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Gear-Tooth Stress Calculation Method for Heavily Crowned Gear*

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Gear rating is decided by tooth strength, surface durability and scoring resistance of gear pair. In Japan, in order to prevent gear-tooth breakage induced by end tooth bearing, gears are often crowned heavily.

The stress calculation method for this heavily crowned gear is already proposed by one of the authors, but this calculation method is very troublesome. So, it is desirable that a simple and easy stress calculation method for crowned gear be established.

For this reason, the authors proposed a simple and easy calculation method for maximum longitudinal tooth stress of crowned gear using the infinite length cantilever plate theory applied with moment-image method introduced Wellauer, E.J. and Seireg, A.. This simple and easy calculation method is a method by which the maximum gear-tooth stress is obtained as the stress of uniformly loaded spur gear divided by the crowning factor. The crowned gear tooth stress is estimated for the gear-tooth load distribution which is obtained by solving the contact problem about two elastic ellipsoids substituted for meshing gear-tooth.

The authors demonstrated by experiments that this crowned gear-teeth stress calculation method is quite proper. Moreover, they proposed the stress calculation method for a special state of end tooth bearing of crowned gear, and proved the practicability of this method by experiments.

1. Introduction

In order to prevent end tooth bearing, heavily crowned spur gears have been used. To calculate tooth stress of heavily crowned gear, using the tooth bending characteristic, gear-tooth load distribution must be obtained. Hayashi⁽¹⁾⁽²⁾ showed that gear-tooth load distribution can be obtained by solving an integral equation of Fred-Holm through numerical calculation. Umezawa et al.⁽³⁾ showed the deflection and bending moment of a finite length thick cantilever plate by solving a finite-difference equation when a concentrated force is applied on it. Further, using the results obtained by the same method, Umezawa⁽⁴⁾ proposed a practical equation which gives the deflection of gear-tooth and tooth stresses induced by a concentrated force. Fujio and Aida⁽⁵⁾⁽⁶⁾ showed that the deflection of an infinite length cantilever plate with varying thickness can be obtained by using Green function or finite element method. Kagawa⁽⁷⁾ obtained an equation to calculate the deflection and fillet stress of a

finite length cantilever plate when a concentrated force is applied on it.

Each solution of elastic problems shown above can be applied to calculate the tooth stress of gear. But the gear-tooth load distribution factor in the "Formulae for Load Capacity of Cylindrical Involute Gears"⁽⁸⁾ proposed by "The Research Committee about the Design Procedures of Power Transmission Gears" in the Japan Society of Mechanical Engineers is given by applying moment-image method⁽⁹⁾ introduced by Wellauer, E.J. and Seireg, A. to the solution⁽¹⁰⁾ of the elastic problem of a very long cantilever plate under a transverse concentrated force. In Japan, for the time being, in order to put the "formulae for Load Capacity of Cylindrical Involute Gears" to practical use, efforts will be made. On the other hand the calculation method of tooth stresses for crowned gears using the very long cantilever plate theory has been proposed by one of the authors⁽¹¹⁾ and by experiments it has been proved that the accuracy of the method is good enough and it has been utilized, but it is very troublesome.

Recently, heavily crowned gears have been often used as the practicality of them has come to be recognized. And it has been widely desired to establish immediately a simple tooth stress calculation method for crowned gear to which the "Formulae for Load Capacity of Cylindrical Involute Gears" can be applied.

