(1 - \cos \theta)$

ii) When it is considered a rigid body about the center of gravity: I_{pg}

$$I_{pg} = m_p K_G^2, \quad (K_G: \text{radius of gyration}) \quad (7)$$

iii) When it is considered a rigid body about the planet axle: I_p

$$I_p = I_{pg} + I_{pg} = m_p r'^2 \quad (8)$$

In Fig. 3, $\mathbf{R}_e = \overrightarrow{O_p Q}$, $\gamma_e = |\mathbf{R}_e|$, $\mathbf{R}_c = \overrightarrow{O_p P}$, $\gamma_c = |\mathbf{R}_c|$, $\mathbf{R}_d = \overrightarrow{O_p P}$ and $\gamma_d = |\mathbf{R}_d|$, and using each force of action and the moment of inertial resistance $-k I_{pg} \ddot{\theta}_p$ about the center of gravity, we have

$$\left. \begin{aligned} \mathbf{F} + \mathbf{W} + \mathbf{F}_c + \mathbf{F}_d + \mathbf{F}_i &= 0 \\ \mathbf{R}_c \times \mathbf{F} + \mathbf{R}_c \times \mathbf{W} + \mathbf{R}_d \times \mathbf{F}_c + \mathbf{R}_d \times \mathbf{F}_d - k I_{pg} \ddot{\theta}_p &= 0 \end{aligned} \right\} \quad (9)$$

$$\mathbf{R}_d \times \mathbf{F}_d = -k I_{pg} \ddot{\theta}_p \quad (10)$$

With Eqs. (5), (8) and $\mathbf{R}_c = -(\gamma_c / \gamma_d) \mathbf{R}_d$, Eq. (9) leads to next equations.

$$\mathbf{R}_c \times \mathbf{F}_c = -k \left\{ -m_p \dot{\theta}_p^2 r r_G \sin \gamma - I_p \ddot{\theta}_p + m_p g r \sin(\varphi - \alpha) + \frac{r_s}{r_d} I_d \ddot{\theta}_p \right\} \quad (11)$$

$$\left. \begin{aligned} \mathbf{R}_c \times \mathbf{F} &= -k \left\{ m_p \dot{\theta}_p^2 r r_G \sin \beta + m_p (2\dot{\theta}_p \dot{\theta}_p + \dot{\theta}_p^2) \times r_s r \sin \theta_p - I_p \ddot{\theta}_p \frac{r_s r}{r'^2} \cos \theta_p \right\} \\ \mathbf{R}_c \times \mathbf{W} &= -k m_p g r_s \sin(\theta - \alpha), \quad \mathbf{R}_d \times \mathbf{F}_d = k \frac{r_s}{r_d} I_d \ddot{\theta}_p \end{aligned} \right\} \quad (12)$$

$$\mathbf{R}_c + \mathbf{F}_i = -[\mathbf{R}_c \times \mathbf{F} + \mathbf{R}_c \times \mathbf{W} + \mathbf{R}_d \times \mathbf{F}_c + \mathbf{R}_d \times \mathbf{F}_d] \quad (13)$$

Substituting Eqs. (11) and (12) in Eq. (13) and using the relation $\mathbf{b} = -(b / \gamma_e) \mathbf{P}_e$, the produced torque in the mechanism and input torque $\mathbf{T}_i$ are obtained. That is

$$\left. \begin{aligned} \mathbf{T}_i = \mathbf{b} \times \mathbf{F}_i &= -k \left[\frac{w r}{g} b \left\{ \frac{b}{r_s} \dot{\theta}_p^2 - (\dot{\theta}_p + \dot{\theta}_p)^2 \right\} \sin \theta_p + I_p \ddot{\theta}_p b \left(\frac{r}{r'^2} \cos \theta_p + \frac{1}{r_s} \right) \right. \\ &\quad \left. - w b \left\{ \frac{r}{r_s} \sin(\varphi - \alpha) + \sin(\theta - \alpha) \right\} + I_d \ddot{\theta}_p \frac{d\theta}{d\theta} \right] \quad (14) \end{aligned} \right\}$$

(1) I-1 type mechanism

With this mechanism, next relations are formed.

