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An Analysis of the Elastic and Plastic Deformations
of Machined Surfaces in Contact^{*}
(1st Report, Deformations of Contacting Rough Surfaces when
the Directions of Surface Asperities Cross Each Other)

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When two rough surfaces with directional asperities are in contact, the elastic and plastic deformations caused by surface asperities vary with cross angle of asperity directions and asperity sizes of two rough surfaces. In this report, the elastic and plastic deformations of machined surfaces with directional asperities were analyzed by considering the following conditions.

- (1) The elastic and plastic deformations of a certain projection in contact.
- (2) The elastic deformation of the above-mentioned projection caused by the loads of surrounding projections in contact.
- (3) The elastic deformation of projections which begin to contact newly by an increase of normal load.

The elastic and plastic deformations were obtained by synthesizing the above-mentioned factors with the statistics of surface asperities in contact. It was revealed that the elastic and plastic deformations are classified by the contact surface coefficient and vary with the cross angle of asperity directions and the ratio of projection slopes of two rough surfaces. The comparison between calculated values and experimental data will be explained in the next report.

1. Introduction

In the previous papers^{(1)~(4)} the authors have explained in detail the elastic and plastic deformations in two cases of rough surfaces in contact; the contact between a flat surface and a rough one with directional asperities and the contact of two surfaces whose asperity directions coincide with each other. In these cases, the characteristics of the asperities in contact were defined by those perpendicular to the asperity direction (e.g. the cutting direction of the abrasive grains in grinding).

As a further step, this report makes an analysis of the deformation mechanism of rough surfaces in contact when the directional asperities intersect each other at an arbitrary angle. In this contact, the characteristics of the asperities can be dealt with as two-dimensional ones. However, asperities in contact (the equivalent asperities which are in contact with a rigid flat surface) become three-dimensional when the whole contacting surface is considered.

Concerning three-dimensional surface asperities (let us call the properties of

the section of the surface asperities two-dimensional, and those including one more direction orthogonal to it three-dimensional), there are a few papers in which the contour lines of surface asperities are drawn by means of an electric computer, or the number and distribution of points in contact are counted⁽⁵⁾ and even they do not make clear the problems quantitatively. Similarly, there are researches analyzing the contact of directional asperities by considering them as perfectly plastic bodies⁽⁵⁾ or elastic deformation of the only one contact point.⁽⁶⁾ But the quantitative explanation on the elastic and plastic deformations of asperities is still insufficient.

This paper analyzes the deformation mechanism of the directional asperities in contact at an arbitrary cross angle, but under the following conditions, the results of this paper should be always consistent with those of the previous paper⁽⁴⁾ which dealt with the contact of directional asperities lying in parallel to each other. These conditions are:

- (1) When the cross angle of the asperity directions is close to 0°.
- (2) When the asperity heights of one surface are negligibly small compared with those of the other surface.

Furthermore, in this paper the following points are different from the previous paper.⁽⁴⁾

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(1) The characteristics of asperities in contact (the asperities of the equivalent surface) are three-dimensional.

(2) The elastic deformation of projections is considered that of the concentrated loads on a semi-infinite body.

Therefore, it is also difficult to include the contact problems in which the asperity directions are identical into this paper's analysis.

Nomenclature

Subscripts 1,2: Two surfaces in contact
 Subscript e: Equivalent asperities when rough surfaces are in contact
 Subscripts α_0, ϕ : Asperities in the directions α_0, ϕ shown in Fig.1 and Fig.7
 σ : Standard deviation of asperity heights
 Z : Height from the center line of asperity heights
 $f(Z)$: Probability density function of the asperity heights
 A : Nominal contact area
 ξ : Slopes of asperity projections obtained by profilometer with stylus
 σ_ξ : Standard deviation of ξ
 $\tan \theta, \tan \theta_a$: Slopes of the projections assumed as triangular wedges and their average
 $\overline{\tan \theta}$: Average of $\tan \theta$ where θ is half the vertical angle of triangular wedge
 β : Cross angle of the asperity directions of two surfaces in contact
 ρ : Ratio of the projection slopes of two surfaces in contact
 t : Mean interval of contact points
 t_T : Mean interval of contact points for a long distance in the direction of $\phi=0^\circ$
 λ, λ_T : Values of $t_{\phi=90^\circ}/t_0$, t_0/t_T in the vicinity of the contact point, where $t_0=t_{\phi=0^\circ}$
 S_1, S_2 : Coefficients considering the distribution of t
 E, ν : Effective Young's modulus and Poisson's ratio
 p_m : Plastic flow pressure (lower Brinell hardness of two surfaces in contact)
 $h, \bar{h}$: Depth to calculate the elastic deformation of projections. $\bar{h}$ is a dimensionless value h/t_0