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ed at the fixed end of the plate is obtained by applying moment-image method introduced by Wellauer, E.J. and Seireg, A. to the solution of elastic problem when a concentrated force P is applied perpendicularly on an infinite length cantilever plate. It is given as follows:

$$\sigma = k_t \frac{6P\beta_F(\xi, \eta)}{S^2} \quad \text{----- (7)}$$

Where $\beta_F(\xi, \eta)$ is the coefficient of bending moment distribution at point B, and it is given as follows:

$$\beta_F(\xi, \eta) = \beta_I(\xi - \xi', \eta) + \beta_I(\xi + \xi', \eta) + \beta_I(2\eta - \xi - \xi', \eta) \quad \text{----- (8)}$$

It may safely be said that the tooth load for crowned gear distributes along a straight line in the lengthwise direction of tooth as well as for spur gear. Therefore, when the tooth load distributes along a straight line as shown in Fig. 3, bending stress at point B located at the fixed end of a finite length cantilever plate is given as follows:

$$\sigma = k_t \frac{6}{S^2} \int_{x_0}^{x_1} P(x') \beta_F(\xi, \eta) dx' \quad \text{----- (9)}$$

(a) Infinite length cantilever plate

(b) Finite length cantilever plate
Fig. 2

Fig. 3 Load distribution

3.2 Tooth load distribution

Tooth load distribution of crowned gear can be obtained by using the method which has been already introduced by one of the authors⁽¹⁴⁾. Namely gear-tooth of crowned gear is substituted for an ellipsoid which is formed by rotating an ellipse about the major axis, the radius of semi-minor axis in said ellipse being equal to the radius of curvature of transverse tooth profile at the contact point and the radius of curvature at the end point of minor axis being equal to the radius of curvature of crowning. And it is assumed that the tooth load distribution of crowned gear on the contact surface is given as the solution of contact problem about two elastic ellipsoids. In this method, as the tooth load distribution is obtained by using Hertzian contact theory about two elastic bodies, so the form of contact area is an ellipse and the surface constructed by enveloping the end point of a perpendicular line having the length proportional to the distributed load forms an ellipsoid surface as shown in Fig. 4(a). If the semi-major axis is ρ_x , the semi-minor axis is ρ_y and the maximum contact stress is p_{0c} , then the distributed load on the tooth surface is given as follows:

$$\frac{x^2}{\rho_x^2} + \frac{y^2}{\rho_y^2} + \frac{p^2}{p_{0c}^2} = 1 \quad \text{----- (10)}$$

When the general meshing state of crowned gear is taken into account, the case that the contact ellipse is not perfect must be covered. In this case, tooth load distribution is assumed as follows:

As in Fig. 4(b), the form of contact area is an ellipse, the end part of major axis of which is broken, and the form of this incomplete contact ellipse is similar to the perfect contact ellipse when the

(a) The case when contact ellipse is perfect

(b) The case when contact ellipse is not perfect

Fig. 4

load is smaller. Furthermore, the surface constructed by enveloping the end point of a perpendicular line having the length proportional to the distributed load forms an ellipsoid surface.

4. Maximum tooth stress of crowned gear

In the case that the contact ellipse is not broken, maximum tooth stress of crowned gear can be obtained from Eqs. (9) and (10). But to calculate tooth stress by this method is troublesome, so it cannot be recommended as the practical calculation method. Therefore to establish a simple and easy method to obtain maximum tooth stress of crowned gear is desired.

If the factor of crowning B_c which is the ratio of the tooth stress σ_u of spur gear caused by the uniformly distributed tooth load to the maximum tooth stress σ_{cmax} of crowned gear having the same dimensions as the spur gear is introduced and if it can easily be obtained for various conditions, maximum tooth stress σ_{cmax} of crowned gear in the lengthwise direction of tooth may easily be obtained as follows:

$$\sigma_{cmax} = \sigma_u / B_c \quad (11)$$

4.1 Factor of crowning

In introducing the factor of crowning, it is assumed that the following conditions hold.

- (1) Radius of curvature of crowning is uniform over face width.
- (2) Form of contact ellipse on the tooth surface of crowned gear is perfect.
- (3) Ratio of the tooth stress of spur gear caused by uniformly distributed tooth load to the tooth stress of crowned gear is equal to the ratio of each bending moment at the fixed end of a finite length cantilever plate obtained by applying moment-image method to the theory about an infinite length cantilever plate.
- (4) It can be assumed that the tooth load of crowned gear is distributed along the line of major axis of contact ellipse, because generally the length of major axis is sufficiently greater than the length of minor axis.