$$n = r_s / r_e, \quad \theta_p = n\theta \quad (15)$$

$$\varphi = \theta + \theta_p = (1+n)\theta, \quad \dot{\theta}_p = n\dot{\theta}, \quad \ddot{\theta}_p = n\ddot{\theta} \quad (16)$$

$$\frac{b}{r_s} = 1+n \quad (17)$$

Substituting Eqs. (15)~(17) in Eq. (14), we have

$$\left. \begin{aligned} \mathbf{T}_i &= -k \left[-\frac{w r}{g} b n (1+n) \dot{\theta}^2 \sin \theta_p - w b \left\{ \frac{r}{r_s} \sin(\theta + n\theta - \alpha) \right. \right. \\ &\quad \left. \left. + \sin(\theta - \alpha) \right\} + I_d \ddot{\theta} \frac{d\theta}{d\theta} \right] \quad (18) \end{aligned} \right\}$$

(2) I-2 type mechanism

With this mechanism, next relations are formed.

$$n' = r_e / r_d, \quad \theta_p = \frac{1}{n'} (\theta - \theta) \quad (19)$$

$$\varphi = \frac{1}{n'} \{ (1+n')\theta - \theta \}, \quad \dot{\theta}_p = \frac{1}{n'} \left(1 + \frac{d\theta}{d\theta} \right) \dot{\theta}, \quad \ddot{\theta}_p = -\frac{1}{n'} \ddot{\theta} \quad (20)$$

$$r_s = \frac{n'}{n' + 1} \frac{d\theta}{d\theta} b \quad (21)$$

Substituting Eqs. (19)~(21) in Eq. (14), we have

$$\left. \begin{aligned} \mathbf{T}_i &= -k \left[\frac{w r}{g} b \dot{\theta}^2 \frac{b}{r_s} \left(1 - \frac{b}{r_s} \right) \sin \theta_p - \frac{1}{n'} I_p b \ddot{\theta} \left\{ \frac{r}{r'^2} \cos \theta_p + \frac{1}{r_s} \right\} \right. \\ &\quad \left. - w b \left\{ \frac{r}{r_s} \sin \frac{1}{n'} (\theta - \theta - n' \alpha) + \sin(\theta - \alpha) \right\} + I_d \ddot{\theta} \frac{d\theta}{d\theta} \right] \quad (22) \end{aligned} \right\}$$

3.1.2 II type mechanism

The relations between the places and forces at each part of II type mechanism are shown in Fig. 4. With the same procedure as the I type mechanism, we have the equation for input torque as follows:

$$\left. \begin{aligned} \mathbf{T}_i &= k \left[\frac{w r}{g} b \left\{ \frac{b}{r_s} \dot{\theta}^2 + (\dot{\theta} - \dot{\theta}_p)^2 \right\} \sin \theta_p + I_p b \ddot{\theta}_p \left(\frac{1}{r_s} - \frac{r \cos \theta_p}{r'^2} \right) \right. \\ &\quad \left. - w b \left\{ \frac{r}{r_s} \sin(\varphi - \alpha) + \sin(\theta - \alpha) \right\} + I_d \ddot{\theta} \frac{d\theta}{d\theta} \right] \quad (23) \end{aligned} \right\}$$

(1) II -1 type mechanism (e, a: spur gears)

$$\left. \begin{aligned} \mathbf{T}_i &= k \left[\frac{w r}{g} b n (n-1) \dot{\theta}^2 \sin \theta_p - w b \left\{ \frac{r}{r_s} \sin(\varphi + \alpha) \right. \right. \\ &\quad \left. \left. + \sin(\theta - \alpha) \right\} + I_d \ddot{\theta} \frac{d\theta}{d\theta} \right] \quad (24) \end{aligned} \right\}$$

(2) II -2 type mechanism (c, d: spur gears)

$$\left. \begin{aligned} \mathbf{T}_i &= k \left[\frac{w r}{g} b \dot{\theta}^2 \frac{b r_s}{r_s^2} \sin \theta_p + \frac{1}{n'} I_p b \ddot{\theta} \left(\frac{1}{r_s} - \frac{r \cos \theta_p}{r'^2} \right) \right. \\ &\quad \left. - w b \left\{ \frac{r}{r_s} \sin(\varphi - \alpha) + \sin(\theta - \alpha) \right\} + I_d \ddot{\theta} \frac{d\theta}{d\theta} \right] \quad (25) \end{aligned} \right\}$$

Fig. 4 Relation between intermeshing and force (II type mechanism)