σ_{co}, σ_{cm} : Normal stress right under the contact points on the plane at the depth h and its mean value
 K_1 : Contact surface coefficient
 z : Plastic deformation of projections
 k : Dimensionless value of the initial contact positions
 δ_0, δ_{oh} : Displacement of the surface at the contact points and that at the depth h
 δ_{oc} : Relative displacement between the free surface and the depth h which is produced by the surrounding contact points
 δ_{ce} : Relative displacement of the projections which begin to contact newly
 δ_{me} : Mean relative displacement between the free surface and that at the depth h
 P : Normal load
 δ_E, δ_S : Elastic and plastic deformations caused by the existence of surface asperities

2. Characteristics of the surface asperities

2.1 Probability density function of asperity heights

When the asperities are in contact at a cross angle β shown by Fig.1, if the projections are approximated by wedge-shaped ones, the contacting asperities will be as shown by Fig.2(a). If one of the surfaces is replaced with a flat one, the deformation of asperities shown by Fig.2(a) will correspond with that of the position ABCD in Fig.2(b). So the problems of the contact surfaces, whose asperity directions cross each other, can be considered one of the contact between a surface with pyramid-shaped projections and a flat one.

In the case of directional asperities, if the probability density function of asperity heights taken in a certain direc-

Fig. 1. Definition of β and α_0

Fig. 2. Form of an equivalent microscopic projection

Fig. 3. State of the surface asperities in contact

tion α_0 in Fig.1 is approximated by Gaussian distribution, the equivalent probability density function $f(Z)$ (where the contact of rough surfaces is replaced with that between an equivalent rough surface and a flat one) along an arbitrary section of surface is given by

$$f(Z) = N(0, \sigma_e^2) \quad (1)$$

$$\sigma_e^2 = \sigma_1^2 + \sigma_2^2$$

Fig.3 shows these equivalent asperities on the surface $L_x \times L_y$. The probability density function of asperity heights in the direction L_x should be given by Eq.(1).

Every probability density function in the direction L_x at a small distance should be also Eq.(1). Therefore, the equivalent probability density function of the whole surface $L_x \times L_y$ (this surface is called the equivalent surface in this report) can be given by Eq.(1). This is shown by Fig.4.

2.2 Distribution of the microscopic projections in contact

When the probability density function

Fig. 4. Abbott's bearing area of a surface in contact

Fig. 5. Displacement of the surface when a concentrated load Q is applied on a semi-infinite body

of asperity heights in contact is given by $f(Z)$ the distribution of the vertexes of projections will be known from Eq.(26) of the reference paper (4). Put θ_1, θ_2 as the half vertical angles (in the direction $\alpha_0=0^\circ$ in Fig.1) of the projections, then the following relations will be established for the pyramid projections in Fig.2(b).

$$\tan \angle OEC = \overline{\tan \theta_2} / \sin \beta \quad (2)$$

$$\tan \angle OED = \overline{\tan \theta_1} / \sin \beta \quad (3)$$

In Fig.2(b), call Z_1 the height of the vertex E of projections and Z the height of a certain point O . Then the area of the parallelogram $ABCD$ is

$$\frac{2(Z_1 - Z)^2 \overline{\tan \theta_1} \overline{\tan \theta_2}}{\sin \beta}$$

Therefore, the increment of the area $ABCD$ when Z becomes $Z - \Delta Z$ is

$$\frac{4(Z_1 - Z) \overline{\tan \theta_1} \overline{\tan \theta_2}}{\sin \beta}$$

