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Studies on the Fatigue Crack Initiation and Propagation*

(Report 1, Effects of Crack Length, Stress Range and Minimum Stress on Fatigue Crack Propagation)

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The fatigue crack propagation behaviors were investigated on HT80 steel plates under net section stress control in which stress range and minimum stresses were widely varied.

The fatigue crack growth rates were proportional to the effective crack length $2a_e$ which was an actual crack length $2a$ modified by the finite width effect of the specimen. When minimum stress $\sigma_{\min}$ was in compression side, the growth rate depended on maximum stress $\sigma_{\max}$ and in tension side, it depended on stress range $\Delta\sigma_n$. As the result, the effects of crack length, stress range and minimum stress on the fatigue crack propagation can unifyingly be shown such that the crack growth rate is proportional to the cyclic plastic zone size at a crack tip $R_p(c) = a_e \left(\sec \frac{\pi \sigma_n}{2 \sigma_y} - 1 \right)$. Where, σ_y is yield strength of material and σ_n is $\sigma_{\max}$ when $\sigma_{\min} \leq 0$ and is $\Delta\sigma_n$ when $\sigma_{\min} \geq 0$.

1. Introduction

In the safe life design of a machine structure, it is essential to understand the fatigue crack propagation behavior. The most basic factors affecting the fatigue crack propagation behavior include crack length, stress amplitude and mean stress, on which quite a few reports have already been made (1)-(4). These reports, however, failed to provide satisfactory general formulations because of the restriction in scope of experiments conducted.

In our study, fatigue tests were conducted by plate specimens of HT80 steel under test conditions involving extensive stress range and minimum stress, whereby to clarify the effects of various mechanical factors on the rate of crack growth. The results of experiments could be satisfactorily rearranged by taking into consideration the cyclic elastic-plastic response phenomenon occurring at the crack tip.

Table 1 Chemical compositions of the material

C	Si	Mn	P	S	Cu	Ni	Cr	Mo	V	Al	B
0.14	0.25	0.77	0.015	0.009	0.25	0.86	0.51	0.46	0.051	0.026	0.0037

%

2. Method of Experiment

2.1 Test materials and test specimens

80 kg/mm² refined high strength steel (HT80 steel) was used as a test material. Its chemical compositions and mechanical properties are given in Tables 1 and 2, respectively. The material steel, hot-rolled to a thickness of 6 mm, was spray-quenched from 870°C and annealed at 680°C for 40 minutes. The test specimen was taken from the material steel in the rolling direction of the material. Then, after machined into a plate with a width of 200 mm as shown in Fig. 1, it was finally polished at its surface. Fig. 1(a) shows a specimen used for investigating the effect of stress range. Provided with a notch at the center measuring about 0.05 mm in root radius, this type of specimen with a plate thickness of 2.5 mm was made in two kinds; one with a notch length of 2 mm and the other with a notch

length of 50 mm. A specimen shown in Fig. 1(b) was designed for the investigation in minimum stress. It measured 5 mm

Table 2 Mechanical properties of the material

kg/mm², %

σ_y	σ_B	σ_T	ϕ	ψ
82.0	87.7	157	25.9	67.9

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in plate thickness, about 0.05 mm in notch root radius, and 3.6 mm or 50 mm in notch length. Notches were cut on these specimens by means of a high quality file.

2.2 Test method

In this test, a Losenhausen universal fatigue testing machine (UHS 100, 100/60 t, 200~700 cpm) complete with a low cycle facility (5 ~ 20 cpm) was employed. The effect of stress range was investigated by the pulsating fatigue test in which the minimum stress was rated at 1 kg/mm², while the effect of minimum stress, was investigated by various partly reversed and pulsating fatigue tests. During the tests, load unbalance of the specimen was adjusted within 5%. Net section stress control was carried out by the following methods. On the tension side, load was adjusted in such a manner that the value

of net section stress σ_n , in which the decrease in sectional area due to its contraction, notch and crack is taken into account, would stay constant. On the compression side, load was adjusted so that the value of stress in which load carried by the crack surface is taken into account would stay constant. Load was reduced each time the crack grew by 0.5~3 mm, whereby to maintain the accuracy of the above control within $\pm 2\%$. Crack length was obtained by measuring at both sides of the specimen and averaging the measured values.