3.1.3 III type mechanism

The relations between the places and forces at each part of III type mechanism are shown in Fig. 5. The equation for input torque of this mechanism coincides with Eq. (23) in II type mechanism. Furthermore, the equation for input torque fluctuation of III-1 type mechanism in which e and a are spur gears coincides with Eq. (24) in II -1 type mechanism. The equation

for input torque fluctuation of III-2 type mechanism in which c and d are spur gears is given as follows :

$$T_i = k \left[\frac{wr}{g} b \dot{\theta}^2 \frac{r_a}{r_s} \sin \theta_p - \frac{1}{n} I_p b \ddot{\theta} \left(\frac{1}{r_s} - \frac{r \cos \theta_p}{r'^2} \right) - w b \left\{ \frac{r}{r_s} \sin(\varphi + \alpha) + \sin(\theta - \alpha) \right\} + I_a \ddot{\theta} \frac{d\theta}{d\theta} \right] \quad (26)$$

3.2 Equation of motion

Studies on the dynamic intermeshin of the spur gears, except the noncircular gears, have been reported^{(2)~(6)}. The model of the mechanism as vibration system is shown in Fig.6. In the figure, k is in case of gear, the resultant spring constant along the intermeshing line of action, and, in case of axle, k is the spring constant which is converted on the planet carrier. f_0 is the backlash of the gear along the intermeshing line of action. In this case, the kinematic energy K is represented as follows :

$$K = \frac{1}{2} \{ (I_s + I_c) \dot{\theta}_s^2 + I_d \dot{\theta}_d^2 + I_{ao} \dot{\theta}_{ao}^2 + I_b \dot{\theta}_b^2 + I_i \dot{\theta}_i^2 \} \quad (27)$$

Fig.5 Relation between intermeshin and force (III type mechanism)

Fig.6 Model of vibrating system of mechanism

In case that this vibration system vibrates a little of conservation system, and the viscous damping and the tooth profile error are neglected, the equation of motion is derived as follows :

$$\left. \begin{aligned} (I_s + I_c) \ddot{\theta}_s &= l_s k_a A - l_c k_{cd} B, & I_d \ddot{\theta}_d &= -l_d k_{cd} B - b k_{do} C, & I_{ao} \ddot{\theta}_{ao} &= b k_{do} C, \\ I_b \ddot{\theta}_b &= -l_b k_a A + l_d k_{cd} B - b k_{bi} D + T_b, & I_i \ddot{\theta}_i &= b k_{bi} D + T_i, \end{aligned} \right\} \quad (28)$$

where,

$$A = \int l_s \dot{\theta}_s dt - \int l_c (\dot{\theta}_s - \dot{\theta}_b) dt, \quad B = \int l_s (\dot{\theta}_s - \dot{\theta}_b) dt - \int l_d (\dot{\theta}_b - \dot{\theta}_i) dt \\ C = b(\theta_s - \theta_{ao}), \quad D = b(\theta_b - \theta_i)$$

Here, each value is denoted as follows ;

$$\left. \begin{aligned} l_1 &= \int l_s \dot{\theta}_s dt, & l_2 &= \int l_c (\dot{\theta}_s - \dot{\theta}_b) dt, & l_3 &= \int l_d (\dot{\theta}_b - \dot{\theta}_i) dt, \\ l_4 &= \int l_d (\dot{\theta}_b - \dot{\theta}_i) dt, & l_5 &= b \theta_{ao}, & l_6 &= b \theta_b, & l_7 &= b \theta_i, & l_4' &= b \theta_d \end{aligned} \right\} \quad (29)$$

Differentiating Eq.(29) and substituting them in Eq.(28), we have

$$\left. \begin{aligned} \ddot{l}_2 &= \frac{l_s}{l_c} \ddot{l}_1 - \frac{l_s}{I_b} (-l_b k_a A + l_d k_{cd} B - b k_{bi} D + T_b) + \frac{l_s}{I_s + I_c} (l_s k_a A - l_c k_{cd} B) \\ \ddot{l}_4 &= \frac{l_d}{l_b} \ddot{l}_1 + \frac{l_d}{I_b} (-l_b k_a A + l_d k_{cd} B - b k_{bi} D + T_b) + \frac{l_d}{I_d} (l_d k_{cd} B - b k_{do} C) \\ \ddot{l}_5 &= \frac{b^2}{I_{ao}} k_{do} C + \frac{b T_{ao}}{I_{ao}}, & \ddot{l}_6 &= \frac{b}{I_b} (-l_b k_a A + l_d k_{cd} B - b k_{bi} D + T_b) \\ \ddot{l}_7 &= \frac{b^2}{I_i} k_{bi} D + \frac{b T_i}{I_i} \end{aligned} \right\} \quad (30)$$

$$l_1 = \frac{l_a}{b} l_6, \quad l_3 = \frac{l_c}{l_s} l_2, \quad l_4' = \frac{l_d}{b} - \frac{l_4}{l_d} \quad (31)$$

