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Transient Creep of Two Dimensional Stress Concentrated Parts*

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The paper is concerned with a transient creep problem of thin plates with stress concentrated parts (notch or hole etc.) subjected to load in their own plane. In analysis of this problem the finite element method is employed. The solutions are obtainable for two hardening creep laws: the time hardening law and the strain hardening law. As numerical examples some problems on thin plates with a hole or a notch are treated, and the difference between the calculating results from two hardening laws is discussed. The results of the prediction method agree well with experimentally determined values for aluminum plates with circular notches at room temperature.

1. Introduction

There are many theoretical and experimental investigations on the creep problem of two-dimensional axi-symmetric structures such as thick walled cylinders⁽¹⁾⁻⁽⁸⁾ and rotating discs⁽⁹⁾⁻⁽¹⁶⁾ because of simplicity. As to general two dimensional creep problems, however, the problems of infinite plates with circular hole⁽¹⁷⁾⁻⁽²²⁾ and the torsional problem of prismatic bar⁽²³⁾ have only been studied.

In the paper the calculating method for a two dimensional creep problem is developed for two hardening creep laws: the time hardening law and the strain hardening law. By the use of this method some problems of thin plates with a hole or a notch are analyzed, and the variations of stresses and strains with time are calculated. Also the difference between the calculation results from two hardening laws is discussed.

There are few experimental studies on creep problems of structures⁽¹⁹⁾. To examine the propriety of the numerical method mentioned above the authors compare the numerical results with the experimentally determined values for aluminum plates with circular notches at room temperature (35°C).

2. Analytical Procedure

The outline of the analytical procedure for the creep problem by a finite element method is given. The relation between the apparent nodal forces $[f_i^*]$ and the nodal displacements $[\delta_i]$ of the element is

$$[f_i^*] = [k][\delta_i] \text{ ----- (1)}$$

where

$$[f_i^*] = [f] + [k][\varepsilon_i]$$

$$[f]: \text{nodal forces}$$

$[k]$: stiffness matrices of the element used in elastic analysis

$[\varepsilon_i]$: creep strains at any time after loading

$$[\varepsilon_i] = \sum_{j=1}^i [d\varepsilon_{ij}]$$

$[\delta_i]$: nodal displacements at any time after loading

$$[\delta_i] = \sum_{j=0}^i [d\delta_{ij}]$$

The strains $[\varepsilon_i]$ and the stresses $[\sigma_i]$ are related to $[\delta_i]$ with the following expression, respectively

$$[\varepsilon_i] = [N][\delta_i] \text{ ----- (2)}$$

$$[\sigma_i] = [D_i][N][\delta_i] - [D_i][\varepsilon_i] \text{ ----- (3)}$$

where $[N]$ is the strain-displacement matrix and $[D_i]$ is the stress-strain matrix used in usual elastic analysis.

Now, if it is assumed that in the transient creep the creep strain in uni-axial state for constant stress may be given by

$$\varepsilon_i = A\sigma^m t^n \text{ ----- (4)}$$

(where A , m , and n are material constants) and Eq.(4) is extended to the plane stress state under the assumptions that the solid is isotropic, that the material obeys the Von Mises flow rule and that the creep strains are incompressible, then the creep strain rates may be written as follows for the time hardening law and the strain hardening law.

Time hardening law: $[\dot{\varepsilon}_i] = [C_i][\sigma] \text{ ----- (5)}$

$$\begin{Bmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ \dot{\gamma}_{xy} \end{Bmatrix} = nA(\sigma_{eq})^{n-1}(t)^{n-1} \begin{Bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 3 \end{Bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

Strain hardening law: $[\dot{\varepsilon}_i] = [C_i][\sigma] \text{ ---- (6)}$

$$\begin{Bmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ \dot{\gamma}_{xy} \end{Bmatrix} = n(A)^{1/n}(\sigma_{eq})^{(n-1)/n}(\varepsilon_{eq})^{(n-1)/n} \times \begin{Bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 3 \end{Bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

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where

$$\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

$$\epsilon_{eq} = \int_0^t \left\{ \frac{4}{3} (\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2 + \dot{\epsilon}_{zz}^2) + \frac{1}{3} \dot{\gamma}_{xy}^2 \right\}^{1/2} dt \quad (7)$$

In Eqs. (4) to (6) A , m and n are obtained from a fit of uniaxial test data.

