Discharge Coefficients of Float-Area-Type Flow Meters*

By Hiroshi MAKI**

In this paper, in order to improve the measuring accuracy of this kind of flow meter, it is experimentally pointed out that the commonly used discharge coefficients of float-area-type flow meters do not always have constant values over a wide range of characteristic Reynolds number even in the region of Re larger than critical characteristic Reynolds number Re_c .

It is shown that only in the case of the floats having special shapes or geometrical dimensions the discharge coefficients of this kind of flow meters have constant values independent on the non-dimensional position of float.

When employing such float, semi-empirical formula to predict the flow rate characteristics is obtained, and the limits of applicability of the formula are experimentally explained.

1. Introduction

The discharge coefficient C of float-area-type flow meters has been defined⁽¹⁾ as

$$C = Q / A \sqrt{2g V_f (\gamma_f - \gamma_o) / A_f \gamma_o} \quad (1)$$

where Q denotes the volume flow rate, A the minimum annular cross sectional area between the tapered tube and the float, g the acceleration of gravity, V_f the volume of the float, γ_f the specific weight of the float, γ_o the specific weight of the working fluid, and d_f the maximum diameter of the float, with $A_f = \pi d_f^2 / 4$. The float with a shaft⁽³⁾ as shown in Figs. 1 (b) and (c) has been widely employed for float-area-type flow meters, since K. Fisher et al.⁽²⁾ (1941) reported that the discharge coefficient defined

Fig. 1 Shapes of the float and types of the shaft as a stabilizer

by Eq. (1) could be made roughly uniform in the region of Reynolds numbers larger than a certain critical value by employing a float with a thin maximum-diameter part.

There has been no published report pointing out correctly that the discharge coefficient defined by Eq. (1) is generally not constant even in the region of Reynolds numbers above the critical one and may change with working conditions. Thus, most studies known so far on the flow-rate characteristics, i.e. on the relations between the flow rate and the position of the float, of this kind of flow meters have been

based on the assumption of constant discharge coefficient⁽⁴⁾.

Recently, the float-area-type flow meters have been widely used in process control systems for flow rate. Therefore, a higher accuracy in the measurement of flow rate is being required. It is obvious, as in the case of differential-pressure-type flow meters, that the constant value of the discharge coefficient over a wide range of Reynolds numbers is meaningful. It is well known that, if the discharge coefficient has not a constant value, much time and efforts are consumed in obtaining accurately the changes in flow rate characteristics corresponding to varying working conditions.

The author has demonstrated⁽⁵⁾⁽⁶⁾, with the aid of theoretical analyses and visualization experiments of the flow in the annular clearance between the float and the tapered tube, that the widely used floats of float-area-type flow meters could be classified into two types, i.e. "the float with large l " and "the float with small l ", where l denotes the length of the part of the maximum diameter of the float, and clarified theoretically and experimentally the number of dimensionless parameters influencing the flow rate characteristics when employing those two types of floats. Furthermore, he reported⁽⁶⁾ that the flow rate characteristics in turbulent region were greatly influenced by vertex angles, θ_1 and θ_2 in Fig. 1 (b), of the cone portions upstream and downstream of the maximum diameter part of the float.

In order to improve the measuring accuracy of float-area-type flow meters, it will be attempted in this report, firstly, to obtain experimentally the geometrical dimensions of floats having constant values of the discharge coefficient over a wide range of Reynolds numbers by utilizing the results reported in References (5) and (6), and, secondly, to propose a semi-empirical formula for predicting the flow rate characteristics when employing such floats and to explain the limits of applicability of the above formula.