Equations (30) and (31) are the equations of motion of the vibration system. Here, the equations are converted to nondimensional form. The representative angular frequency is

$$\omega_0^2 = \frac{b^2 k_{bi}}{I_i} \quad (32)$$

Equation (30) is converted with τ , where, $\tau = \omega_0 t$, and each value is set as follows :

$$\left. \begin{aligned} \xi_i &= \frac{l_i}{b} (i=1, 2, \dots, 7) \\ j_s &= \frac{I_i}{I_s + I_c}, & j_d &= \frac{I_i}{I_d}, & j_{ao} &= \frac{I_i}{I_{ao}}, & j_b &= \frac{I_i}{I_b}, & k_a &= \frac{k_a}{k_{bi}}, \\ k_c &= \frac{k_{cd}}{k_{bi}}, & k_d &= \frac{k_{do}}{b^2 k_{bi}}, & l_{ao} &= \frac{T_{ao}}{b^2 k_{bi}}, & l_b &= \frac{T_b}{b^2 k_{bi}}, & l_i &= \frac{T_i}{b^2 k_{bi}}, \\ b_a &= \frac{l_{ao}}{b}, & b_s &= \frac{l_s}{b}, & b_c &= \frac{l_c}{b}, & b_d &= \frac{l_d}{b} \end{aligned} \right\} \quad (33)$$

l_{ao} , l_{eo} , l_{co} and l_{do} are l_a , l_e , l_c and l_d at the rotating angle of the input axle flywheel $\theta_i = 0$, respectively, and we have

$$l_a = l_{ao} l_a', \quad l_e = l_{eo} l_e', \quad l_c = l_{co} l_c', \quad l_d = l_{do} l_d' \quad (34)$$

$$\begin{aligned}
\frac{d^2\dot{\xi}_2}{d\tau^2} &= \frac{1}{l_1'} \frac{dl_1'}{d\tau} \frac{d\dot{\xi}_2}{d\tau} - j_s(-b_s b_{s1} l_1' k_a A' + b_s b_{s1} l_1' k_s B' - b_s l_1' D' + b_s l_1' t_s) \\
&\quad + j_s(b_s^2 l_1'^2 k_a A' - b_s b_{s1} l_1' k_s B') \\
\frac{d^2\dot{\xi}_4}{d\tau^2} &= \frac{1}{l_4'} \frac{dl_4'}{d\tau} \frac{d\dot{\xi}_4}{d\tau} + j_s(-b_s b_{s1} l_4' k_a A' + b_s^2 l_4'^2 k_s B' - b_s l_4' D' + b_s l_4' t_s) \\
&\quad + j_s(b_s^2 l_4'^2 k_s B' - b_s l_4' k_s C') \\
\frac{d^2\dot{\xi}_5}{d\tau^2} &= j_{s0} k_s C' + j_{s0} t_{s0} \\
\frac{d^2\dot{\xi}_6}{d\tau^2} &= j_s(-b_s l_{s1}' k_a A' + b_s l_{s1}' k_s B' - D' + t_s), \quad \frac{d^2\dot{\xi}_7}{d\tau^2} = D' + t_s
\end{aligned} \quad (35)$$

where

$$\begin{aligned}
A' &= \dot{\xi}_1 - \dot{\xi}_2, & B' &= \dot{\xi}_3 - \dot{\xi}_4 \\
C' &= \dot{\xi}_4' - \dot{\xi}_5, & D' &= \dot{\xi}_5 - \dot{\xi}_7
\end{aligned}$$

Substituting Eqs.(33) and (34) in Eq.(31), we have

$$\left. \begin{aligned}
\frac{d\dot{\xi}_1}{d\tau} &= b_s l_{s1}' \frac{d\dot{\xi}_6}{d\tau} \\
\frac{d\dot{\xi}_3}{d\tau} &= \frac{b_s}{b_s} \frac{l_{s1}'}{l_1'} \frac{d\dot{\xi}_2}{d\tau} \\
\frac{d\dot{\xi}_4'}{d\tau} &= \frac{d\dot{\xi}_6}{d\tau} - \frac{1}{b_s l_{s1}'} \frac{d\dot{\xi}_1}{d\tau}
\end{aligned} \right\} \quad (36)$$