3. Experimental Results

3.1 Fatigue crack propagation from a small notch

Fatigue crack propagation curves of a specimen with a short notch are shown in

Fig. 1 Details of test specimens.

Fig. 2 Fatigue crack growth curves for short crack.

Fig. 2. Figs. 2(a) and 2(b) show the results of the pulsating fatigue test, in which the minimum stress $\sigma_{\min}$ applied was 1 kg/mm^2 . In these figures σ_n and σ_g represent the net section stress and gross stress at the maximum stress, respectively. In Fig. 2(c), by $\sigma_n = -25 \leftrightarrow 15 \text{ kg/mm}^2$ it is meant that the specimen is in the condition of stress cycling whose minimum stress is -25 kg/mm^2 and maximum stress is 15 kg/mm^2 . In Figs. 2(a) and 2(c), the transition points are observed. This point, as indicated by the arrow in Fig. 3, represents a point where the shape of fracture begins to change from flat, tensile type to shear type having a shear lip, and beyond this folding point the straight line is more inclined. From Fig. 2, in the area where crack length $2a$

(a) $\sigma_n = 1 \leftrightarrow 20 \text{ kg/mm}^2$. (b) $\sigma_n = -25 \leftrightarrow 15 \text{ kg/mm}^2$. (c) $\sigma_n = 35 \leftrightarrow 50 \text{ kg/mm}^2$. Arrows correspond to the transition points shown in Fig. 2.

Fig. 3 Examples of fracture surface.

is short or where the infinite plate is considered to have a crack almost with certainty, $\log a$ and the number of cycles N are in the linear relation if the fracture shape is the same. Consequently, in this case as in the experiments reported in the past (1)-(4), the following relation will hold:

$$\frac{da}{dN} = k a \dots \dots \dots (1)$$

where; k is proportional constant. In the case of $\sigma_n = 98 \text{ kg/mm}^2$ in Fig. 2(b), crack starts to grow rapidly at $a = 12.65 \text{ mm}$, ending up in unstable fracture.

Fig. 4 shows a crack propagation curve under net section stress control. Under any stress condition, the crack propagation curve can be divided into the next three areas. That is, an area where, within the scope of $a < 30 \text{ mm}$ ($2a/W < 0.3$), crack growth rate da/dN increases with crack length; an area where, within the scope of $30 \text{ mm} < a < 60 \text{ mm}$ ($0.3 < 2a/W < 0.6$), da/dN does not show any variation even if crack length increases; and an area where, within the scope of $a > 60 \text{ mm}$ ($2a/W > 0.6$), da/dN diminishes as crack length increases.

3.2 Crack propagation under changing stress range.

As aforementioned under the net sectional stress control, within the scope of $0.3 < 2a/W < 0.6$, the crack growth rate stays constant irrespective of crack length. Consequently, if stress and other conditions in this area are changed in various ways, the effect of the factor of change on crack growth can be grasped firmly. From this view point, we conducted pulsating fatigue tests with various maximum stress values using a test specimen shown in Fig. 1(a) which has a notch length of $2a_0 = 50 \text{ mm}$ ($2a_0/W = 0.25$). The test results are shown in Fig. 5. As this diagram has employed various scales on the axis of abscissa to

Fig. 4 Fatigue crack propagation under net section stress control.

suit each stress level, direct comparison between each stress level cannot be made, but it can be seen from the diagram that the crack growth rate stays constant at every stress level, except in its initial stage.

If the stress level is raised in step, the crack growth rate immediately becomes larger than the value in the following steady state, and if the level is raised each time by 10 kg/mm², this phenomenon is more obvious as the level is higher. A large crack growth rate was recorded too immediately after the crack was initiated from notch at a high stress level ($\sigma_n = 80$ kg/mm²). If the stress level is lowered, in the low stress range there comes an interposing period immediately after the change in which crack growth is withheld for a while. In the high stress range, however, the crack growth rate becomes large immediately after the change, and after a transitory period of decrease, it resumes to increase until it turns into a constant rate corresponding to each stress level. This indicates the existence of two areas in or around the crack tip sustaining the effect of stress; one area undergoes the compressive residual stress, while the other is located in close proximity to the crack tip and sustains extensive plastic fatigue damage. In the high stress range, the damaged area is large.