Thus the numerical solution in a transient creep problem of two dimensional structures proceeds as follows.

Step 1: Determine the elastic solution by vanishing $[\dot{\epsilon}_{ij}]$ in Eqs. (1) to (3).

Step 2: Assume that the stresses obtained in Step 1 remain constant over a very small initial increment of time δt_0 ($=10^{-40}$ hour), and calculate the increments of creep strains, $[d\epsilon_{ij}] = [\dot{\epsilon}_{ij}] \delta t_0$ that occur in the increment time $\Delta t_1 = \delta t_0$ by the use of the time hardening law, Eq. (5). Next by substitution of the increments of creep

strains into Eqs. (1)-(3) and (7) calculate the stresses, the strains and so on at $t = \Delta t_1$ for time hardening law. In the case of using the strain hardening law, Eq. (6), since the creep strain rates $[\dot{\epsilon}_{ij}]$ diverge at $t=0$ because of $\epsilon_{eq}=0$, only in the initial routine Eq. (5) is employed. The propriety of values of δt_0 may be examined by considering the state of convergency of stress distributions.

Step 3: Choose a suitable increment of time Δt and determine the stresses at the end of the interval Δt by using Eqs. (1) to (7). The time increment Δt is controlled as the increments of creep strain for each step become $1/\beta$ of initial elastic strain where the elastic strain is maximum. The values of β in calculation may be decided from the convergency of the solutions.

Fig.1 Shapes of plates used in calculation
(reference stress $\sigma_0 = 4 \text{ Kg/mm}^2$)

Fig.2 Division of plate into triangular elements (Shape A)

Step 4: Continue in this manner until a desired time interval has been achieved or until a steady state has been reached, whichever occurs first. The authors regard it as the steady state when $|\dot{\sigma}_{max}/\sigma_{max}| \leq 10^{-6}$ where σ_{max} is the maximum in the most variational stress component with time.

3. Numerical results and discussions

The shapes of the plates employed in calculation and their material constants are shown in Fig.1 and Table 1, respectively. Figure 2 illustrates one example of the division of plates into triangular elements. Table 2 shows the effect of β on the convergency of values of σ_{ij} and ϵ_{ij} in the element at hole edge near the point A for the case of numerical example 1. It is found from Table 2 that in the case of time hardening law the differences in calculating results when taking 400 and 500 as β are less than 0.3 percent each time after loading and in the case of strain hardening law the differences when employing 200 and 300 as β are less than 0.1 percent. It is

Table 1 Material constants used in numerical examples

Numerical example	Shape	E kg/mm ²	ν	A (kg/mm ²) ^{-m} h ⁻ⁿ	m	n
1	A	7,870	0.3	3.85 10	3.5	0.2
2		21,000	0.3	4.36 10	3,4,5	0.2
3	B	7,600	0.3	4.64 10	4.43	0.273
4	C					
5	D					

Table 2 Effect of β on convergency of solutions(example 1)

(a) Time hardening law

β \ time	10 ⁻⁴	10 ⁻³	10 ⁻²	10 ⁻¹	1	10	10 ²
100	2.098 2 2.512 9	2.029 9 2.575 5	1.944 5 2.665 0	1.843 9 2.792 9	1.734 8 2.973 9	1.627 7 3.229 6	1.534 4 3.593 2
200	2.132 4 2.484 3	2.063 2 2.544 6	1.975 1 2.632 1	1.870 3 2.757 8	1.755 7 2.936 6	1.642 7 3.189 8	1.544 0 3.549 9
300	2.145 7 2.473 6	2.076 0 2.533 1	1.986 8 2.619 9	1.880 4 2.744 9	1.763 7 2.923 0	1.648 4 3.175 3	1.547 6 3.534 3
400	2.153 0 2.467 7	2.083 1 2.526 8	1.993 4 2.613 2	1.885 9 2.737 9	1.768 1 2.915 7	1.651 5 3.167 6	1.549 5 3.526 1
500	2.157 7 2.464 0	2.087 7 2.522 8	1.997 5 2.609 0	1.889 5 2.733 5	1.770 8 2.911 1	1.653 4 3.162 8	