2. Experimental apparatus and procedure

Water and air were used as working fluids. Since the experimental apparatus was the same as reported in Reference (6), no description will be made here. Tapered tubes employed were made of pyrex glass and the distribution of the inner diameters of the tubes was accurately measured by the same method as reported in Reference (6). The mean inner diameters and the gradients of the tapered

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** Assistant Professor, Science University of Tokyo, Noda.

tubes employed are listed in Table 1. Floats employed in this experiment were those with small ℓ as defined in Reference (6), with the maximum

Table 1 Tapered tubes employed

No.	Gradient	Mean inner diameter	Diameter of the shaft
1	7.221×10^{-3}	22.49 mm	3 mm
2	8.768×10^{-3}	28.57 mm	4 mm
3	9.911×10^{-3}	34.10 mm	5 mm
4	1.250×10^{-2}	45.11 mm	6 mm
5	1.875×10^{-2}	56.50 mm	6 mm
6	3.909×10^{-2}	142.3 mm	14 mm

diameter part of the float contained in the separated region. Therefore, in order to prevent the instability for the rotational vibration of the float, a shaft as shown in Fig. 1 (c) was employed as the stabilizer. The outer diameters d_s of the shafts are also included in Table 1. When employing a shaft as the stabilizer, the flow rate characteristics was varied, if the centers of the tapered tube and of the float did not coincide with each other. Therefore, a means was taken, as reported in Reference (7), of fixing the shaft at the same position for all tapered tubes employed.

Since the results of preliminary investigations revealed that the vertex angle θ_2 of the cone portion downstream of the maximum diameter part of the float did not influence the flow rate characteristics for $\theta_2 >$ about 120° , floats with $\theta_2 = 180^\circ$ as shown in Fig. 1 (c) and $\theta_1 = 60^\circ, 80^\circ, 90^\circ, 95^\circ, 96^\circ, 100^\circ$ and 120° were employed in this investigation. These angles θ_1 were obtained by machining within the rotational accuracy for the setting angle of the swivel base of the lathe. The experimental results for $\theta_1 = 180^\circ$ were taken from Reference (6). Since the preliminary investigations showed, furthermore, that the flow rate was indicated smaller with larger diameter d_1 of the cylinder portion of the float (see Fig. 1 (c)), all floats employed in this experiment were so manufactured as to have $d_1/d_f = 0.6 \sim 0.7$.

The tapered tubes listed in Table 1 were combined with floats which were made of stainless steel, duralumin and ebonite and the relations between the flow rate and the position of the float were studied. The measurement of d_f and θ_1 of the float was performed by means of a universal measuring machine. As a result, the manufacturing accuracy of θ_1 was found to be within $\pm 5'$. The weight and the volume of the float were measured by means of a balance. The physical properties of the working fluids were taken from appropriate physical tables.

3. Experimental results and discussions

As reported before⁽⁶⁾, the general relation for the flow rate characteristics of a float-area-type flow meter employing a float with small ℓ may be written as $Q/\nu d_f = f_0(Re, \beta_1, \gamma_f)$, which reduces, in the region of Re above the transition characteristic Reynolds number Re_c , to

$$\frac{Q}{\nu d_f} = C_1 4\sqrt{2\pi} f(\beta_1, \gamma_f) Re^{C_2} \quad (2)$$

In Eq. (2), $f(\beta_1, \gamma_f)$ is influenced by the ex-

istence of a stabilizing shaft of the type shown in Fig. 1 (c), so that a correction for the decrease in the effective cross sectional area of the float due to the shaft is made to give

$$f(\beta_1, \gamma_f) = \frac{\beta_1(1+2\beta_1)}{\sqrt{\left[1 - \frac{(1+2\beta_1)^2 (8\beta_1)^2}{\{(1+4\beta_1)^2 - \gamma_f^2\}^2}\right]} (1-\gamma_f^2)} \quad (3)$$

Similarly, Re is written as⁽⁸⁾

$$Re = \left[\frac{W - F}{\nu^2 \rho (1 - \gamma_f^2)} \right]^{1/2} \quad (4)$$

where $\beta_1 = (D_t - d_f)/4 d_f$, D_t denotes the inner diameter of the tapered tube at the minimum annular clearance at the equilibrium position of the float, $\gamma_f = d_s/d_f$, ν and ρ denote the kinematic viscosity and the density of the working fluid, respectively, W the weight of the float, F the buoyant force acting on the float and C_1 and C_2 the correction factors for the theoretical results depending on the values of θ_1 and θ_2 , respectively.