If P is tooth load, the following equation are obtained.

For crowned gear:

Point B shows the position of maximum tooth stress in the lengthwise direction of tooth

Fig. 5

$$P = \frac{2}{3} \pi \rho_x \rho_y \rho_{\theta c} \quad (12)$$

For spur gear:

$$P = \frac{\pi}{2} b \rho_u \rho_{\theta u} \quad (13)$$

where ρ_u and $\rho_{\theta u}$ are the value of half contact width and maximum contact stress respectively when a uniformly distributed tooth load is applied on the tooth surface of spur gear.

If the coordinate system is taken as in Fig. 5, maximum bending moment M_{cmax} at the tooth-fillet of crowned gear in the lengthwise direction of tooth is obtained from Eqs. (10) and (12) as follows:

$$M_{cmax} = \frac{3}{4} \frac{P \cos \omega}{\lambda} \int_{-\lambda}^{\lambda} \left(1 - \frac{\xi^2}{\lambda^2}\right) \left\{ \beta_1(\xi_1 - \xi_{max} + \xi, \eta) + \beta_2(\xi_1 + \xi_{max} - \xi, \eta) + \beta_3(2\eta - \xi_1 - \xi_{max} + \xi, \eta) \right\} d\xi \quad (14)$$

In the case when the tooth load of spur gear is uniformly distributed, bending moment at the tooth-fillet in the lengthwise direction of tooth is uniform. If the bending moment in the middle of face width is taken as representative, it is given as follows:

$$M_u = \frac{P \cos \omega}{n} \int_{-n/2}^{n/2} \left\{ \beta_1(\xi, \eta) + \beta_2(n - \xi, \eta) + \beta_3(n + \xi, \eta) \right\} d\xi \\ = \frac{2 P \cos \omega}{n} \int_0^{(3/2)n} \beta_1(\xi, \eta) d\xi \quad (15)$$

Fig. 6 $\int_0^{\lambda} \left(1 - \frac{\xi^2}{\lambda^2}\right) \beta_1(\xi, \eta) d\xi$

Calculation condition: when $\xi_1 \leq 1.3$; $\lambda = \xi_1$, $n = 2.6$

when $1.3 < \xi_1 \leq n/2$;

$\lambda = \xi_1$, $n = 2\xi_1$

Fig. 7 Minimum value of $K_c(\lambda, \xi_1, n, \eta)$ ($n \geq 2.6$, $0 < \xi_1 \leq n/2$)

From Eqs. (14) and (15), factor of crowning B_c is obtained by introducing factor $K_c(\lambda, \xi, \eta, \gamma)$ as follows:

$$B_c = \frac{4}{3} K_c(\lambda, \xi, \eta, \gamma) \frac{\lambda}{\eta} \frac{\int_0^{(3/2)\eta} \beta_r(\xi, \eta) d\xi}{\int_0^{\lambda(1-\frac{\xi^2}{\lambda^2})} \beta_r(\xi, \eta) d\xi} \quad (16)$$

Then, all values which are utilized to obtain the factor of crowning in the case of $\eta \geq 2.6$ must be given.

$$\int_0^{(3/2)\eta} \beta_r(\xi, \eta) d\xi = -0.5037 + 0.503 (\eta \leq 0.8) \quad (17)$$

The value of $\int_0^{\lambda(1-\frac{\xi^2}{\lambda^2})} \beta_r(\xi, \eta) d\xi$ is obtained by Fig. 6. In Fig. 7 minimum value of $K_c(\lambda, \xi, \eta, \gamma)$ is shown. By the way, the influence of η on $K_c(\lambda, \xi, \eta, \gamma)$ is less than 3% in the case of $\eta \geq 2.6$.

