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Thermoelastic Behavior of a Cylindrical Shell Subjected to an Instantaneous Hot Spot*

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As an example of the non-stationary problems of thermoelasticity for shells, a thin cylindrical shell of finite length subjected to an instantaneous plane hot spot at some point is discussed. Regarding the intensity of a hot spot at some point in the initial state as a delta function and applying Fourier transform, the temperature distribution is determined by solving the non-stationary equation of heat conduction for a shell. The associated thermal displacements and stresses due to this temperature distribution are obtained in the form of double Fourier series by employing the differential equation for thermoelasticity of the cylindrical shell introduced by K.Mizoguchi. The numerical calculations are carried out for a cylindrical shell subjected to an instantaneous plane hot spot at its center, and the temperature distribution, the thermal deflection, and the thermal membrane and bending stresses with the lapse of time are shown in the figures.

1. Introduction

Solutions due to an instantaneous hot spot may be considered to be Green's function for the non-stationary thermoelastic problems and applied widely to the practical ones. Some investigations on thermal stresses in plates (or disks) and spheres under a hot spot have been published⁽¹⁾, but as for the shell structures, although the problem in the presence of a heat source is rather important, there are few studies even for a thin cylindrical shell.

Flügge-Conrad⁽²⁾, Jahanshahi⁽³⁾ and the present author⁽⁴⁾, who discussed on cylindrical shells under a hot spot, have treated this kind of problem as the so-called quasi-static one. Mizoguchi⁽⁵⁾ obtained the solutions for an infinitely long cylindrical shell with a moving hot spot along the circumference by taking into consideration the cooling effect.

It has been pointed out in the static problems that a cylindrical shell, especially a comparatively short one, shows interesting and very important behaviors of displacements and stresses, which might not be expected from our common sense⁽⁶⁾.

Taking the above-mentioned things into consideration, a non-stationary thermoelastic problem of a thin cylindrical shell of finite length is treated in this paper, for the case that a cylindrical shell is heated instantly by a hot spot and immediately afterwards cooled naturally. The temperature distribution is first determined in consideration of temperature

field in the initial state by using the non-stationary equation of heat conduction for a shell and regarding the intensity of a hot spot as a delta function. The associated thermal displacements and thermal stresses due to this temperature distribution are obtained by applying Fourier transform to the fundamental equations for thermoelasticity of the cylindrical shell.

Numerical examples are shown for a comparatively short cylindrical shell made of mild steel and the variation of the temperature distribution, the thermal deflection and the thermal stresses along any cross section with the lapse of time are made clear.

The problem of a plane hot spot is dealt with in the present paper.

2. Theoretical analysis

Let us consider a thin cylindrical shell of finite length l , which is locally heated by a plane hot spot at $x = \xi$, a distance from the left end of a cylinder, at time $t = 0$ instantly and immediately afterwards cooled naturally (Fig.1). The method of analysis for the equation of heat conduction for shell and the derivation of the thermal displacements and the thermal stresses will be described in the following.

Fig.1 Cylindrical shell subjected to instantaneous hot spot

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2.1 Temperature distribution

It is assumed that the temperature state of the surroundings on the inner and the outer surfaces of a shell and the heat transfer conditions on both surfaces are the same. Since it seems that a plane hot spot does not cause any temperature gradient through the thickness, the equation of heat conduction for the cylindrical shell may be given in the case of the mean temperature rise T as follows^{(7), (8)}:

$$\frac{\partial T}{\partial t} - \chi \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{a^2 \partial \theta^2} \right] + \frac{2\beta}{c\varphi\kappa} (T - \Theta) = 0 \quad (1)$$

where t : time, Θ : temperature of the surrounding medium, c : specific heat of cylindrical material, φ : density of cylindrical material, β : heat transfer coefficient, $\chi = \lambda / c\varphi$: coefficient of heat conduction, a : radius of cylindrical shell, κ : thickness of shell.

The equation of heat conduction (1) is solved under the following conditions.