3.1 Relations between C_1 and C_2 in Eq. (2) and θ_1

In the first place, the relations between the correction factors C_1 and C_2 in Eq. (2) and θ_1 are explored experimentally. Figure 2 shows the experimental results obtained by the following procedures. Combining various floats, machined

Fig. 2 Relations between θ_1 and both C_1 and C_2

with respective values of θ_1 as mentioned above, with the tapered tubes listed in Table 1, the relations between the flow rate and the position of the float are obtained for $Re > Re_c$ and $10^{-2} < \beta_1 < 5 \times 10^{-2}$. The results are plotted on the logarithmic scales with $Q/\nu d_f$ and Re as rectangular coordinates and with β_1 as parameter. Since the experimental data for a fixed value of β_1 lie along a straight line, the value of C_2 is obtained from the gradient of the latter. Substituting C_2 into Eq. (2), C_1 is obtained as an arithmetic mean of a number of experimental values. It is seen from Fig. 2 that C_1 and C_2 are increase and decrease, respectively, with an increasing θ_1 . Though a more precise hydrodynamic study is required for making these phenomena clearer, the general flow rate characteristics of the float-area-type flow meter may be semi-empirically assessed from Fig. 2 and Eq. (2) when employing the float with small ℓ .

3.2 The value of θ_1 giving a constant discharge coefficient and the semi-empirical formula for estimating the flow rate characteristics

As seen from Fig. 2, C_2 becomes unity when employing the floats with small l and machined with $\theta_1 = 96^\circ$. In this particular case, by equating Eq. (1) and Eq. (2), we obtain

$$C = \frac{C_1}{\sqrt{\left[1 - \frac{(1+2\beta_1)^2 (8\beta_1)^2}{\{(1+4\beta_1)^2 - \gamma_f^2\}^2}\right] (1-\gamma_f^2)}} \quad (5)$$

and, therefore, the discharge coefficient C defined by Eq. (1) may be expressed as a function of β_1 and γ_f . As seen from the definition (Eq. (1)), the constancy of the discharge coefficient means that the flow rate characteristics, i.e. the relation between the flow rate and the pressure difference in the case of differential-pressure-type flow meters, is unaffected by a change in the kinematic viscosity of the working fluid. Therefore, the float-area-type flow meter independent of the change in the kinematic viscosity of the working fluid can be manufactured only when employing a float having $\theta_1 = 96^\circ$. Moreover, as seen from Fig. 2, when employing a widely used float having $\theta_1 = 180^\circ$, the discharge coefficient C is a function of β_1 , γ_f and Re .

As mentioned above, the relation expressed by Eq. (5) may be applied to the case where the shaft of the type as shown in Fig. 1 (c) is employed. On the other hand, with one of the type as shown in Fig. 1 (b), we may put $\gamma_f = 0$ in Eq. (5), since the decrease of the pressure force acting on the float is approximately eliminated, so that C in Eq. (5) may be regarded as a function of β_1 only. Figure 3 shows the influence of γ_f on Q/vd_f calculated from

Fig. 3 Effect of existence of the shaft on the flow rate characteristics

Eq. (2) when $Re > Rec$. The ordinate E in Fig. 3 is defined by

$$E = \frac{(Q/vd_f) - (Q/vd_f)_0}{(Q/vd_f)_0} \quad (6)$$

where (Q/vd_f) denotes the value of Q/vd_f for $\gamma_f = 0$. As seen from Fig. 3, the values of E are not affected so much by β_1 and the change of Q/vd_f for varying values of γ_f may easily be estimated. Since it tells us the range of γ_f for which Eq. (5) is regarded as a function of β_1 alone according to the measuring accuracy of the flow rate, Fig. 3 serves as a standard in determining the diameter of the shaft when adopting the float of the type shown in Fig. 1 (c).

As described in the previous section, the relations shown in Fig. 2 were obtained for $10^{-2} < \beta_1 < 5 \times 10^{-2}$, which corresponds to

widely-used conditions in the float-area-type flow meters. In order to study more precisely the limits of applicability of Eq. (5), we put $C_2 = 1$ in Eq. (2) and obtained the relations between C_1 and β_1 for typical values of Re in the region of $Re > Rec$, which are given in Fig. 4. As seen from Fig. 4, Eq. (2) may be rewritten in the range $10^{-2} < \beta_1 < 6 \times 10^{-2}$, to an accuracy of $\pm 2\%$, as