(a) At low stress level ($\sigma_{\min} = 1 \text{ kg/mm}^2$)

(b) At high stress level ($\sigma_{\min} = 1 \text{ kg/mm}^2$)

Fig. 5 Fatigue crack propagation under the condition of changing the stress levels.

In Fig. 5(b), at $\sigma_n = 90$ kg/mm² no change was observed in the rate of crack growth when the hold time at maximum stress was increased from 2 seconds to 600 seconds at the arrow mark.

3.3 Crack propagation under changing stress range and minimum stress.

Fig. 6 shows the results of tests with a specimen with a long notch shown in Fig. 1(b). When the stress range and maximum stress are increased, i.e. when $[(\sigma_n = -15 \leftrightarrow 5 \text{ kg/mm}^2) \rightarrow (-10 \leftrightarrow 10), (-10 \leftrightarrow 10) \rightarrow (-20 \leftrightarrow 20), (-35 \leftrightarrow 5) \rightarrow (-20 \leftrightarrow 40)]$, the crack growth rate becomes large immediately after change as in the case of 3.2. Under pulsating, in case the stress range is constant and the maximum stress is increased, there is no change so far as the maximum stress and stress range are relatively small, i.e., $[(0 \leftrightarrow 20) \rightarrow (20 \leftrightarrow 40), (20 \leftrightarrow 40) \rightarrow (40 \leftrightarrow 60)]$, but the crack growth rate immediately after change shows an appreciable increase so long as the maximum stress is increased, if the maximum stress and stress range are relatively large, i.e., $[(0 \leftrightarrow 60) \rightarrow (20 \leftrightarrow 80), (20 \leftrightarrow 80) \rightarrow (30 \leftrightarrow 90)]$. In case the stress range and maximum stress are large, if the maximum stress is increased the crack will open wide at the initial charge of load and develop into ductile crack in extreme cases. Damages due to the above as well as the increase in strain range at the crack tip that continues for some period after change are considered responsible for the enhancement of crack growth.

In case the maximum stress is constant and the stress range is reduced, i.e. $[(-30 \leftrightarrow 60) \rightarrow (-50 \leftrightarrow 60)]$, there occurs almost no noticeable change, and in case the maximum stress is constant and the stress range is increased, no change occurs at low stress levels, i.e. $[(15 \leftrightarrow 20) \rightarrow (10 \leftrightarrow 20), (10 \leftrightarrow 20) \rightarrow (0 \leftrightarrow 20)]$. When the stress range is constant and the maximum stress is reduced, i.e. $[(-30 \leftrightarrow 10) \rightarrow (-35 \leftrightarrow 5)]$, crack growth is checked by the compressive residual stress at the crack tip caused by the high stress that existed before change.

In case the minimum stress is essentially a large compressive stress, i.e. $[(-30 \leftrightarrow 60), (-30 \leftrightarrow 10), (-35 \leftrightarrow 5)]$, the crack growth rate is noticeably increased in the neighborhood of notch root. This may be ascribed to the fact that at the notch root, different from the crack tip, sizable plastic deformation can occur due to compressive stress. In the specimens shown in Figs. 5 and 6, the fracture surface assumed a tensile type when the crack growth rate was $0.03 \mu\text{/c}$ or less, and a shear type when the rate was above $0.15 \mu\text{/c}$, irrespective of stress range or minimum stress.

3.4 Microscopic observation of crack growth.

The behavior of a crack tip on the specimen surface held at the steady state at $\sigma_n = 90$ kg/mm² as given in Fig. 5 (b)

was observed repeatedly at the maximum and minimum stress level by replica method. Fig. 7 shows the results of observation. When a load was applied the crack tip underwent large deformation, and when the deformation exceeded a certain value the crack gained in its length, but the crack opening was closed at the minimum stress. This process was repeated for each loading. Fig. 8 shows such behavior observed many a time at 70 kg/mm² or lower stress levels. In this observation it was confirmed that a new, minute crack was initiated first

some distance apart from the crack tip and it increased its size as the main crack approached. When the two cracks were brought close to each other, shear fracture was caused to unite the two cracks into one with a longer length. Generally, the striation spacing of the fracture surface is said to correspond to the macroscopic growth rate, so the above phenomenon is possible only on the surface of a specimen.