(b) Strain hardening law

β \ time	10 ⁻⁴	10 ⁻³	10 ⁻²	10 ⁻¹	1	10	10 ²
100	2.187 7 2.446 6	2.129 2 2.500 1	2.056 2 2.578 4	1.971 5 2.690 7	1.879 8 2.851 1	1.787 4 3.080 4	1.700 1 3.412 8
200	2.190 9 2.444 1	2.131 9 2.497 7	2.058 7 2.575 7	1.973 7 2.687 8	1.881 7 2.847 7	1.788 8 3.076 8	1.701 1 3.408 7
300	2.192 1 2.443 1	2.133 0 2.496 7	2.059 6 2.574 8	1.974 4 2.686 8	1.882 3 2.846 6	1.789 3 3.075 5	1.701 5 3.407 4
400	2.192 7 2.442 6	2.133 5 2.496 3	2.060 0 2.574 3	1.974 8 2.686 3	1.882 7 2.846 0	1.789 6 3.074 8	1.701 6 3.406 6
500	2.193 1 2.442 3	2.133 8 2.496 0	2.060 3 2.574 0	1.975 1 2.685 9	1.882 9 2.845 7	1.789 7 3.074 6	1.701 8 3.406 2

Upper line: σ_t/σ_0 , Lower line: ϵ_t/ϵ_0
 (σ_0 : mean value of σ_t in section AB, 4 Kg/mm², $\epsilon_0 = \sigma_0/E$)

confirmed from the result of inspection that the above also holds approximately in other elements. Thus in the further numerical calculations the authors employ 400 as β for the time hardening law and 200 for the strain hardening law considering computing time.

Figures 3(a)-(d) illustrate the results of the numerical example 1. Figure (a) shows the variation of stress σ_t in section AB with time. The stress σ_t reduces abruptly at hole edge and its vicinity with the lapse of time, while σ_t increases a little in the range of $x/P \geq 0.35$. The stresses greatly change in initial one hour and in 5000 hours reach the stationary state. The circle in the figure denotes the elastic rigorous solution by Howland.⁽¹²⁾ Figures (b) and (c) show the variations of the stress ϵ_t and the creep strain ϵ_{cr} in section AB with time, respectively. They increase monotonously.

At point A the creep strain ϵ_{cr} in 100 hours takes 56 percent of the strain ϵ_t , and becomes 70 percent until steady state. Figure (d) shows the stresses and the strains from two hardening laws at 100 hours after loading. The difference in the stresses from two hardening laws is about ten percent at the hole edge and the difference in the strains is small.

Figures 4(a)-(e) show how the distribution of σ_t in section AB for example 2 changes with time depending on the values of m . It is evident from Fig. (a)-(d) that the stresses at hole edge reduce rapidly for initial time and the point of the maximum stresses which exist at hole edge in a moment of loading is somewhat displaced from the hole edge with the lapse of time. This tendency is more striking with the values of m as pointed out by Penny and Hayhurst.⁽¹³⁾ Figure(e) shows the distribution of σ_t .

and σ_{xx} in the case of $m=5$. It is found that at 3×10^{-12} hours the point of maximum in σ , moves away from the hole edge while the point of maximum in σ_{xx} stays at the hole edge.

The results of numerical examples 3 to 5 are illustrated in Figs.5. It is seen from Figs.5 that the initial elastic stress concentration factors become smaller in the order of the hyperbolic

(a) σ_y (c) ϵ_{cy} (b) ϵ_y (d) σ_y and ϵ_y at 100 hours

Fig.3 Numerical example 1

notch, the circular hole and the semi-circular notch, and the order does not change up to stationary state while the rates of stress reduction are different

among them. The elastic stress concentration factors by the authors coincide with the results by Howland⁽²³⁾ and Ishida⁽²⁴⁾.