(i) Initial condition ($t=0$);

A plane hot spot with the intensity Q generates at any point $(\xi, 0)$ on a cylindrical shell.

(ii) Boundary condition;

At both ends of a cylindrical shell the temperature rise is equal to the temperature state of the surrounding medium because of the goodness of heat transfer, that is, at $x=0$ and $x=l$ $T - \Theta = 0$. The analysis is made under the above conditions without prescribing the temperature gradient at both ends of the cylinder. However, it will be shown later that the solutions obtained in this analysis may be also applicable for the case that the temperature gradient at both ends is zero, except that a hot spot comes into existence in the proximity of the end of the cylinder (refer to Fig.4 (d)).

In order to find the solution of Eq. (1), we put

$$T - \Theta = T_n(t) X_n(x) \Theta_n(\theta) \quad (2)$$

Substituting Eq.(2) into Eq.(1), we obtain

$$\frac{1}{\chi T_n} \frac{dT_n}{dt} - \frac{1}{X_n} \frac{d^2 X_n}{dx^2} - \frac{1}{a^2 \Theta_n} \frac{d^2 \Theta_n}{d\theta^2} + \frac{2\beta}{\lambda \kappa} = 0$$

In the above equation, if we put

$$(1/\chi T_n) (dT_n/dt) \equiv -g_n^2 \quad (3)$$

$$(1/\Theta_n) (d^2 \Theta_n/d\theta^2) \equiv -n^2 \quad (4)$$

the following expression can be obtained.

$$\frac{1}{X_n} \frac{d^2 X_n}{dx^2} + g_n^2 - \frac{n^2}{a^2} - \frac{2\beta}{\lambda \kappa} = 0 \quad (5)$$

where g_n is the coefficient determined later from the boundary condition and n is taken as integer values, because θ is a periodic function. The solutions of Eqs. (3)~(5) can be readily found as

$$\left. \begin{aligned} T_n(t) &= c' \exp(-\chi g_n^2 t) \\ X_n(x) &= A_n' \cos K_n x + B_n' \sin K_n x \\ \Theta_n(\theta) &= A_n \cos n\theta + B_n \sin n\theta \end{aligned} \right\} \quad (6)$$

where $K_n = \sqrt{g_n^2 - n^2/a^2 - 2\beta/\lambda \kappa}$, c' , A_n , B_n , A_n' and B_n' are the constants of integration. Therefore from the expression (2), the equation of the temperature rise is given for the general problem as follows:

$$T - \Theta = \sum_{n=0}^{\infty} c' \exp(-\chi g_n^2 t) (A_n' \cos K_n x + B_n' \sin K_n x)$$

$$\times (A_n \cos n\theta + B_n \sin n\theta)$$

Now, as the solution of our present problem the following equation is adopted.

$$T - \Theta = \sum_{n=0}^{\infty} A_n \exp(-\chi g_n^2 t) \sin K_n x \cos n\theta \quad (7)$$

From the boundary condition (ii) we find

$$\sin K_n l = 0$$

Therefore

$$K_n = m\pi/l \quad m = 1, 2, 3, \dots \quad (8)$$

If we write g_{mn} for g_n on account of the dependence on the integer m of g_n , we find from Eq.(6)

$$g_{mn}^2 = (m\pi/l)^2 + (n/a)^2 + 2\beta/\lambda \kappa \quad n = 0, 1, 2, 3, \dots \quad (9)$$

A_{mn} depends also on m , and Eq.(7) becomes

$$T - \Theta = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} A_{mn} \exp(-\chi g_{mn}^2 t) \sin \frac{m\pi x}{l} \cos n\theta \quad (7')$$

A_{mn} is determined from the initial condition (i). Let us consider an infinitely long cylindrical shell as shown in Fig.2, in which the boundary condition (ii) is automatically satisfied.

$$A_{mn} = \frac{2}{\pi l} \int_0^l \int_0^\pi (T - \Theta)_{t=0} \sin \frac{m\pi x}{l} \cos n\theta dx d\theta$$