If the probability density for vertexes of projections to appear just at an arbitrary height Z is $\Psi(Z)$ and that for its bottom to appear is $\Psi_R(Z)$, then for a horizontal section of asperities at the height Z , we have

$$A \int_Z^\infty f(Z) dZ = N_T \int_Z^\infty \int_Z^\infty \left[\frac{4 \overline{\tan \theta_1} \overline{\tan \theta_2}}{\sin \beta} (Z_1 - Z) \times \{\Psi(Z) - \Psi_R(Z)\} \right] dZ_1 dZ \quad (4)$$

where, A : nominal contact area, N_T : total number of the projection vertexes belonging to A . By differentiating Eq.(4) with respect to Z , the total number N_Z of the projection vertexes, which are higher than

Z and are involved in the contact area A is

$$N_z = \frac{\sin \beta}{4 \tan \theta_1 \tan \theta_2} \frac{AZ}{\sigma_e^2} f(Z) \quad (5)$$

2.3 The mean distance of the contact points

The definition of the interval of contact points in three-dimensional asperities is difficult. Fig.5 shows the displacement δ of the surface when a concentrated load Q is applied on a semi-infinite body, where r is distance on a horizontal plane from the loading point.⁽⁷⁾ It is known from Fig.5 that the range clearly deformed by the load Q is relatively narrow, in other words, the properties of asperities in neighborhood of a contact point play an important role. From now on, therefore, the analysis may be limited to the interval near the contact points.

Fig.6 shows the standard deviation of asperity heights in an arbitrary direction

SK3, S38C, FC15, ground surfaces,
measured length=0.5mm

Fig. 6. Standard deviations of σ and σ_{ξ} in the direction α_0

Fig. 7. Directions and cross angles of the asperities

α_0 . Here, the experimental values were obtained by measuring within 0.5mm of a ground surface. σ is the value taken in the direction $\alpha_0=0^\circ$ over a long measuring length. When the measured length is short, $\sigma_{\alpha_0=90^\circ}/\sigma \neq 0$. So, the characteristics of asperities within a small range coincide with those of wedge-shaped projections. As seen in Fig.6, σ_{α_0} can be approximated as

$$\sigma_{\alpha_0} = \sigma \cos \alpha_0 \quad (6)$$

Let the slope of wedge-shaped projection be $\tan \theta$. Then the slope $\tan \theta_{\alpha_0}$ in an arbitrary direction α_0 in Fig.1 is

$$\tan \theta_{\alpha_0} = \tan \theta \cos \alpha_0 \quad (7)$$

The standard deviation σ_{ξ} of the slope ξ which is measured by the profilometer with stylus is proportional to $\tan \theta$.⁽¹⁾ If this relation can be applied in an arbitrary direction α_0 , then

$$\sigma_{\xi\alpha_0} = \sigma_{\xi} \cos \alpha_0 \quad (8)$$

where σ_{ξ} is the value for $\alpha_0=0^\circ$. The marks $\bullet$ and the solid curve in Fig.6 show the experimental values and Eq.(8) respectively. The experimental values are approximated well by Eq.(8).

Call the direction perpendicular to the asperity direction (which is the grinding direction for ground surfaces) the base-line of asperities, then σ , $\tan \theta$, σ_{ξ} in Eqs.(6)(7)(8) will be the values on this base-line. The state of the directional asperities ① and ② being in contact with each other at cross angle β as shown in Fig.7(a) can be expressed on the equivalent flat surface as Fig.7(b). Let the surface ① with bigger asperities be the base surface, and the ridge line 1 (base line of the asperity ①) be its base line. Then the standard deviation $\sigma_{e\phi}$ of the asperities in the direction L of angle ϕ and the standard deviation $\sigma_{\xi e\phi}$ of slopes obtained by the profilometer with stylus are given as follows.

$$\sigma_{e\phi}^2 = \sigma_1^2 \cos^2 \phi + \sigma_2^2 \cos^2 (\phi + \beta) \quad (9)$$

$$\sigma_{\xi e\phi}^2 = \sigma_{\xi 1}^2 \cos^2 \phi + \sigma_{\xi 2}^2 \cos^2 (\phi + \beta) \quad (10)$$