On the actually used crowned gears, the values of $K_c(\lambda, \xi, \eta, \gamma)$ may be taken as unity, so the factor of crowning can be obtained by the next equation when the practical use of it is taken into account.

$$B_c = \frac{4\lambda}{3\eta} \frac{\int_0^{(3/2)\eta} \beta_r(\xi, \eta) d\xi}{\int_0^{\lambda(1-\frac{\xi^2}{\lambda^2})} \beta_r(\xi, \eta) d\xi} \quad (18)$$

where $\eta \geq 2.6$, $\lambda + 0.8 \leq \xi \leq \frac{\eta}{2}$ (0.8 is the dimensionless value of end relief (15)).

4.2 Tooth stress of spur gear

Tooth stress of spur gear can generally be given by using the tooth stress calculation method introduced by Lewis, but here the tooth stress is obtained by the infinite length cantilever plate theory to which moment-image method is applied.

Now if k_t is taken as the stress-concentration factor at tooth-fillet, then stress of spur gear caused by uniformly distributed load is given as follows:

$$\sigma_u = 6k_t \frac{M_u}{S^2} \quad (19)$$

Substituting M_u in Eq. (19) by Eq. (15), we obtain the following equation.

$$\sigma_u = 12k_t \frac{P \cos \omega}{\pi S^2} \int_0^{(3/2)\eta} \beta_r(\xi, \eta) d\xi \quad (20)$$

In the case of $\eta \geq 2.6$, the value of integral $\int_0^{(3/2)\eta} \beta_r(\xi, \eta) d\xi$ is given by Eq. (17).

5. Experiment

Table 1 shows the principal dimensions of test gears. Fig. 8 shows the form

Table 1 Dimensions of test gears

	Spur gear		Crowned gear	
	A	B	C	D
Module	10	11	10	12
Cutter pressure angle	20°			
Number of tooth	23	15	25	18
Addendum modification coefficient	+0.3	+0.36	+0.3	+0.5886
Face width mm	115	140	80	135
Pitch diameter mm	230	165	250	192
Outside diameter mm	256	195	276	230
Material	S40C induction hardened	SNCM26 carburized	S40C induction hardened	SNCM26 carburized
Hardness at the tooth crest Hc	74~76	69~75	74~77	78~81
Form of crowning		Fig. 8(a)	Fig. 8(b)	Fig. 8(c)

of crowning. The lengthwise distribution of tooth stresses was measured by electric wire resistance strain gauges which were 3 mm in gauge length and were pasted at several points in the lengthwise direction of tooth at the tension side of tooth-fillet corresponding to the critical section determined by Hofer's 30° tangent method. By the way tooth stresses at compression side were not measured, because it is generally accepted that, when bending strength of gears is dealt with, tooth stresses at tension side have only to be considered.

Moreover, in order to know the distribution of tooth stresses as accurately as possible, we carried out the comparison (16) of out-put between the strain gauges by applying a concentrated force at a position on the tooth surface which corresponds to the position of each strain gauge pasted at tooth-fillet.

As the experimental apparatus, an Amsler type universal testing machine and a static strain meter were used. Fig. 9 shows the situation of loading on the tooth surface by using rack type loading tool.

(a) Gear B

(b) Gear C

(c) Gear D

Fig. 8 Form of crowning

6. Experimental results and consideration

6.1 Stress-concentration factor at tooth-fillet

Stress-concentration factor at tooth-fillet is defined as the ratio of experimental values to calculated values when a concentrated force is applied at a position of addendum which corresponds to the position of strain gauge pasted at tooth-fillet.

Table 2(a) shows the stress-concentration factor related with the dimensionless distance η of a loaded point from the tip of the tooth which is measured along the tooth center line. It may be said that k_t is not affected by the loaded point and its value is nearly constant. Therefore, in this paper, as the stress-concentration factor at tooth-fillet, we use the value when a concentrated force is applied at the tip of the tooth.

Table 2(b) shows experimental values of stress-concentration factor and radii of curvature of tooth-fillet per unit module at each critical section determined by Hofer's 30° tangent method. For gear D which has the minimum radius of curvature per unit module, the value of stress-concentration factor is maximum, but there is not so much difference as a whole.