Fig. 9 shows how the crack tip opens and closes at the steady state at $(-20 \leftrightarrow$

(a)

(c)

Fig. 6 Fatigue crack propagation under the condition of changing the stress range and the minimum stress.

N=498

N=499

N=500

200μ $\sigma_{\max} = 90 \text{ kg/mm}^2$ $\sigma_{\min} = 1 \text{ kg/mm}^2$

Fig. 7 Microscopic observations of fatigue crack propagation.

$\sigma_{\max} = 70 \text{ kg/mm}^2$ $\sigma_{\min} = 1 \text{ kg/mm}^2$

Fig. 8 Fatigue crack propagation which accompanied with joining to micro crack.

(a) $\sigma_n = -20 \leftrightarrow 40 \text{ kg/mm}^2$ (b) $\sigma_n = 20 \leftrightarrow 60 \text{ kg/mm}^2$

Fig. 9 The cyclic behavior of crack tips under partly reversed and pulsating loading.

40) and (20↔60) given in Fig. 6(c). In the partly reversed condition of (-20↔40), the crack closes its opening completely at the point where stress is zero, and hardly any change is observed in the crack tip condition even if the stress is further augmented on the compression side. On the other hand, in the pulsating condition of (20↔60) the crack tip is entirely closed as soon as the minimum stress is reached. As a result, the crack tip opening displacement at the maximum stress becomes almost the same in the above two conditions. This is in conformity with the fact that, as shown in Fig. 6(c), the crack growth rates in the two conditions are almost the same. From Fig. 9, it can be said that the plastic strain range in close proximity to the crack tip depends on the maximum stress when the minimum stress is located on the compression side, and on the stress range when the minimum stress is located on the tension side.

4. Discussions

4.1 Effect of crack length on crack growth

To generalize the result of experiment with a test specimen its finite width effect must be corrected. When there is a crack with a length of a on

Fig. 10 Effective crack length in center cracked plate subject to tension.

the finite width plate carrying net section stress as large as σ_n , the crack length of an infinite plate which would present the same stress strain condition as the said finite width plate at its crack tip is defined here as effective crack length a_e . Strictly, the crack tip is in the cyclic elastic-plastic condition. Here we base the study on the theory of elasticity for convenience' sake. Stress intensity factors K_{inf} and K_{fin} , for infinite plate and finite width plate respectively, are expressed as follows:

$$K_{inf} = \sigma_n \sqrt{\pi a_e}$$

$$K_{fin} = \sigma_n \sqrt{\pi a f}$$

where; f : Factor representing finite width effect.

Consequently, if $K_{fin} = K_{inf}$, effective crack length a_e will be expressed as follows.

$$a_e = af = \frac{K_{fin}^2}{\pi \sigma_n^2} \quad \dots \dots \dots (2)$$

For a strip with a center crack under tension, the formula for f is as given below.

$$f = \left(1 - \frac{2a}{W}\right)^2 \sec \frac{\pi a}{W} \quad \dots \dots \dots (3)$$

The change of a_e due to $2a/W$ shown in Fig. 10 is in a trend similar to that of crack growth rate as shown in Fig. 4 and 3.1. It follows that between effective crack length a_e and crack growth rate da/dN there exists such a relation as shown in Fig. 11. Accordingly,

$$\frac{da}{dN} = k a_e \quad \dots \dots \dots (4)$$

Eq. (1) is included in Eq. (4) as a special case. In the case of $\sigma_n = 90 \text{ kg/mm}^2$ long cracks cease to be rectilinear, and when $\sigma_n = 98 \text{ kg/mm}^2$ fracture took place at $a = 12.65 \text{ mm}$. This means that, under high

Fig. 11 The relations between effective crack length $2a_e$ and crack growth rate.

stress, if a crack attains too great length it approaches unstable fracture strength.