We regard a hot spot with the intensity Q at $(\xi, 0)$ as the delta function.

$$(T - \Theta)_{t=0} = Q \delta(x - \xi) \delta(\theta) / a$$

Substituting this expression into the above equation and using the relation of the delta function of

$$\int_0^l f(x) \delta(x - \eta) dx = f(\eta) \quad ,$$

we obtain

$$\begin{aligned} A_{mn} &= \frac{2Q}{\pi l a} \int_0^\pi \delta(x - \xi) \delta(\theta) \sin \frac{m\pi x}{l} \cos n\theta dx d\theta \\ &= \frac{2Q}{\pi l a} \sin \frac{m\pi \xi}{l} \end{aligned}$$

(This relation may be found by the same procedure as in the case of the statical concentrated load such that the intensity Q of a hot spot is defined as the limiting case of $A_0 \rightarrow 0$ and $T_0 \rightarrow \infty$, where A_0 is a small area element on a cylindrical shell and T_0 distributed on A_0 is the mean temperature rise through the thickness of shell). Therefore, taking into consideration the term of time and using the relation $A_{m0} = (A_{mn})_{n=0} / 2$, Eq.(7)' becomes

$$\begin{aligned} T - \Theta &= \frac{Q}{\pi l a} \sum_{m=1}^{\infty} \exp(-\chi g_{m0}^2 t) \sin \frac{m\pi \xi}{l} \sin \frac{m\pi x}{l} \\ &+ \frac{2Q}{\pi l a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \exp(-\chi g_{mn}^2 t) \sin \frac{m\pi \xi}{l} \sin \frac{m\pi x}{l} \cos n\theta \end{aligned} \quad (10)$$

This is an expression of the temperature rise after time t for a plane hot spot at a point $(\xi, 0)$.

Fig.2 Imagined infinitely long cylindrical shell

2.2 The relation between the intensity of hot spot and heat quantity

A heat quantity stored in a cylindrical shell after time t may be written as

$$M = \iiint_V c \rho (T - \Theta) dV = \int_0^l \int_{-\pi}^{\pi} c \rho (T - \Theta) R a dx d\theta \quad (11)$$

Substituting Eq.(10) into Eq.(11), we obtain after the performance of integration

$$M = \frac{4c\rho RQ}{\pi} \sum_{m=1,3,5,\dots}^{\infty} \exp(-\chi g_m^2 t) \frac{\sin m\pi x/l}{m} = \frac{4c\rho RQ}{\pi} \exp(-2kt/c\rho R) \sum_{m=1,3,5,\dots}^{\infty} \exp[-\chi(m\pi/l)^2 t] \frac{\sin m\pi x/l}{m} \quad (12)$$

Equation(12) gives the total heat quantity stored in a cylindrical shell, which decreases gradually as the time elapses and becomes zero as the time tends to infinity. A heat quantity M_0 given for the initial state is obtained by putting $t=0$ in Eq.(12).

$$M_0 = \frac{4c\rho RQ}{\pi} \sum_{m=1,3,5,\dots}^{\infty} \frac{\sin m\pi x/l}{m} = c\rho RQ \quad \therefore Q = M_0/c\rho R \quad (13)$$

This gives the relation between the intensity of heat source and the heat quantity in the initial state. A heat quantity M and the temperature rise $T - \Theta$ may be also written with M_0 as

$$M = \frac{4M_0}{\pi} \exp(-2kt/c\rho R) \sum_{m=1,3,5,\dots}^{\infty} \exp[-\chi(m\pi/l)^2 t] \frac{\sin m\pi x/l}{m} \quad (12)'$$

$$T - \Theta = \frac{M_0}{\pi l a c \rho R} \exp(-2kt/c\rho R) \sum_{m=1}^{\infty} \exp(-\chi u^2 t/a^2) \sin \frac{m\pi x}{l} \sin \frac{m\pi \chi}{l} [1 + 2 \sum_{n=1}^{\infty} \exp(-\chi n^2 t/a^2) \cos n\theta] \quad (10)'$$

