Investigation of Strength and Paths of Vortices
Shed from Vortex Generators in a Pipe and
Application to Conical Diffusers*

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A simple linearised theory is presented to investigate the strength and the paths of trailing vortices being shed from vortex generators in a pipe, and the prediction is experimentally proved. Then various types of vortex generators are applied to conical diffusers. According to the tests, the one recommended by the theory makes the best pressure recovery.

1. Introduction

It is proved by the experiment described in the previous paper(1) that vortex generators are effective to prevent flow from separating inside conical diffusers. In these cases vortex generators are mounted on the inner wall of a pipe as shown in Fig.1, and the pipe is inserted immediately upstream of diffusers. Vortices are shed from the tips of the blades of vortex generators and the trailing vortices serve to mix the main flow with the boundary layer fluid. The effectiveness of vortex generators depends upon the strength of vortices and the distance of vortices from the wall. Therefore, investigation of the behavior of vortices in diffusers is very important, if application of vortex generators is planned to improve the performance of conical diffusers.

J. P. Jones(2) studied the paths of vortices produced by a row of wing-type vortex generators which were mounted on a flat plate, and the results were compared with his analysis. In the present study vortex generators are made of blades with rectangular planform, as shown in Fig.1, and the angles of incidence of any two adjacent blades are equal and opposite. As shown in Fig.1, the ratio of the distance between every other blades to the distance between adjacent blades is called pitch ratio. Pitch-chord ratio, aspect ratio, pitch ratio and blade height-pipe radius ratio are changed in the analysis, and the predicted paths and strength of trailing vortices in a circular pipe are compared with the experimental results measured with a vortometer.

2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a₀</td>
<td>radius of a vortometer</td>
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<tr>
<td>A</td>
<td>coefficient in Eq. (10)</td>
</tr>
<tr>
<td>AS</td>
<td>aspect ratio of a blade of vortex generators</td>
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<td>b</td>
<td>blade height</td>
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<td>C</td>
<td>blade chord</td>
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<td>Cₚ</td>
<td>pressure-recovery coefficient of a diffuser</td>
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<td>L</td>
<td>axial length from center of vortex generators</td>
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<tr>
<td>n</td>
<td>rotational speed of a vortometer</td>
</tr>
<tr>
<td>n₀</td>
<td>rotational speed of a vortometer measured at L=0.105 m</td>
</tr>
<tr>
<td>N</td>
<td>number of blades of vortex generators</td>
</tr>
<tr>
<td>N₀</td>
<td>number of vortex pairs, N₀=N/2</td>
</tr>
<tr>
<td>R₀</td>
<td>pipe radius</td>
</tr>
<tr>
<td>rₜ</td>
<td>radius of vortex core</td>
</tr>
<tr>
<td>Rᵣ</td>
<td>inlet radius of a diffuser, Rᵣ=R₀</td>
</tr>
<tr>
<td>Sᵣ</td>
<td>arc length at the root between two blades with the same configuration</td>
</tr>
<tr>
<td>Sₜ</td>
<td>arc length between two blades which form a diverging pair</td>
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<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>U</td>
<td>main flow velocity</td>
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<tr>
<td>V₀</td>
<td>vortex generators</td>
</tr>
<tr>
<td>vₘ</td>
<td>downwash velocity</td>
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w: circumferential component of velocity in a vortex
W: complex potential in Z-plane
W₀: complex potential in Z₀-plane
α: blade setting angle measured from the axial direction
β: angle of incidence
Γ: strength of vortex
Γ₀: defect of vortex strength due to wall friction
δ*: displacement thickness of boundary layer
θ: momentum thickness of boundary layer
θ₀: a parameter, θ₀ = (2π/N₀)(1 - 1/(S₁/S₂))
ν: molecular viscosity
ν₀: turbulent viscosity
ν₀*: divergence angle of a diffuser

3. Analysis

The present analysis is made to examine the behavior of vortices downstream of vortex generators. A linearized theory is developed with the following main assumptions.