Equations (35) and (36) are the equations of motion which are represented with non-dimensional values. In case of rotating the input axle flywheel with constant angular velocity ω , next equation is obtained from Eq.(29) by denoting $\omega/\omega_0 = \Omega$.

$$\theta_i = \omega t = \Omega \tau, \quad \dot{\xi}_7 = \Omega \tau \quad (37)$$

In I-type mechanism, by denoting

$$\left. \begin{aligned}
y_1 &= \dot{\xi}_2, & y_2 &= \dot{\xi}_4, & y_3 &= \dot{\xi}_5, & y_4 &= \dot{\xi}_6, & y_5 &= \dot{\xi}_8, & y_6 &= \dot{\xi}_4', \\
y_7 &= \frac{d\dot{\xi}_2}{d\tau}, & y_8 &= \frac{d\dot{\xi}_2}{d\tau}, & y_9 &= \frac{d\dot{\xi}_3}{d\tau}, & y_{10} &= \frac{d\dot{\xi}_6}{d\tau}
\end{aligned} \right\} \quad (38)$$

we have

$$\begin{aligned}
\frac{dy_1}{d\tau} &= y_7, & \frac{dy_2}{d\tau} &= y_8, & \frac{dy_3}{d\tau} &= y_9, & \frac{dy_4}{d\tau} &= y_{10}, \\
\frac{dy_5}{d\tau} &= \frac{b_s}{b_s} \frac{l_{s1}'}{l_1'} y_7, & \frac{dy_6}{d\tau} &= y_{10} - \frac{1}{b_s l_{s1}'} y_9 \\
\frac{dy_7}{d\tau} &= -j_s(-b_s b_{s1} k_a A' + b_s b_{s1} l_1' k_s B' - b_s D' + b_s t_s) + j_s(b_s k_a A' - b_s b_{s1} l_1' k_s B') \\
\frac{dy_8}{d\tau} &= \frac{1}{l_4'} \frac{dl_4'}{d\tau} y_8 + j_s(-b_s b_{s1} l_4' k_a A' + b_s^2 l_4'^2 k_s B' - b_s l_4' D' + b_s l_4' t_s) \\
&\quad + j_s(b_s^2 l_4'^2 k_s B' + b_s l_4' k_s C') \\
\frac{dy_9}{d\tau} &= j_{s0} k_s C' + j_{s0} t_{s0}, & \frac{dy_{10}}{d\tau} &= j_s(-b_s k_a A' + b_s l_{s1}' k_s B' - D' + t_s)
\end{aligned} \quad (39)$$

where

$$A' = b_s y_4 - y_1, \quad B' = y_5 - y_2, \quad C' = y_6 - y_3, \quad D' = y_4 - \Omega \tau \quad (40)$$

In I-2 type mechanism, l_{c1}' and l_{d0}' are set as 1 in Eq.(34), and then by setting

Fig.7 Equivalent inertia system

$\gamma_{11} = \xi_1$ in the obtained equation, we have next equation from Eq.(38).

$$\begin{aligned}
\frac{dy_1}{d\tau} &= y_7, & \frac{dy_2}{d\tau} &= y_8, & \frac{dy_3}{d\tau} &= y_9, & \frac{dy_4}{d\tau} &= y_{10} \\
\frac{dy_5}{d\tau} &= \frac{b_s}{b_s} \frac{1}{l_1'} y_7, & \frac{dy_6}{d\tau} &= y_{10} - \frac{1}{b_s} y_9 \\
\frac{dy_7}{d\tau} &= \frac{1}{l_1'} \frac{dl_1'}{d\tau} y_7 - j_s(-b_s b_{s1} l_1' k_a A' + b_s b_{s1} l_1' k_s B' - b_s l_1' D' + b_s l_1' t_s) \\
&\quad + j_s(b_s^2 l_1'^2 k_a A' - b_s b_{s1} l_1' k_s B') \\
\frac{dy_8}{d\tau} &= j_s(-b_s b_{s1} l_4' k_a A' + b_s^2 l_4'^2 k_s B' - b_s D' + b_s t_s) + j_s(b_s^2 l_4'^2 k_s B' + b_s l_4' k_s C') \\
\frac{dy_9}{d\tau} &= j_{s0} k_s C' + j_{s0} t_{s0} \\
\frac{dy_{10}}{d\tau} &= j_s(-b_s l_{s1}' k_a A' + b_s l_{s1}' k_s B' - D' + t_s), & \frac{dy_{11}}{d\tau} &= b_s l_{s1}' y_{10}
\end{aligned} \quad (41)$$

where

$$A' = y_{11} - y_1, \quad B' = y_5 - y_2, \quad C' = y_6 - y_3, \quad D' = y_4 - \Omega \tau \quad (42)$$