where $u = m\pi a/l$, $m = 1, 2, 3, \dots$, $n = 1, 2, 3, \dots$

2.3 Thermal displacements and thermal stresses

A cylindrical shell is assumed to be supported simply at both ends, that is, the boundary condition is at $x=0$ and $x=l$ (refer to Fig.3)

$$W = 0, \quad U = 0, \quad T_x = 0, \quad M_x = 0$$

The fundamental differential equation in terms of the thermal deflection W for thermoelasticity of the cylindrical shell may be written for the mean temperature rise T (corresponding to $T - \Theta$ in Eq.(10))⁽⁷⁾.

$$\begin{aligned} A \frac{\partial^8 W}{\partial x^8} + \frac{4B}{a^2} \frac{\partial^6 W}{\partial x^6 \partial \theta^2} + \frac{6C}{a^4} \frac{\partial^4 W}{\partial x^4 \partial \theta^4} \\ + \frac{8-2\nu^2}{a^4} \frac{\partial^4 W}{\partial x^4 \partial \theta^2} + \frac{6L}{a^4} \frac{\partial^4 W}{\partial x^4} \\ + \frac{1}{a^6} \frac{\partial^2}{\partial \theta^2} \left(4 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \theta^2} \right) \left(\frac{\partial^2}{\partial \theta^2} + 1 \right) W \\ = \frac{12(1+\nu)\alpha}{a R^2} \left[\frac{(2-\nu)\epsilon^2}{12} \frac{\partial^6 T}{\partial x^6 \partial \theta^2} + \frac{(3-\nu)\epsilon^2}{12 a^2} \frac{\partial^6 T}{\partial x^2 \partial \theta^4} \right. \\ \left. + \frac{\epsilon^2}{12 a^4} \frac{\partial^6 T}{\partial \theta^6} + (1-\nu)A \frac{\partial^4 T}{\partial x^4} + \frac{(1-\nu)\Gamma'}{a^2} \frac{\partial^4 T}{\partial x^2 \partial \theta^2} \right. \\ \left. + \frac{\epsilon^2}{12 a^4} \frac{\partial^4 T}{\partial \theta^4} \right] \quad (14) \end{aligned}$$

where α : linear coefficient of thermal expansion of the material, ν : Poisson's ratio, $\epsilon = R/a$, $[A, B, C, \Gamma'] = 1 + \epsilon^2 [1/3, 1/12, (1-\nu^2)/72, (2-\nu)/6(1-\nu)]$, $L = 2(1-\nu^2)A/\epsilon^2$.

In order to obtain the solutions satisfying the boundary conditions of simply supported ends, an infinitely long cylindrical shell, as shown in Fig.2, is again considered. A double Fourier transform is applied to both sides of Eq.(14).

$$\bar{W}(m, n) = \int_{-\pi}^{\pi} \int_0^l W(x, \theta, t) \sin \frac{m\pi x}{l} \cos n\theta dx d\theta$$

Since $T(x, \theta, t)$ is given in Eq.(10)' in the form of $T - \Theta$, we get

$$\int_{-\pi}^{\pi} \int_0^l T(x, \theta, t) \sin \frac{m\pi x}{l} \cos n\theta dx d\theta = \frac{2M_0}{c\rho R} \exp[-\{(2k/c\rho R) + (\chi u^2/a^2) + (\chi n^2/a^2)\}t] \sin \frac{m\pi x}{l}$$