(1) The flow is steady and incompressible.
(2) The influence of the wall boundary layer is negligible on the behavior of vortices in the pipe.
(3) The axial component of velocity is uniform everywhere in a cross section perpendicular to the pipe axis and it is equal to the main flow velocity.
(4) The motion of vortex core in the cross section is described as a two-dimensional potential flow.*
(5) The circulation around a blade of vortex generators is calculated considering the boundary layer along the wall and the influence of the downwash induced by the trailing vortices.

Fig. 2(a) shows the distribution of the vortices being shed from the vortex generators. The vortices having equal strength Γ₀ are distributed equi-spaced at a distance b from the wall or at the positions marked A, and the vortices having the strength -Γ₀ are also distributed equi-spaced at the positions marked B. A vortex A and a vortex B form a pair, and there are N₀ pairs, or the total number of vortices is N = 2N₀. In the analytical model the wall effect is represented by N₀ pairs of the vortices A' and B' which are the mirror images of N₀ pairs of A and B vortices. This plane is called Z-plane. The flow in Z-plane is mapped to Z₀-plane of Fig. 2(b) to reduce the number of vortices to four. If the complex potential of these four vortices is determined in Z₀-plane, the induced velocity at the vortices in Z-plane can be calculated. Therefore, the position of each vortex after a short period of time is determined. As the flow pattern is axially transported by the main flow, the path of each vortex is obtained by this analysis.

3.1 Calculation of the induced velocity
N₀ pairs of vortices and their mirror images in Z₀-plane are respectively mapped to a pair of vortices in Fig. 2(b) by the relationship

\[ Z₁ = Z₂^N₀ \]  
\[ W₁ = (\Gamma₀/2\pi) \log(Z₀ - Z₁A)(Z₀ - Z₁B) \]  
\[ -\log(Z₀ - Z₁A')(Z₀ - Z₁B') \]  

(1)

(2)

where \( Z₁A, Z₁B, Z₁A', \) and \( Z₁B' \) indicate the positions of the vortices \( A₁, B₁, A₁', \) and \( B₁' \) respectively. Attention is now paid to the specific vortex \( A₁ \), which is positioned on x-axis. The velocity at the point in Z-plane is obtained by differentiation of the next equation and putting \( Z = Z₀ \).

\[ W = W₁ - (\Gamma₀/2\pi) \log(Z - Zₐ) \]  

(3)

* The trailing vortices extend only downstream from the vortex generators and the flow in the cross section is not two-dimensional near the vortex generators. However, a numerical example made by the present authors shows that the paths predicted by a two-dimensional vortex model is little different from those by a three-dimensional vortex model.
The x- and y-components of velocity \( u \) and \( v \) are respectively expressed as

\[
\begin{align*}
\frac{u}{v} &= -\frac{\Gamma_0}{4\pi r} \frac{N_0}{(r'N_0 - rN_0)^2 + 2r'N_0(1 - \cos N_0 \theta_0)} \\
v &= \frac{\Gamma_0}{4\pi r} \frac{2r'N_0}{(rN_0 + 1 + r'N_0 (N_0 - 1))(1 - \cos N_0 \theta_0) - (r'N_0 - rN_0)^3} \\
\end{align*}
\]

where the radial distance of the mirror images \( r' \) in Z-plane is given as the ratio of the square of the pipe radius \( r_0 \) to the radial distance of the vortices \( r \), i.e., \( r' = r_0^2 / r \). \( \theta_0 \), the angular distance between a vortex marked A and one marked B, is expressed by the number of pairs \( N_0 \) and the pitch ratio \( S_0 / S_2 \) as shown in Fig.2(a), i.e., \( \theta_0 = (2\pi/N_0)(1 - 1/(S_0 / S_2)) \). If the strength of the vortex \( \Gamma_0 \) is known, the induced velocity is determined from Eq.(4).