As can be seen from Fig. 11(a) and (c), at the transition of fracture type from tensile type to shear type the crack growth rate is $0.03 \sim 0.08 \mu/\text{cycle}$. This property is same as the result obtained by D. Broek, et al⁽⁶⁾ in their experiments with aluminium alloy.

4.2 Effect of stress range on crack growth

Fig. 12 shows the relation between normalized crack growth rate $k=(da/dN)/a_e$ and stress range obtainable from Fig. 5 and Fig. 11 (a) and (b) when minimum stress is 1 kg/mm^2 . As shown in the Figure, in the following equations that have already been established⁽¹⁾⁻⁽⁴⁾, (the inclination of the curve shown in the Figure) is not constant but is a function of stress range. In other words, at $\Delta\sigma_n \leq 25 \text{ kg/mm}^2$ ($\Delta\sigma_n/\sigma_y \leq 0.3$, σ_y : yield strength) $m \approx 2$, at $25 \leq \Delta\sigma_n \leq 60 \text{ kg/mm}^2$ ($0.3 \leq \Delta\sigma_n/\sigma_y \leq 0.7$) $m \approx 3$, and over this limit m increases.

$$\frac{da}{dN} = C'' \Delta\sigma_n^m a_e \quad \dots\dots\dots (f')$$

Fig.12 Effect of stress range on fatigue crack growth rate.

Fig. 13 shows the following equation⁽⁷⁾ and experimental values⁽¹⁾. The equation expresses the plastic zone size R_p of a crack tip of an infinite plate consisting of a perfectly elastic-plastic body, which undergoes tensile stress σ in the state of plane stress. Fig. 13 and Fig. 12 are in the analogous trend.

$$R_p = a \left(\sec \frac{\pi\sigma}{2\sigma_y} - 1 \right) \quad \dots\dots\dots (5)$$

Since the crack tip closes its opening completely at the minimum stress as shown in Figs. 7 and 8, cyclic plastic zone size $R_p(c)$ is considered a parameter representing a cyclic plastic strain field in the proximity of the crack tip. The equation expressing $R_p(c)$ can be obtained by substituting $\Delta\sigma_n$ into σ and a_e into a in Eq. (5)

$$R_p(c) = a \left(\sec \frac{\pi\Delta\sigma_n}{2\sigma_y} - 1 \right) \quad \dots\dots\dots (6)$$

The curve shown in Fig. 12 expresses the following equation.

$$\frac{da}{dN} = C R_p(c) \quad \dots\dots\dots (7)$$

Eq. (7), being $\Delta\sigma_n < 70 \text{ kg/mm}^2$ ($\Delta\sigma_n/\sigma_y < 0.9$), is in good agreement with the results of experiment, and constant C is calculated at 1.25×10^{-4} for tensile type fracture and at 2.5×10^{-4} for shear type fracture. Also, since crack opening displacement COD⁽⁸⁾ is roughly proportional to R_p , cyclic crack opening displacement ΔCOD can be obtained by the same process as for $R_p(c)$. By using this ΔCOD the following equation can be obtained.

Fig.13 Theoretical plastic zone size⁽⁷⁾ and experimental values⁽¹⁾.

$$\frac{da}{dN} = C' (\Delta COD) \dots\dots\dots (7')$$

where;

$$COD = \frac{4\sigma_a e}{\pi E} \ln \frac{\pi \Delta \sigma_n}{2\gamma} = \frac{4\sigma_y}{\pi E} R_P(c) \dots (6')$$

Contact C' is calculated at 0.025 for tensile type fracture and at 0.050 for shear type fracture. Eqs. (7) and (7') can be considered reasonable if the distribution of cyclic plastic strains causing fatigue damage in the crack tip is assumed to be analogous with $R_P(c)$ or ΔCOD .

In the past, various of m were obtained by arranging experimental results according to Eq. (1'), but these values can be covered by Eqs. (7) and (7'), as exemplified in Fig. 14. The inclination in the Figure indicates the value of m obtained by researchers from their experimental points. Frost⁽¹⁾ rated it at 3 for mild steel, and Yokobori, et al.⁽³⁾ rated it at 3.25 for mild steel and at 2.0 ~ 2.56 for HT80 steel. So far as constant C in Eq. (7) is concerned, high strength steel gives a smaller value than mild steel.