3.3 Dynamic characteristic by equivalent moment of inertia

Denoting the moment of inertia as I_b in case of assuming the planetary gear and counterweight to the planet carrier to be able to rotate freely about their axle, we have

$$I_s' = I_s + 2 \frac{\omega}{g} b^2 \quad (43)$$

The equivalent moment of inertia I_{eq} , which is converted on the axle by using $\dot{\theta}_b$ and $\dot{\theta}_d$, is obtained from the relation of the kinematic energy as follows:

$$\begin{aligned}
\frac{1}{2} I_{eq} \dot{\theta}_b^2 &= \frac{1}{2} I_s' \dot{\theta}_b^2 + \frac{1}{2} (I_s + I_i) \dot{\theta}_i^2 + \frac{1}{2} \dot{\theta}_d^2 I_d \\
\therefore I_{eq} &= I_s' + \left(\frac{\dot{\theta}_i}{\dot{\theta}_b} \right)^2 (I_s + I_i) + \left(\frac{\dot{\theta}_d}{\dot{\theta}_b} \right)^2 I_d \quad (44)
\end{aligned}$$

Bearing in mind that the moment of inertia I_d of the output axle is small as compared with those of others, the equation of motion is presented as follows:

$$\left. \begin{aligned}
I_i \ddot{\theta}_i &= k_{bi} (\theta_b - \theta_i) + T_i \\
I_{eq} \ddot{\theta}_b &= -k_{bi} (\theta_b - \theta_i) - k_{d0} (\theta_b - \theta_{d0}) + T_b \\
I_{d0} \ddot{\theta}_{d0} &= k_{d0} (\theta_b - \theta_{d0}) + T_{d0}
\end{aligned} \right\} \quad (45)$$

where, T is a constrained force.

Equation (45) is considered to represent the vibration of the shafting with three disks as shown in Fig.7. Neglecting the term with constrained force, the equation of free vibration is obtained as follows:

$$\left. \begin{aligned}
\ddot{\theta}_b &= -\frac{k_{bi}}{I_{eq}} (\theta_b - \theta_i) - \frac{k_{d0}}{I_{eq}} (\theta_b - \theta_{d0}) \\
\ddot{\theta}_{d0} &= \frac{k_{d0}}{I_d} (\theta_b - \theta_{d0})
\end{aligned} \right\} \quad (46)$$

4. Computation of dynamic characteristic

Each value of the apparatus of the intermittent and rotate mechanism in I-1 type mechanism which is shown in Table 1 and Fig.2, is:

$$\begin{aligned}
n &= \gamma_a / \gamma_e = 8/9 \\
\gamma_e &= 50.625 \text{ mm}
\end{aligned}$$

weight of planetary gear part : $\mathcal{W}=1790$ g
 unbalance value at planet part : $\mathcal{W}r=1903$ g mm
 eccentric value of the center of gravity at planet part : 1.063 mm

$I_d=104$ g mm s^2
 output load : I_{d0}
 no-load : $I_{d(0)}=167$ g mm s^2
 load 1 : $I_{d(1)}=457$ g mm s^2
 load 2 : $I_{d(2)}=874$ g mm s^2

where, $I_{d0}=I_d+62.7$ (moment of inertia of the slitted disk and coupling at output).

4.1 Computation of the input torque fluctuation

The reference place α at the apparatus is zero, and the displacement curve is

$$\left. \begin{aligned} \text{i) } \phi=0, \quad \left(-\frac{9}{16}\pi \leq \theta \leq \frac{9}{16}\pi\right) \\ \text{ii) } \phi=-\frac{\pi}{8}+\frac{2}{9}\theta+\frac{1}{8}\sin\frac{16}{9}\theta \end{aligned} \right\} \dots \dots \dots (47)$$