Applying these expressions to Eq.(14),

$$\begin{aligned} \left[A \left(\frac{m\pi}{l} \right)^8 + \frac{4B}{a^2} \left(\frac{m\pi}{l} \right)^6 n^2 + \frac{6C}{a^4} \left(\frac{m\pi}{l} \right)^4 n^4 - \frac{(8-2\nu^2)}{a^4} \left(\frac{m\pi}{l} \right)^4 n^2 + \frac{6L}{a^4} \left(\frac{m\pi}{l} \right)^4 + \frac{n^2}{a^6} \left\{ 4 \left(\frac{m\pi}{l} \right)^2 + \left(\frac{n}{a} \right)^2 \right\} (n^2 - 1)^2 \right] \bar{W}(m, n) \\ = \frac{2M_0}{c\rho R} \exp[-\{(2k/c\rho R) + (\chi u^2/a^2) + (\chi n^2/a^2)\}t] \sin \frac{m\pi x}{l} \end{aligned}$$

From this $\bar{W}(m, n)$ can be calculated. Since the inversion formula is given as

$$W(x, \theta, t) = \frac{1}{2\pi l} \sum_{m=1}^{\infty} \bar{W}(m, 0) \sin \frac{m\pi x}{l} + \frac{1}{\pi l} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \bar{W}(m, n) \sin \frac{m\pi x}{l} \cos n\theta,$$

we finally obtain the expression for the thermal deflection.

$$\begin{aligned} W(x, \theta, t) = \frac{12(1+\nu)\alpha a^2 M_0}{\pi l c \rho R^3} \exp(-2kt/c\rho R) \sum_{m=1}^{\infty} \exp(-\chi u^2 t/a^2) \sin \frac{m\pi x}{l} \sin \frac{m\pi \chi}{l} \\ \times \left[\frac{1-\nu}{u^4 + 6L'} + 2 \sum_{n=1}^{\infty} \frac{\exp(-\chi n^2 t/a^2)}{H} \left[-\frac{\epsilon^2 n^2}{12} \{ (2-\nu)u^4 + (3-\nu)u^2 n^2 - n^2(n^2-1) \} \right. \right. \\ \left. \left. + (1-\nu)u^2(Au^2 + \Gamma'n^2) \right] \cos n\theta \right] \quad (15) \end{aligned}$$

where

$$L' = 2(1-\nu^2)/\epsilon^2, \quad u = m\pi a/l$$

$$H = A u^8 + 4B u^6 n^2 + 6C u^4 n^4 - (8-2\nu^2) u^4 n^2 + 6L u^4 + n^2(4u^2 + n^2)(n^2-1)^2$$

Fig.3 Displacements, stress resultants and couples on shell element

The components of axial and circumferential displacement u and v are obtained from $w \sim u$ and $w \sim v$ relations under the temperature rise $T^{(4), (5)}$.

$$u = -\frac{(1+\nu)\alpha M_0}{\pi l c \rho h} \exp(-2\beta t/c\rho h) \sum_{m=1}^{\infty} \exp(-\chi \mu^2 t/a^2) \sin \frac{m\pi x}{l} \cos \frac{m\pi x}{l} \times \left[\frac{\mu^4 + 6L''}{\mu(\mu^4 + 6L')} + 2 \sum_{n=1}^{\infty} \frac{\exp(-\chi n^2 t/a^2)}{H} \mu [A\mu^6 + 3A''\mu^4 n^2 + 3\mu^2 n^4 + n^6 - 3(2-\nu)\mu^2 n^2 - 3n^4 + 6L''A\mu^2 + 2n^2] \cos n\theta \right] \quad (16)$$

(a)

(b)

(c) $a/h = 50$ (d) $a/h = 50$

Fig.4 Temperature distribution

$$U = \frac{2(1+\nu)\alpha M_0}{\pi \ell c \rho R} \exp(-2\ell t/c\rho R) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\exp\{-\chi(\mu^2+n^2)t/a^2\}}{H} n[(\mu^2+n^2)^3 - 2(2-\nu)\mu^4 - (4-\nu)\mu^2 n^2 - n^4 - 6L''\mu^2] \sin \frac{m\pi \xi}{\ell} \sin \frac{m\pi x}{\ell} \sin n\theta \quad (17)$$

$$A'' = 1 + (1+\nu)\varepsilon^2/18, \quad L'' = 2(1-\nu)/\varepsilon^2$$

The components of stress-resultants and -couples in thermoelastic problems for the cylindrical shell under the temperature rise T may be expressed as⁽⁷⁾ (refer to Fig.3).