Fig.4 Numerical example 2

Fig.5 Numerical examples 3~5 (Strain hardening law)

4. Experiment

4.1 Test piece

The test pieces used in experiment are shown in Fig.6. Figure(a) shows the round bar with which the creep constants employed in calculation are determined and (b) shows the plate with circular notch made of the same material in which the authors compare the theoretical results with experimental ones. As the material of the test pieces, aluminum is employed the creep deformation of which

Fig.6 Test piece used in experiments

Fig.7 Schematic view of experiments

considerably appears at room temperature. The mechanical properties and the chemical compositions of the materials are shown in Table 3.

Table 3 Material of test pieces

(a) Mechanical properties
(with JIS No.5 test pieces)

Tensile strength	Yield point	Young's modulus
12 kg/mm ²	11 kg/mm ²	7600 kg/mm ²

(b) Chemical compositions (%)

Cu	Si	Fe	Mn	Zn	Al
0.03	Si+Fe<0.03	—	—	—	99.5

4.2 Experimental procedure

The creep test should be performed at a constant temperature, since the creep deformation is significantly influenced by the variation in temperature. Then a constant temperature room (width 1.40 m, height 1.80 m, depth 3.60m) is constructed with plates of heat insulator. The temperature change in experiment is $\pm 0.2^\circ\text{C}$ for set temperature 35°C . A schematic view of experiments is shown in Fig.7.

As shown in Fig.7 the plate test piece is loaded by 2640 kg through a special load apparatus. The values of strains in longitudinal direction at section AA (Fig.6) are measured with strain

gauges with 1 mm gauge length. The authors stick 30 strain gauges on either plane, 2 gauges on the notch edge (point A) shown in Fig.8 and in addition to these 4 gauges on the longitudinal symmetrical line far from the circular notch in order to correct the eccentricity of loading.

Fig.8 Strain gauges stuck on section 'AA'.

4.3 Experimental results

The creep constants of aluminum used in experiment are determined as follows by the tensile creep test.

$$A = 4.64 \times 10^{-8} (\text{kg/mm}^2)^{-4.43} (\text{h})^{-0.273}$$

$$m = 4.43, n = 0.273$$

The creep curves of the material are shown in Fig.9. Figure 10 shows the comparison between the theoretical and the experimental results on the variation of ϵ_p with time at points A, B and C. A, B and C are taken as the points where stress decreases considerably with time, where the variation of stress is small, and where the stress increases with time, respectively. It is seen from Fig.10 that the theoretical results agree fairly well with the experimental ones and at point A where the variation of stress is remarkable the theoretical results based on strain hardening law are nearer to the experimental values than the ones from time hardening law.

Figures 11(a)-(d) show the strain distributions in AA each time after loading. It seems from Figs.11 that the theoretical values based on strain hardening law coincide with the experimental ones.

5. Conclusions

In the paper the authors described the calculating method on the variations of stress and strain due to creep in plane stress concentrating part by the use of finite element method. As numerical examples several plates with a notch or a hole are treated. The propriety of this prediction method is examined by comparison with experimental results.

Fig.9 Creep curve of material

Fig.10 Variation of ϵ_p with time at points A, B and C

It is found from the numerical results and experimental ones that

(1): The creep index m is larger, the variations of stress with time are remarkable and the stresses more rapidly arrived at stationary state. Also the point when the stress reaches maximum is displaced from the hole edge or the notch edge more quickly.

(2): Most of the variations of stress occur in the initial time after loading.

(3): The difference between the results based on time hardening theory and strain hardening theory is a little, and from comparison between the theoretical values and the experimental ones it seems that the results from strain hardening law agree well with experimentally determined values.

In numerical calculations FACOM 230-60 in Data Processing Center, Kyoto University is used.

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Fig.11 Variation of ϵ_y distribution with time