6.2 Tooth stress of spur gear

Here the tooth stress of spur gear calculated by the method which is given Eq. (20) when a uniformly distributed tooth load is applied (from now on this method will simply be called the moment-image method) was compared with the values calculated by Niemann's calculation method⁽¹⁷⁾ and Aida-Terauchis' equation^{(18)~(20)}, and it was clarified that the moment-image method is quite practicable.

Table 2 Stress-concentration factor at tooth-fillet

(a)							
Gear	A			C			
η	0.06	0.16	0.44	0.06	0.17	0.37	0.51
Stress-concentration factor	1.2	1.2	1.1	1.3	1.4	1.3	1.3

(b)				
Gear	A	B	C	D
Stress-concentration factor	1.2	1.3	1.3	1.4
Radius of curvature of tooth-fillet (in module unit)	0.467	0.461	0.465	0.371
Tooth height to the critical section mm	21.28	23.93	21.22	27.70

Fig. 9 Loading on the tooth surface by using rack type loading tool

According to Niemann's method, tooth stress at tension side is given as follows:

$$\sigma_u = k_t \sqrt{(\sigma_{Nb} - \sigma_{Nc})^2 + (a \tau_{Nt})^2} \quad \text{----- (21)}$$

where σ_{Nb} is bending stress, σ_{Nc} is compressive stress, τ_{Nt} is shearing stress and k_t is the stress-concentration factor at tooth-fillet.

Aida-Terauchis' tensile tooth stress equation is given as follows:

$$\sigma_u = (1 + 0.08 S/\rho) (0.66 \sigma_{Nb} + 0.40 \sqrt{\sigma_{Nb}^2 + 36 \tau_{Nt}^2}) \quad \text{----- (22)}$$

where S is tooth thickness at the critical section determined by Hofer's 30° tangent method and ρ is the radius of curvature of tooth-fillet at the critical section.

Fig. 10 shows tensile tooth stress of spur gear A when a uniformly distributed tooth load is applied. Abscissa gives dimensionless distance of loaded position η and contact ratio ϵ corresponding with the loaded position which are considered the critical loading point.

As shown in Fig. 10, values calculated by the moment-image method agree quite well with experimental values. On the other hand the values calculated by Niemann's method and Aida-Terauchis' equation are about 10~20% greater than the experimental values. From this example it is known that the moment-image method may be adopted as the tooth stress calculation of spur gear when a uniformly distributed tooth load is applied.

6.3 Tooth stress of crowned gear

Experiments about crowned gear were carried out in the tooth surface range of each test gear shown in Fig. 8 whose radius of curvature of crowning is uniform.

Fig. 11 shows an example of tooth stress distribution when contact ellipse is perfect. In the case of gear D which has

(ϵ is the value of contact ratio when η represents the value corresponding to the critical loading position)

Fig. 10 Tooth stress of spur gear A when lengthwise load distribution is uniform

larger radius of curvature of crowning, tooth stress distribution of experimental results differs somewhat from that of calculated ones. Experimental results have smaller maximum tooth stress than the calculated ones and the stress distribution is gentle. Furthermore the greater the tooth load is, the more remarkable the difference becomes. On the other hand, in the cases of gears B and C having smaller radii of curvature of crowning, experimental values comparatively well agree with calculated values.

By the way, as the result of examining the state of tooth bearing by painting minium on the tooth surface, it was found that the length of major axis of contact ellipse was in all gears 10~20% larger than the calculated values.