The mean distance t_{ϕ} of contact points in the direction L in Fig.7 is given as⁽³⁾

$$t_{\phi} = 2 \overline{\tan \theta_{e\phi}} / f_{\phi}(Z) \quad (11)$$

Here, the subscript ϕ implies the characteristics of asperities along the line L and $f_{\phi}(Z)$ is the probability density function having $\sigma_{e\phi}^2$ of Eq.(9). The value $\overline{\tan \theta_{e\phi}}$ is given by

$$\overline{\tan \theta_{e\phi}} = 0.93 / \sigma_{\xi e\phi} \quad (12)$$

The relation between the mean distance t_0 along the base line 1 and t_ϕ in the direction ϕ is given as follows from Eq.(11) and $f_\phi(Z)=N(0, \sigma_{e\phi}^2)$.

$$\frac{t_\phi}{t_0} = \frac{\overline{\tan \theta_{e\phi}} \sigma_{e\phi}}{\overline{\tan \theta_{e0}} \sigma_{e0}} = \frac{\sigma_{\xi e0} \sigma_{e\phi}}{\sigma_{\xi e\phi} \sigma_{e0}} \quad (13)$$

Here, the subscript 0 implies that $\phi=0^\circ$. σ and σ_{ξ} are in an intimate relation to each other. When ρ is defined as

$$\rho = \frac{\overline{\tan \theta_1}}{\overline{\tan \theta_2}} = \frac{\sigma_{\xi 2}}{\sigma_{\xi 1}} \quad (14)$$

the relation between σ_2/σ_1 ($\sigma_1 \geq \sigma_2$) and ρ for ground surface is shown as Fig.8. Now, let this relation be roughly approximated by the following equation

$$\sigma_2/\sigma_1 = \rho^{3/2} \quad (15)$$

Substituting Eq.(15) into Eq.(13),

$$\frac{t_\phi}{t_0} = \left[\frac{1+\rho^2 \cos^2 \beta}{1+\rho^3 \cos^2 \beta} \frac{\cos^2 \phi + \rho^3 \cos^2 (\beta+\phi)}{\cos^2 \phi + \rho^2 \cos^2 (\beta+\phi)} \right]^{1/2} \quad (16)$$

Fig.9 shows an example of this relation. To simplify the analysis, let the curves in Fig.9 be approximated by an ellipse having the distances along $\phi=0^\circ$ and $\phi=90^\circ$ as its axes. Then, the ratio λ of the minor axis to the major one is given as follows.*

Fig. 8. Relation between ρ and σ_2/σ_1

Fig. 9. Relation between ϕ and t_ϕ/t_0

* The ellipse is chosen such that its area is equal to that calculated from Eq.(16) when the major axis is t_0 .

$$\lambda = \frac{1}{\pi t_0^2} \int_0^{2\pi} \int_0^\phi t_\phi dt_\phi d\phi \quad (17)$$

The relation between λ and β is shown by Fig.10. The degree of above approximation drops in the region of small values of ρ . The relation of Eq.(16) has been defined for the average plane of $f_\phi(Z)$. But actually this relation is supposed to be satisfied at any height.

The mean distance t_T of the contact points defined over a long range along the base line 1 in Fig.7 is defined as follows in the same manner as Eq.(11).

$$\begin{aligned} t_T &= 2 \overline{\tan \theta_{e0}} / f(Z) \\ &= 2 \overline{\tan \theta_1} / \sqrt{1+\rho^2 \sin^2 \beta} f(Z) \end{aligned} \quad (18)$$

Here, $f(Z)=N(0, \sigma_{e0}^2)$. Let the ratio of t_0 to t_T be replaced by λ_T , then

$$\lambda_T = \sqrt{\frac{1+\rho^3 \cos^2 \beta}{1+\rho^3}} \quad (19)$$

The relation between λ_T and ρ is shown in Fig.11. If Eq.(19) is valid for all heights of asperities, then

$$t_0 = \lambda_T t_T = 2 \tan \theta_1 \lambda_T / \sqrt{1+\rho^2 \sin^2 \beta} f(Z) \quad (20)$$

So, the mean distance within the neighborhood of a certain contact point can be obtained.