3.2 Strength of trailing vortex

In the present analysis the lifting-line theory is adopted to estimate an approximate value of the vortex strength generated by the blades, for it is difficult to determine the accurate strength due to the complicated flow phenomena. It is assumed that trailing vortices are only shed from the tips of the blades and they axially extend downstream as shown in Fig.3. Owing to the downwash velocity induced by these vortices the direction of flow at the blades is different from the main flow direction. For simplicity it is assumed that the downwash velocity is uniform spanwise and that it is equal to the theoretical value at the blade root \( \omega \). The angle of incidence varies by \( \Delta \alpha \) due to the downwash and the circulation around the blade \( \Gamma_0 \) is

\[
\Gamma_0 = \pi(C_0 - \Delta) \lambda U
\]

where

\[
\Delta = \Delta \theta \omega / U
\]

The downwash velocity \( \omega \) is calculated from Eq.(2) by putting \( Z = \alpha \) and taking one half of the circulation, because the vortex line extends downstream only. That is,

\[
\omega = (\Gamma_0 / 2\pi) \lambda
\]

where

\[
\lambda = \frac{N_0 r'N_0 - 1}{(rN_0 + 1 + r'N_0 (N_0 - 1))(1 - \cos N_0 \theta_0) - (r'N_0 - rN_0)^3}
\]

From Eqs.(5) and (6)

\[
\Gamma_0 = \pi(C_0 - \Delta) \lambda U / (1/\lambda)^2
\]

The strength of the vortex is determined by Eq.(8) providing that the dimensions and the arrangement of vortex generators are given.
4. Experimental apparatus and procedures

4.1 Test rig
The test rig is shown schematically in Fig.4(a). Air is admitted into a test pipe of 153.5 mm diameter through a bell-mouth attached to a settling chamber which contains a honey-comb and fine mesh screens. The length of the pipe between the bell-mouth and the vortex generators is adjustable so that the boundary layer thickness at the location of the vortex generators varies. For measurements of strength and paths of the trailing vortices, a vortmeter is mounted in a short pipe which is inserted between two pipe flanges at any desired distance from the vortex generators.

4.2 Vortmeter and measuring procedure
Fig.4(b) shows the dimensions and the configuration of a vortmeter used in the present experiment. The rotational speed of the vortmeter is measured with a reflecting type photo pick-up which is mounted on the pipe wall at the diametrically opposite position to the vortmeter as shown in Fig.4(a). The vortmeter is movable both in the circumferential and in the radial directions to search the position of the vortex center, where the vortmeter indicates the maximum rotational speed. Fig.5 shows the characteristics of the vortmeter. This preliminary experiment has been performed at stations where the axial distances from the vortex generators are 5.24, 26.2 and 45.2 times the blade chord respectively. The rotational speed is proportional to the main flow velocity as expected.

5. Results and Discussion

5.1 Strength of trailing vortices
The strength of trailing vortices being shed from the vortex generators is measured as the rotational speed of the vortmeter at the station L/C=5.24, where it is expected that the trailing vortex is fully rolled up.

5.1.1 Comparison of theory and experiment
The effects of the aspect ratio and the number of blades on the strength of vortices are investigated keeping the ratio of the blade height to the pipe radius b/r0=0.13 and the ratio of the blade height to the displacement thickness δ*/b=0.14. The blades are equally spaced or the pitch ratio is 2.0 in this experiment. The results are shown in Fig.6. The rotational speed of the vortmeter is made dimensionless by dividing it with the rotational speed at the reference condition, where the configuration of vortex generators is C=30 mm and N=2N0=12, and the ratio is presented as the ordinate. The abscissa is the reciprocal of the aspect ratio, or the ratio of the blade chord to the blade height. The open circles and the closed circles show the results of N=12 and N=18 respectively. The solid lines in the figure indicate the theoretical results calculated from Eq.(8). Agreement is satisfactory.