As these experimental results show,

(a) Gear B

(b) Gear C

(c) Gear D

Fig. 11 Tooth stress distribution of crowned gear when contact ellipse is perfect

the form of contact area is nearly equal to the calculated one, but in the case of gear D tooth stress distribution differs considerably from calculated one. The reason is that, as shown in paragraph 3.2, tooth load distribution of crowned gear was obtained as the solution of problem about two elastic ellipsoids of rotation which contact with each other without considering lengthwise tooth bending deflection distribution. Therefore, in the cases of gears B and C where the radii of curvature of crowning are small and the state of tooth load distribution is comparatively local, good tooth stress distribution was obtained by applying the tooth load distribution which was obtained as the solution of a contact problem about two elliptic bodies directly. On the other hand in case where the radius of curvature of crowning is large and load distributes widely on face width, when the tooth stress distribution is calculated, the lengthwise bending deflection characteristic of tooth which affects the load distribution must be taken into account. But in this paper it is not considered.

Fig. 12 shows tooth stress distribution of gears C and D in the case when contact ellipse was broken and end tooth bearing occurred. Experimental results for these gears agree comparatively well

(a) Gear C

(b) Gear D

Fig. 12 Tooth stress distribution of crowned gear when contact ellipse is broken

(ϵ is the value of contact ratio when η represents the value corresponding to the critical loading position)

(a) Gear B

(ϵ is the value of contact ratio when η represents the value corresponding to the critical loading position)

(b) Gear C

(ϵ is the value of contact ratio when η represents the value corresponding to the critical loading position)

(c) Gear D

Fig.13 Maximum tooth stress of crowned gear

with calculated ones. Especially it is known that even for gears having large radius of curvature of crowning like gear D, tooth load distribution may be calculated by the method shown in paragraph 3.2 when end tooth bearing takes place.

6.4 Maximum tooth stress of crowned gear

In Fig.13 experimental values of maximum tooth stresses in the lengthwise direction of tooth and the value calculated by Eq.(11) in which the factor of crowning is introduced are shown. Regardless of calculation method for tooth stress σ_u of spur gear at uniformly distributed tooth load, Eq.(11) is useful to calculate maximum tooth stress σ_{max} of crowned gear in the lengthwise direction of tooth.

In the experiments of this time, dimensionless values λ of semi-major axis of contact ellipse on tooth surface are 0.4~0.7 for gear B, 0.7~1.1 for gear C and 0.9~1.5 for gear D. The ratio $\sigma_{max}/\sigma_u = 1/B_c$ which is obtained by transforming Eq.(11) means the degree of increase of tooth stress on the crowned gear compared with the tooth stress of spur gear when a uniformly distributed tooth load is applied, and in the experiments of this time the calculated values of $1/B_c$ are 2.8~4.0 for gear B, 1.6~2.4 for gear C and 1.8~2.6 for gear D.

As is shown in Fig.13, for gears B and C experimental values comparatively well agree with calculated values wherever the loading position on tooth surface is. However, for gear D having large radius of curvature of crowning, in spite of the calculated values of λ and σ_{max}/σ_u not being so different from the values for gear C as shown above, experimental values are 20% smaller than calculated values for the reason shown in paragraph 6.3. These calculated tooth stress values of crowned gear D which were obtained by using the method shown in chapter 4 were a little greater than the experimental values, and this gives safe design procedure. So, to calculate maximum tooth stress of crowned gear in the lengthwise direction of tooth, using the factor of crowning is quite practicable.

7. conclusions

The results obtained from the study about the calculation method of tooth stress for heavily crowned gears are as follows:

(1) Maximum tooth stress of crowned gear in the lengthwise direction of tooth can easily be obtained by dividing the tooth stress of spur gear having uniformly distributed tooth load by the factor of crowning. That is, load capacity of crowned gear is obtained by using the factor of crowning instead of the load distribution factor in the calculation equation of load capacity for spur gear.

(2) Tooth stress distribution of crowned gear is comparatively accurately obtained by applying moment-image

method to the solution of elastic problem when a transverse concentrated force is applied on the surface of an infinite length cantilever plate, even if the contact ellipse is broken and end tooth bearing occurs.

(3) Stress-concentration factor at the tooth-fillet of tension side is 1.2~1.4 in the case when tooth stress is calculated using the infinite length cantilever plate theory which is applied with moment-image method.

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