Fig. 4 Relations between C_1 and β_1

$$\frac{Q}{vd_f} = 0.771 \times 4\sqrt{2\pi} f(\beta_1, \gamma_f) Re \quad (7)$$

Therefore, it becomes evident that the general flow rate characteristics of the float-area-type flow meters can be estimated semi-empirically and that the discharge coefficient can be obtained by putting $C_1 = 0.771$ in Eq. (5), when using a float with small l and having $\theta_1 = 96^\circ$. Moreover, as seen from Fig. 4, Eqs. (5) and (7) may be modified into a more accurate form that applies even outside the range $10^{-2} < \beta_1 < 6 \times 10^{-2}$, by putting C_1 as a function of β_1 by fixing the value of γ_f . We will, however, just point out this possibility here.

In order to verify the values of Rec experimentally, we show the general flow rate characteristics in Fig. 5, where the solid lines show the semi-empirical values calculated from Eq. (7) by

Working fluid, Water Working fluid, Air

Fig. 5 General flow rate characteristics

putting $r_f = 0.16$. As seen from Fig. 5, Re_c is a function of β_1 . Here, if we assume the lower limit of β_1 to be 10^{-2} , $Re_c \approx 3 \times 10^3$ and it becomes evident that the range of applicability of Eq. (7) is $3 \times 10^3 < Re < 1.65 \times 10^5$, the last value being the maximum characteristic Reynolds number in this experiment.

The foregoing experimental results have determined the geometrical shape of a float which gives a constant value of the discharge coefficient of the float-area-type flow meter. Also, a semi-empirical formula for estimating the flow rate characteristics has been obtained. Thus, when such floats are employed, the density of the working fluid is the only quantity that varies with varying working conditions. This fact greatly simplifies the associated equipment including computing devices required when we want to have an automatic correction apparatus for the flow meter or to construct an indirect-type mass-flow meter using float-area-type flow meters.

There are cases where the discharge coefficient defined by Eq. (1) is found seemingly constant even when we employ a Reynolds number defined in a different way from here (see, e.g., Reference (1)). However, the discharge coefficient obviously changes when we vary the working conditions in those cases, and we have to be cautious.

In summarizing this study, we show the value of C calculated from Eq. (5) in Fig. 6. In calculating C , the value of C_f in Eq. (5) was not

Fig. 6 Relations between the discharge coefficient and β_1

fixed at 0.771 but was obtained from Fig. 4 for each value of β_1 and the value of r_f was taken tentatively as 0.16. As is evident from Fig. 3, however, Q/vd_f is little influenced by r_f , so that Fig. 6 is considered to have a wide applicability. Therefore, it becomes evident from Fig. 6 that the discharge coefficient of the float-area-type flow meter, when using a float with small l and with vertex angle $\theta_f = 96^\circ$ and employing the type of a float shown in Fig. 1 (c), is estimated to be 0.788 within an accuracy of $\pm 1\%$ irrespective of Re and β_1 for $3 \times 10^3 < Re$ and $10^{-2} < \beta_1 < 6 \times 10^{-2}$.

4. Conclusions

In an effort to improve of the measuring accuracy of the float-area-type flow meter, an investigation was performed on the discharge coefficient of the latter and the following results were obtained.

- (1) It was pointed out that the discharge coefficient of the float-area-type flow meter was generally not constant for varying working conditions. It was also shown experimentally that the discharge coefficient changed appreciably with the vertex angle of the cone portion upstream of the maximum-diameter part of the float.
- (2) It was found experimentally that the discharge coefficient remained constant over a wide range of characteristic Reynolds numbers only in the case where a float with small l and a particular vertex angle $\theta_f = 96^\circ$ was employed and the flow in the annular clearance between the tapered tube and the float was in the turbulent region. A semi-empirical formula was proposed which might be used to estimate the values of the discharge coefficient within an accuracy of $\pm 2\%$. Furthermore, the range of applicability of the formula was examined experimentally.
- (3) The discharge coefficient of a float-area-type flow meter with a shaft as shown in Fig. 1 (c) was predicted to be 0.788 within an accuracy of $\pm 1\%$ irrespective of Re and β_1 over a wide range of values of the two parameters, and this was verified by experiments carried out with water and air as the working fluids.

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