If the blades are equally spaced, Eq.(7) becomes simple and Eq.(8) is expressed as
\[ \frac{\Gamma_s}{\Gamma_0} = \frac{N \cdot \frac{b}{r_0} \cdot \frac{1}{1 - \left(\frac{1}{b/r_0}\right)^N}} {AS \cdot \frac{1}{1 - \left(\frac{1}{b/r_0}\right)^N}} \]

To study the influences of the pair-number N and the ratio of the blade height to the pipe radius b/r0 on the vortex strength, these are used as the coordinate system in Fig.7 and the third parameter is the second term in the denominator of Eq.(9). The smaller the parameter is, the stronger the vortex is. According to the numerical results shown in Fig.7, the strength of vortex...
and the local value of the vortex strength should be used for calculation of the vortex paths. To study the paths and the decay rate of the strength of trailing vortices with the vortometer, vortex generators with configuration of C=20 mm, AS=1.0, N=12, α=14 degrees and various pitch ratios are used.

5.2.1 Rotational speed of vortometer

It is generally accepted that the core radius of a trailing vortex r_0 grows due to turbulent viscosity and the relation is expressed as r_0=2/ν/Vτ=2L(V/UL)^1/2. According to the literature, the turbulent viscosity is 10^3 times the molecular viscosity for the cases of trailing vortices. An increase in the diameter of vortex core causes a decrease in the rotational speed of the vortometer even though the circulation at a large radius remains constant.

The trailing vortex shed from a single rectangular blade of AS=1.0 and C=30 mm protruding from the pipe wall is measured with the vortometer, where the wall boundary layer is very thin. In the experiment, the change of the rotational speed with respect to the axial distance from the blade agrees well with the prediction based on the theory shown in the appendix, where it is assumed that the circulation round the vortex at r=∞ is constant and that the turbulent viscosity is 15 times the molecular viscosity.

The trailing vortices being shed from vortex generators with C=20 mm, AS=1.0 are tracked with the vortometer. The ratio of the rotational speed of the vortometer at a section to that at L=0.105 m is adopted as the ordinate in Fig.9. Marks shown in the figure are experimental values. The chain line indicates the predicted rotational speed assuming Vτ=15ν and the circulation at a large radius is constant. The wide discrepancy between the chain line and the measured values means that the circulation decreases itself due to the wall friction as the vortices proceed downstream.
5.2.2 Decay rate of vortex

The ratio of the solid line which represents the rotational speed measured with the vortometer to the chain line shows the decay rate of the trailing vortex. The decay rate is expressed by the following equation:

$$\Gamma/\Gamma_0 = 1 - \exp\left(-\frac{2}{A}\right)$$  \hspace{1cm} (10)

The dotted line in Fig.9, indicates the relation of Eq.(10), where $A=2\times10^{-2}$. In other words, the variation of the rotational speed of the vortometer is represented by the solid line, but the decay rate of the trailing vortex is given by the dotted line. Therefore, the dotted line or Eq.(10) must be taken into account for the calculation of vortex paths.

5.2.3 Paths of vortices

Fig.10 shows the paths of vortices for three different pitch ratios of vortex generators. The abscissa is the axial distance $L$, and the ratio of the distance of vortex from the wall to the pipe radius is adopted as the ordinate. In the figure square marks, circular marks and triangular marks indicate the experimental results for the cases of vortex generators with the pitch ratios of 1.5, 2.0 and 3.0 respectively.

For prediction of the path of vortex, Eq.(10) is used to specify the local strength of trailing vortices. The strength of trailing vortices being shed from the vortex generators is assumed to be 0.4 times the strength given by Eq.(8) allowing 0.8 for the ratio between the actual and the theoretical lift coefficients of a blade in a two-dimensional flow, 0.87 for the effect of wall boundary layer mentioned in section 5.1.2, and 0.58 for the ratio of the circulation around the rolled up trailing vortex to the circulation around the blade. This shedding vortex loses the strength along the vortex line as shown by the dotted line in Fig.9.