As $\ddot{\theta} = \ddot{\phi} = (d^2\phi/d\theta^2) \dot{\theta}^2$ and $d\dot{\theta}/d\theta_b = d\phi/d\theta$, we have

$$\left. \begin{aligned} \ddot{\theta}=0, \quad \frac{d\phi}{d\theta}=0 \quad \left(-\frac{9}{16}\pi \leq \theta \leq \frac{9}{16}\pi\right) \\ \ddot{\theta}=-\frac{32}{81}\left(\sin\frac{16}{9}\theta\right)\dot{\theta}^2, \\ \frac{d\phi}{d\theta}=\frac{2}{9}\left(1-\cos\frac{16}{9}\theta\right) \end{aligned} \right\} \dots \dots \dots (48)$$

Fig.8 Theoretical value of input torque fluctuation

Fig.9 Fluctuating curve of input torque (1)

substituting Eqs.(47), (48) and each value in Eq.(18), the value of the input torque fluctuation is obtained. The computed result is shown in Fig.8. The computed result is case of rotating the input axle with 400 rpm is 20 Kg cm at maximum, where the moment of inertia of load $I_d=874$ g mm s^2 . Then, to obtain the value of the input torque fluctuation in case of changing the input rotational frequency and the moment of inertia of load, the computation was carried out with the analog computer. The results are shown in Figs.9 and 10.

4.2 Computation of the equation of motion

The varying radius of the pitch curve and the displacement angle of the noncircular gears c and d are given as follows :

$$\left. \begin{aligned} r_c &= \frac{7+2\cos(16/9)\theta_b}{15+2\cos(16/9)\theta_b} b, \quad \theta_c = -\frac{8}{9}\theta_b \\ r_d &= \frac{8}{15+2\cos(16/9)\theta_b} b, \\ \theta_d &= \frac{7}{9}\theta_b + \frac{1}{8}\sin\frac{16}{9}\theta_b \quad \left(0 \leq \theta_b \leq \frac{9}{8}\pi\right) \\ r_c &= \frac{9}{17}b, \quad \theta_c = -\frac{8}{9}\theta_b, \quad r_d = \frac{8}{17}b, \\ \theta_d &= \theta_b - \frac{\pi}{4} \quad \left(\frac{9}{8}\pi \leq \theta_b \leq \frac{9}{4}\pi\right) \end{aligned} \right\} \dots \dots \dots (49)$$

have

$$\left. \begin{aligned} l_c &= \frac{7+2\cos(16/9)\theta_b}{15+2\cos(16/9)\theta_b} b \cos\alpha_b \\ l_d &= \frac{8}{15+2\cos(16/9)\theta_b} b \cos\alpha_b \\ &\quad \left(0 \leq \theta_b \leq \frac{9}{8}\pi\right) \\ l_c &= \frac{9}{17}b \cos\alpha_c, \quad l_d = \frac{8}{17}b \cos\alpha_c \\ &\quad \left(\frac{9}{8}\pi \leq \theta_b \leq \frac{9}{4}\pi\right) \end{aligned} \right\} \dots \dots \dots (50)$$

where, α_p is the operating pressure angle of the noncircular gears c and d, and α_c is the cutter pressure angle, and α_b is given by the sum of α_c and α_p . When θ_b is zero, α_b and α_c are equal and we have

$$l_{c0} = \frac{9}{17}b \cos\alpha_c, \quad l_{d0} = \frac{8}{17}b \cos\alpha_c \dots \dots \dots (51)$$

Then, on the spur gears a and e, we have

Fig.10 Fluctuating curve of input torque (2)

$$I_s = \frac{8}{17} b \cos \alpha_s, \quad I_e = \frac{9}{17} b \cos \alpha_e \quad (52)$$

Therefore, from Eq.(50), we have

$$\left. \begin{aligned} I_e' &= \frac{1}{I_{e0}} \frac{7+2\cos(16/9)\theta_s}{15+2\cos(16/9)\theta_s} b \cos \alpha_s \\ I_d' &= \frac{1}{I_{d0}} \frac{8}{15+2\cos(16/9)\theta_s} b \cos \alpha_s \\ &\quad \left(0 \leq \theta_s \leq \frac{9}{8}\pi\right) \\ I_e' &= 1, \quad I_d' = 1 \quad \left(\frac{9}{8}\pi \leq \theta_s \leq \frac{9}{4}\pi\right) \end{aligned} \right\} \quad (53)$$