$$T_x = (12D/R^2) \left[\frac{\partial U}{\partial x} + \nu \left(\frac{\partial V}{\partial \theta} + W \right) / a - (1+\nu)\alpha T \right]$$

$$T_\theta = (12D/R^2) \left[\nu \left(\frac{\partial U}{\partial x} \right) + \left(\frac{\partial V}{\partial \theta} + W \right) / a - (1+\nu)\alpha T \right]$$

$$S = \{6(1-\nu)D/R^2\} \times [\partial U / \partial \theta + \partial V / \partial x]$$

$$M_x = D \left[\frac{\partial^2 W}{\partial x^2} + \nu \left(\frac{\partial^2 W}{\partial \theta^2} - \frac{\partial V}{\partial \theta} \right) / a^2 \right]$$

$$M_\theta = D \left[\nu \left(\frac{\partial^2 W}{\partial x^2} \right) + \left(\frac{\partial^2 W}{\partial \theta^2} - \frac{\partial V}{\partial \theta} \right) / a^2 \right]$$

$$N_x = \{(1-\nu)D/a\} \times [\partial^2 W / \partial x \partial \theta - \partial V / \partial x]$$

$$S' = S - N_x / a$$

$$N_x = \partial M_x / \partial x + \partial N_x / \partial \theta$$

$$N_\theta = \partial M_\theta / \partial \theta + \partial N_x / \partial x$$

(18)

The quantities obtained from these relations, which are omitted here for brevity's sake, satisfy all the boundary conditions of simply supported ends, and are all uniform convergent series at time $t > 0$, which make it easy to perform the numerical computations. In the initial state, $t=0$, however, even the thermal deflection w in Eq.(15) becomes no longer finite at the point of application of a hot spot, which is the so-called thermal singularity. Other displacements, the stress-resultants and -couples come to have much higher order of singularities than in the case of the static loadings, which teaches us the difficulty to obtain the numerical results. A treatment of such a problem has been already published by the author⁽⁴⁾, who dealt with the problem as a quasi-static one. Since the convergence of series becomes slow during the short time after the initial state, it must be advantageous to know the values at time $t=0$.

3. Numerical examples

Numerical examples are presented for a cylindrical shell with a plane hot spot at the center of the cylinder ($\xi=l/2$). Two comparatively short cylindrical shells are selected and their dimensions are; $\ell/a=4$, $a/R=50$ and 100 , $a=1$ m. The material constants made of mild steel are $\nu=0.3$, $R=20$ kcal/m²R°C, $\lambda=50$ kcal/m R°C; $t'=(\chi/a^2)t$ containing material constants is adopted as the time parameter.

(a)

(b)

(c)

Fig.5 Thermal deflection

The nondimensional values for the temperature distribution, the thermal deflection and the thermal membrane and bending stresses are given in the forms:

$$\left. \begin{aligned} \text{temperature distribution:} \\ T' = (T - \Theta) / (M_0 / \pi l a c \rho h) \\ \text{thermal deflection:} \\ w' = w / \{ (1 + \nu) \alpha M_0 / \pi l c \rho h \} \\ \text{thermal membrane stress:} \\ (t_x, t_\theta) = (T_x, T_\theta) / (E \alpha M_0 / \pi l a c \rho) \\ \text{thermal bending stress:} \\ (m_x, m_\theta) = (M_x, M_\theta) / (E \alpha M_0 \epsilon / 6 \pi l c \rho) \end{aligned} \right\} \dots (19)$$

The dimensionless values of the surface stresses can be directly obtained from the above expressions.

$$\left. \begin{aligned} t_x - m_x, \quad t_\theta - m_\theta \quad (\text{outer surface}) \\ t_x + m_x, \quad t_\theta + m_\theta \quad (\text{inner surface}) \end{aligned} \right\} \dots (20)$$