As shown in Fig. 12, the crack growth rate for tensile type fracture (resembling the plane strain condition) is smaller than that for shear type fracture (resembling the plane stress condition). This is consistent with the fact that the plastic zone size of the crack tip in the plane strain condition is smaller than the value in the plane stress condition expressed by Eq. (5),

due to the tri-axial stress condition. The transition of fracture type from tensile type to shear type occurs at the point where $R_P(c) = 0.3$ mm, from Eq. (7). Compared with the value of plate thickness that is 2.5 mm this is a very small value.

4.3 Effect of minimum stress on crack growth

Fig. 15 shows the relationship between the minimum stress and the normalized crack growth rate obtained from the experimental results shown in Figs. 12 and 6. When the minimum stress is on the tension side, the crack growth rate depends solely on stress range $\Delta \sigma_n$, and picks up almost no influence from the minimum stress. This is in agreement with the experimental results obtained with steel⁽²⁾ (3). When the minimum stress is shifted to the compression side under the identical stress range, the crack growth rate shows a precipitant drop as indicated by a solid line in the Figure. In this case, the crack growth rate, as indicated by a broken line in the Figure, is in good agreement with the value obtained when $\Delta \sigma_n$ for positive minimum stress values is substituted by maximum stress σ_{max} . In other words, if the minimum stress is on the compression side the crack growth rate depends on the maximum stress. Accordingly, by expressing the cyclic plastic zone size and cyclic crack opening displacement in equations as follows based on Fig. 15, Eqs. (7) and (7') will become applicable to various minimum stress conditions.

Fig.14 The arrangements of other experimental results with Eq. (7).

$$\left. \begin{aligned}
 &\text{If } \bar{\sigma}_{\min} \leq 0 \\
 &R_p(c) = a_e \left(\sec \frac{\pi \bar{\sigma}_{\max}}{2 \bar{\sigma}_y} - 1 \right) \\
 &\Delta COD = \frac{4 \bar{\sigma}_y a_e}{\pi E} \ln \frac{\pi \bar{\sigma}_{\max}}{2 \bar{\sigma}_y} \quad \dots\dots (8) \\
 &\text{If } \bar{\sigma}_{\min} \geq 0 \\
 &R_p(c) = a_e \left(\sec \frac{\pi \Delta \bar{\sigma}_n}{2 \bar{\sigma}_y} - 1 \right) \\
 &\Delta COD = \frac{4 \bar{\sigma}_y a_e}{\pi E} \ln \frac{\pi \Delta \bar{\sigma}_n}{2 \bar{\sigma}_y}
 \end{aligned} \right\}$$

In Eq. (8), application of Eqs. (7) and (7') does not make sense so long as $\Delta \bar{\sigma}_n$ or $\bar{\sigma}_{\max}$ is larger than $\bar{\sigma}_y$. However, the situation will be changed if an analytical representation of the correct volume of crack tip's opening and closing can be obtained.

If the growth rate of fatigue crack, da/dN , depends on the cyclic behavior of plastic deformation at the crack tip, and a geometric similarity relation can hold between them, the following equations can be obtained by assuming that the size of the area where plastic deformation is actually repeated and the resultant stress-strain relation draws a hysteresis loop is $R_p(c)^*$, and the actual volume of crack tip's opening and closing is $(\Delta COD)^*$.

$$\frac{da}{dN} = C_1 R_p(c)^* \quad \dots\dots\dots (9)$$

$$\frac{da}{dN} = C_1' (\Delta COD)^* \quad \dots\dots\dots (10)$$

Since it has been proved by the results of this experiment that Eqs. (7) and (7') can hold, the following equations can be derived from Eqs. (9) and (10).