From Eq.(53), we obtain $dl_d'/d\tau$, that is

Fig.11 Dynamic displacement curve of intermittent and rotate mechanism

Fig.12 Vibrating curve by equivalent moment of inertia

Fig.13 Fluctuating curve of input torque

$$\left. \begin{aligned} \frac{dl_d'}{d\theta_s} &= \frac{1}{I_{d0}} \left[-\frac{8}{15+2\cos(16/9)\theta_s} \sin \alpha_s \frac{d\alpha_s}{d\theta_s} \right. \\ &\quad \left. + \frac{d}{d\theta_s} \left\{ \frac{8}{15+2\cos(16/9)\theta_s} \right\} \cos \alpha_s \right] \\ &\quad \left(0 \leq \theta_s \leq \frac{9}{8}\pi\right) \\ \frac{dl_d'}{d\theta_s} &= 0 \quad \left(\frac{9}{8}\pi \leq \theta_s \leq \frac{9}{4}\pi\right) \end{aligned} \right\} \quad (54)$$

and, with Eq.(37), we have

$$\frac{dl_d'}{d\tau} = Q \frac{dl_d'}{d\theta_s} \quad (55)$$

From Eq.(50), (51) and (52), we have

$$\left. \begin{aligned} b_s &= \frac{8}{17} \cos \alpha_s, \quad b_e = \frac{9}{17} \cos \alpha_e \\ b_s &= \frac{9}{17} \cos \alpha_s, \quad b_e = \frac{8}{17} \cos \alpha_e \end{aligned} \right\} \quad (56)$$

The spring constant of each axle is decided by experiment, and that of the gear is decided as follows :

$$\begin{aligned} k_{ae} &= 6.05 \times 10^4 \text{ Kg/mm}, \\ k_{cd} &= 4.58 \times 10^4 \text{ Kg/mm} \end{aligned}$$

Substituting these equations and coefficients in Eq.(39), the computation was carried out. The results are shown in Fig.11.

4.3 Computation of the dynamic characteristic by the equivalent moment of inertia

When $I_{d(2)}$ is used as the load, we have next equation from Eq.(46).

$$\left. \begin{aligned} \ddot{\theta}_s &= -0.624 \times 10^5 (\theta_s - \theta_{s0}) \\ &\quad - 1.25 \times 10^5 (\theta_s - \theta_{d0}) \\ \ddot{\theta}_{d0} &= 11.03 \times 10^5 (\theta_s - \theta_{d0}) \end{aligned} \right\} \quad (57)$$

The computed result is the vibration of sign curve as shown in Fig.12. As this vibration is supposed to be contained in the input torque in section 4.1, it was added. The result is shown in Fig.13.

5. Experiment and discussion

The experimental apparatus is shown in Fig.14. In Fig.14 a disk of which moment of inertia of load is 874 g mm s^2 is set to the output axle. The experimental results in case of rotating the input axle with 400 rpm and the load $I_{d(1)}$ and $I_{d(2)}$ are shown in Figs.18 and 16. In the experimental results, the vibration curve caused by various influences mentioned in section 4.2 is to be seen. And, the amplitude of the vibration is large at the repose interval of the output axle, as shown in Fig.11. Comparing the results of the computation and experiment, their curves are found to coincide as shown in Fig.17.

6. Conclusions

The results obtained in the present study are summarized as follows :

(1) On the mechanism classified in three types, the equation of the input torque was decided with the balance of the force and moment caused to the planetary gears. As the result, it was made clear that, in I-1 type mechanism, the fluctuation of the input torque was not generated as the planetary gears rotated with constant speed, and the fluctuation by the moment caused by the gravity could be elim-

Fig.14 Experimental apparatus

Fig.15 Experimental result of torque fluctuation

Fig.16 Experimental result of torque fluctuation

Fig.17 Comparison of computing and experimental results of torque fluctuation

inated by the counterweight at the planetary gears.

(2) The input torque equation of III-1 type mechanism coincided with that of II-1 type mechanism.

(3) On the intermittent and rotate mechanism of I-1 type, the fluctuation curve of the theoretical input torque in case of varying the rotational frequency and the load was compared with the experimental results and the results coincided with each other.

(4) As the natural vibration curve was contained in the experimental results of the input torque fluctuation, the model was made by considering the mechanism as a vibration system. Then, the equation of motion was decided after the dynamic intermeshing of the noncircular gear and the dynamic fluctuation curve were obtained. As the result, even in the repose intervals of the intermittent motion, the vibration appeared due to the backlash of each gear and the torsion of each axle and it was made clear the input torque fluctuation was influenced by the vibration.

(5) Converting the mechanism to an equivalent system of the moment of inertia, the vibration curve of the system was computed. This curve was added to one obtained in (3), and the composed curve coincided with one of experimental results.

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