3.1 Temperature distribution

The numerical results of the temperature distribution are shown in Figs.4 (a)~(d). Figs.(a) and (b), which represent the temperature distribution with the lapse of time at each of the cross sections, make it clear that the temperature falls at the point of application of a hot spot, being infinity in the initial state, and that the temperature of the initially unheated portions varies with the lapse of time according to the heat conduction. Figs.(c) and (d) represent the temperature distribution ($a/h=50$) with time parameter t'

along the circumference and generator. From Fig.(d) it is seen that the temperature gradient in the axial direction of the cylinder may be considered not to be so large, because for short time t' it falls rapidly in the vicinity of a hot spot and for long time t' the ratio of change of the temperature is so small. This fact verifies the validity of the boundary condition (ii) and the type of solution (7) in the course of analysis, excepting a hot spot being very near the end of the cylinder.

3.2 Thermal deflection

Figs.5 (a) and (b) represent the thermal deflection with the lapse of time at each cross section of the circumference and the generator through the point of application of a hot spot. The values at the initial state $t'=0$ come from those in the quasi-static state in reference (4), in which it is pointed out that the deflection at the heated point becomes infinitely large. From these figures the behavior of the deflection at the heated and the unheated portions can be understood. Fig.5 (c) shows the variation of the thermal deflection along the circumference through the point of the hot spot for any chosen time t' . We recognize that a cylindrical shell deflects inwards at a small distance from the point of the hot spot, although it does of course deflect outwards in the vicinity of the hot spot. This phenomenon of hollow may be considered to be of great importance in the practical problem, such as the fatigue failure, in relation to the position and the magnitude of hollow with respect to a/h and time t'

3.3 Thermal stresses

The membrane stresses t_x , t_θ and the bending stresses m_x , m_θ are shown in Figs.6 (a), (b) and Figs.7 (a), (b), respectively. In any circumferential cross section at a small distance from the point of the hot spot, the tensile stress occurs in a cylindrical shell, although it is evident that a compressive stress occurs at the heated point. Now, if we turn to the stress at time $t'=0.001$, it will be seen that, as for the maximum value of the tensile stress, t_x is much greater than other stresses and the position of it is rather near the heated point. Comparing the maximum tensile stresses among all for the ratio of $a/h=50$ and 100, we find that those for t_θ , m_x and m_θ are much larger for $a/h=100$ than for $a/h=50$, and their position comes

Fig.6 Thermal membrane stress

closer to the heated point, while the value and the position of the maximum tensile stress t_x do not make any remarkable change for a/h . The effect of a hot spot on the behavior of the stresses may be so small in the circumferential cross section $\theta \geq 60^\circ$. As has been described in Eq.(20), the surface stresses on a cylindrical shell (dimensionless values) can be readily obtained by direct addition or subtraction of the values already shown in the figures.

4. Conclusions

As an example of the non-stationary thermoelastic problems of thin shell in the presence of a heat source, an analysis was made of a thin cylindrical shell of finite length subjected to a plane hot spot occurring instantly at any point on the cylinder. The method of analysis of the equation of heat conduction and the derivation of the thermal displacements and stresses were described. Numerical calculations, which were carried out for a hot spot generated at the center of the cylinder, made clear the behaviors of the temperature distribution, the thermal deflection and the thermal membrane and bending stresses with the lapse of time in a cylindrical shell with the radius to thickness ratios a/h of 50 and 100. A practically important behavior of the thermal deflection and the thermal stresses occurs in the circumferential cross section in the neighbourhood of a hot spot, where they come to have extreme values with the opposite sign to the value of the heated point. This kind of phenomenon seems to be an important factor for the thermal fatigue failure.

The solutions obtained in this paper may be considered as Green's function of the non-stationary problem of thermoelasticity for a thin cylindrical shell and applied widely to the practical problems.

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(a) m_x

(b) m_θ

Fig.7 Thermal bending stress

3400 at University of Osaka Prefecture.

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