The solid lines in Fig.10 indicate the prediction of paths of vortices, which agree well with the experimental data.

In Fig.11 the paths of trailing vortices shed by vortex generators in a pipe are compared with those by vortex generators on a flat plate. The comparison is made for three types of vortex generators, $S_1/S_2=1.5$, 2.0 and 3.0, and the location of vortices at the section of vortex generators is also illustrated. In the figure the solid lines represent the paths of the vortices in a pipe and the broken lines represent those on a flat plate which are calculated using the equation proposed by Jones. For vortex generators on a flat plate, pitch ratio of three or four is recommended, because in
these cases the vortex moves towards the wall. The pitch ratio less than two is not desirable because the vortices move away from the wall. However, when vortex generators are applied to a pipe flow, the trailing vortices produced by vortex generators with pitch ratio of two remain at a constant distance from the wall. If the pitch ratio is larger than two, the vortices approach the wall and the strength of vortices decays faster due to the wall effect as shown in Fig.9. Therefore, the pitch ratio of two is recommended for vortex generators in a pipe.

5.3 Application of vortex generators to conical diffusers

Vortex generators are applied to four conical diffusers having divergence angles of 8, 12, 16 and 20 degrees respectively. The area ratio of these diffusers is four.

5.3.1 Effect of pitch ratio on diffuser performance

To examine the recommendation proposed in section 5.2.3, vortex generators are applied to conical diffusers and the effect of pitch ratio on the pressure-recovery in the diffusers is studied. Fig.12 shows the results. As it is expected from the theory, vortex generators of pitch ratio 2 give the best performance. In the case of thin boundary layer, diffusers with vortex generators of pitch ratio 3 demonstrate good pressure recovery. But in the case of thick boundary layer, the effect of pitch ratio 3 is not so much different from that of pitch ratio 1.5, and the pressure recovery coefficients are little superior to those of conical diffusers without vortex generators. Solid marks in Fig.12 indicate that the flow in the diffuser is not stable and the pressure fluctuates radically.

(a) thin boundary layer

Fig.12 Effect of pitch ratio on pressure-recovery coefficient of conical diffusers

(b) thick boundary layer

Fig.13 Effect of aspect ratio and number of blades on pressure-recovery coefficient

5.3.2 Effect of aspect ratio and number of blades on diffuser performance

Fig.13 shows the effect of aspect ratio of vortex generators on the pressure-recovery coefficients of conical diffusers. In this experiment the pitch-chord ratio and the ratio of blade height to the boundary layer thickness are kept constant. The circles represent a case of vortex generators with $N=12$ and $AS=1.0$ and the triangular marks represent the case of those with $N=6$ and $AS=0.5$, i.e. the chord length of the latter vortex generators is twice that of the former. In the latter case of $AS=0.5$, the vortex generators do not contribute to improvement of the pressure recovery coefficient.

The difference of the two cases may be explained from Fig.14, which indicates the denominator of Eq. (9) or the correction coefficient of the strength of vortices due to the downwash velocity. Aspect ratio is adopted as the abscissa and the number of vortex pairs $N_o$ is used as the contours, and two configurations tested in Fig.13 are indicated as the open circles. As the relative strength of trailing vortices is inversely
Fig. 14 Effect of aspect ratio and pair-number on downwash

proportional to the values of ordinate, it is expected that the arrangement of vortex generators of $AS=0.5$ is not effective enough to prevent the flow from separating in a diffuser. As the effective angle of incidence of the blades is highly reduced by the downwash velocity in this case, however, it is expected that the strength of vortices may be increased without stall of blades by increasing the angle of incidence of the blades.

6. Conclusions

Various arrangements of blades for counter-rotating type vortex generators are investigated both theoretically and experimentally and the following results are obtained.

(1) The strength and the paths of vortices can be predicted by the present theory.

(2) The circulation of blade is reduced by the downwash velocity. The effect becomes more significant in cases of less blades, smaller aspect ratio and smaller ratio of the blade height to the pipe radius under the present arrangement of vortex generators.