$$R_p(c)^* = C_2 R_p(c) \quad \dots\dots\dots (11)$$

$$(\Delta COD)^* = C_2' (\Delta COD) \quad \dots\dots\dots (12)$$

where; $C_2 = C/C_1$ and $C_2' = C'/C_1'$

$R_p(c)^*$ is approximately expressed as an area where, first, due to static tensile stress load, tensile plastic deformation is caused at the crack tip, and then during unloading process, owing to the restraint imposed by the deformation of the surroundings, compressive plastic deformation is caused. In the structural steel used in our experiment, the limit at which the part yielded due to tensile stress draws a hysteresis loop assisted later by cyclic stress, has a stress range nearly equal to $2\bar{\sigma}_y$ instead of $2\bar{\sigma}_y(9)$. This will provide a basis on which the results of experiments can be rearranged satisfactorily by applying $\bar{\sigma}_y$ to Eqs. (6) and (6') as in the case of (7) and (7'). As a result of measurement of the size of an area where a persistent slip band is formed at the crack tip, $R_p(c)^*$ was found to bear nearly a proportional relation with $R_p(c)$ calculated by Eq. (8), and C_2 of Eq. (11) was about 0.1(11)(12). Rice(10), paying no attention to the

Fig.15 Effect of minimum stress on fatigue crack growth rate.

Fig.16 The relations between the range of stress intensity factor and crack growth rate ($\bar{\sigma}_{\min}=1 \text{ kg/mm}^2$).

abovementioned Bauschinger effect of material, carried out his analysis assuming the yielding point under cyclic stress to be $2\sigma_y$, with the result that, under pulsating loading, C_2 of Eq. (11) is 0.25 when $\sigma/\sigma_y \ll 1$, and even after unloading the crack tip stays half-opened. The crack opening behavior like this is usually observed when load is removed after stress is increased by large steps from the low level. However, under the condition where a fixed value of stress is repeatedly applied, the crack opening is entirely closed at the minimum stress as shown in Figs. 7 ~ 9, contrary to the result of calculation by Rice. Further, the scope of Rice's model in which the crack tip's elastic plastic response for cyclic stress can be represented by the range of stress intensity factor ΔK is $\sigma/\sigma_y < 0.6$, wider than $\sigma/\sigma_y < 0.3$ for the case of static loads. It is when the inclination of the curve m is 2 that the experimental results of Fig. 12 and Fig. 14 can be represented by ΔK , and this corresponds to $\sigma/\sigma_y < 0.3$. Moreover, Fig. 16 shows the result of experiment conducted at the minimum stress of 1 kg/mm^2 which is rearranged by the relation between stress intensity factor range ΔK and crack growth rate da/dN . This indicates that ΔK based on the theory of elasticity is not sufficient for the large stress range compared with yield strength.

When the opening and closing behaviors of crack tip differ from Fig. 9, or in the state closely preceding the unstable fracture accompanied by local brittle fracture, or when the crack is kept from growing by the grain boundary in the neighborhood of fatigue limit⁽¹²⁾, it is not advisable to apply Eqs. (7), (7').

5. Conclusion

Fatigue tests were conducted under various stress range and minimum stress conditions using a plate made of HT80 steel provided with a center notch. Crack growth behaviors were observed in the tests with conclusions that follow:

(1) Crack growth rate da/dN under net section stress control is proportional to effective crack length $2a_e$ which can be obtained by correcting the effect of finite width effect as in Eq. (2).

(2) The fatigue crack growth rate depends on stress range $\Delta\sigma_n$ when minimum stress is on the tension side, and on maximum stress $\sigma_{\max}$ when it is on the compression side.

(3) When $\sigma_n/\sigma_y \leq 0.3$ ($\sigma_n: \Delta\sigma_n$ if $\sigma_{\min} \geq 0$, $\sigma_{\max}$ if $\sigma_{\min} \leq 0$, σ_y : yield strength) the crack growth rate is proportionate to the square of σ_n , and when $0.3 \leq \sigma_n/\sigma_y \leq 0.7$, to the cube of it.

(4) That is, when $\sigma_n/\sigma_y < 0.9$, the crack growth rate is proportionate to cyclic plastic zone size $R_p(c)$ at the crack tip as well as to cyclic crack opening displacement ΔCOD .

(5) The type of fracture depends on the crack growth rate. In the slower rate range it assumes a tensile type, while in the faster rate range a shear type.

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