(3) The measured strength of trailing vortex is 0.4 times the potential flow prediction. The ratio consists of three factors: 0.8 is the ratio of the real to the ideal lift coefficients in two-dimensional flow, 0.87 is the effect of the boundary layer on the pipe wall and the amount of roll up to form the trailing vortex is 0.58 of the shed vorticity.

(4) The trailing vortices decay due to the boundary layer on the wall. The rate of decay is described as $1 - \exp(-At/L)$, where the factor $A$ is about $2 \times 10^{-2}$ in the present experiment.

(5) The vortex strength is related to the rotational speed of the vortimeter, but the relationship is not straightforward due to an increase in the diameter of the vortex core. This relationship is described in the appendix.

(6) When vortex generators in counter-rotating arrangement are applied to a conical diffuser, pitch ratio of 2 or equal spacing of blades is recommended. The pitch ratio is different from the optimum pitch ratio of vortex generators on a flat surface.

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Appendix

Relationship between vortex strength and rotational speed of vortometer

For estimation of the strength of a vortex, measurement of velocity distribution around the vortex is required. However, the measurement is not practical if the scale of the vortex is small. As handy means, a vortimeter is widely used for locating the center of vortex and for qualitative comparison of vortex strength. 

In the present study, an analysis is made to investigate the relationship between the size of vortex and the rotational speed of a vane-type vortimeter as shown in Fig. 15.

The rotational speed of a vortimeter is maximum when the axis coincides with the center of vortex, and such a case is examined. If the mechanical loss is negligible, the work done by vanes of the vortimeter is zero. It is assumed that the fluid leaving the rotating circle of the vortimeter rotates at the angular velocity of the vortimeter retaining the original uniform axial velocity. Hence, Eq. (11) is deduced.

$$\int_0^{2\pi} \int_0^{\alpha_0} U(r-d\theta)drd\theta = 0$$

or

$$n = \frac{3}{2ma} \int_0^{\alpha_0} r dr$$

To describe the circumferential velocity distribution in the vortex before the vortimeter, Newman’s equation (4) is applied.

Fig. 15 A vortimeter and velocity distribution of a vortex
\[ n = \frac{2 \pi \Gamma_0}{(2 \pi a_0)^2} \left( 1 - \frac{r_1}{a_0} \right) \int_0^{a_0} a_0 \left( \frac{r}{r_1} \right)^2 e^{-\left( \frac{r}{r_1} \right)^2} \frac{d(r)}{r_1} \]  

or

\[ n = \frac{3 \Gamma_0}{(2 \pi a_0)^2} \left( a_0 / r_1 \right)^2 \int_0^{a_0} \frac{a_0}{r_1} e^{-\left( \frac{r}{r_1} \right)^2} d(r) \]

In the case of free vortex flow pattern, \( r_1 = 0 \) and

\[ n = \frac{3 \Gamma_0}{(2 \pi a_0)^2} \]  

The above equation indicates that the rotational speed \( n \) is proportional to the vortex strength \( \Gamma_0 \), and besides it is influenced by the ratio of the radius of vortex core to the radius of the vortmeter. Fig.16 shows the results calculated with Eq.(14). Fig.16(a) shows the variation of the rotational speed of a vortmeter when the radius of vortex core changes. The ordinate is the brace of Eq.(14a) or the ratio of the rotational speed \( n \) to that obtained for the case of \( r_1 = 0 \). Fig.16(b) shows the variation of the rotational speed when vortometers with different radii are used to measure a vortex core. In this case the ordinate is the ratio of the rotational speed of vortmeter to that of an infinitesimal vortmeter. If it is assumed that the flow pattern of the vortex is the Rankine type, that is, a solid vortex core is surrounded with a free vortex as shown by the dotted line in Fig.15, the expected rotational speed of the vortmeter becomes the dotted lines in Fig.16.

References