3. Analysis of the elastic and plastic deformations of the contacting rough surfaces

3.1 Elastic and plastic deformations of microscopic projections

First, the following assumptions are to be made.

- (i) The microscopic projections are wedge-shaped and their vertical angles are 2θ . The elastic deformation can be discussed as a problem of semi-infinite bodies.
- (ii) The probability density function of asperity heights is a Gaussian distribution.

(iii) The contact pressure at the contact points equals to the plastic flow pressure p_m . The lateral plastic flow is negligibly small.

(iv) p_m does not change before and after contact.

(v) When the contact of two rough surfaces is replaced by that between an equivalent rough surface and a rigid flat one, the effective Young's modulus E obtained by the following equation is used.

$$\frac{1}{E} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \quad (21)$$

When two wedge-shaped asperities contact with each other as in Fig.12(a), the normal view of contact point on the effective plane is as shown in Fig.12(b).

Making reference to the coordinates shown in Fig.13, the normal displacement δ caused by the concentrated force Q on the semi-infinite body is calculated from the

following equation. (7)

$$\delta = \frac{1+\nu}{2\pi E} Q \left\{ 2(1-\nu) \frac{1}{(r^2+h^2)^{3/2}} + \frac{h^2}{(r^2+h^2)^{5/2}} \right\} \quad (22)$$

$$r^2 = x^2 + y^2$$

When the plastic flow pressure is applied to the parallelogram ABCD of Fig.12(b), the displacement δ_0 at the center O is,

$$\delta_0 = \int_0^{2\pi} \int_0^{r_0} \delta_{h=0} r dr d\phi_r$$

$$= \frac{1-\nu^2}{\pi E} p_m \frac{l_1}{\rho} S(\beta, \rho) \quad (23)$$

Let the displacement caused by the plastic deformation be z , then

$$l_1 = z \tan \theta_1 / \sin \beta,$$

$$\rho = \tan \theta_1 / \tan \theta_2 = l_1 / l_2$$

$$S(\beta, \rho) = \sin \beta_0 \ln \left\{ \frac{1-\cos(\beta_0+\beta)}{1+\cos(\beta_0+\beta)} \frac{1+\cos \beta_0}{1-\cos \beta_0} \right\}$$

$$+ \sin \bar{\beta}_0 \ln \left\{ \frac{1+\cos(\bar{\beta}_0-\beta)}{1-\cos(\bar{\beta}_0-\beta)} \frac{1+\cos \bar{\beta}_0}{1-\cos \bar{\beta}_0} \right\}$$

$$\tan \beta_0 = \rho \sin \beta / (1-\rho \cos \beta)$$

$$\tan \bar{\beta}_0 = \rho \sin \beta / (1+\rho \cos \beta)$$

When the area ABCD of the contact point is small, the elastic displacement δ_{oh} at the depth h right under the point O is given as follows from Eq.(22).

$$\delta_{oh} = \frac{1+\nu}{\pi E h} p_m \tan^2 \theta_1 (3-2\nu) z^2 \frac{1}{\rho \sin \beta} \quad (24)$$

Fig. 10. Relation between ρ and λ

Fig. 11. Relation between ρ and λ_T

The contact surface coefficient K_1 is defined as follows.

$$K_1 = \frac{4 \tan \theta_1 p_m}{\pi E \sqrt{1+\rho^2}} \quad (25)$$

This K_1 means the conditions of surfaces in contact. In Eq.(25), since $\tan \theta_1 / \sqrt{1+\rho^2}$ is the value of $\tan \theta_e$ for $\beta=0^\circ$, K_1 can be calculated independently of β . So the results of deformations can be compared with those of the contact in which the directions of asperities of two surfaces coincide with each other. The increments $\Delta \delta_0$, $\Delta \delta_{oh}$ by the increase Δz of the plastic deformation are derived from Eq.(23), (24) as follows.

$$\Delta \delta_0 = K_1 \frac{(1-\nu^2) S(\beta, \rho)}{4} \frac{\sqrt{1+\rho^2}}{\rho \sin \beta} \Delta z \quad (26)$$

$$\Delta \delta_{oh} = K_1 \frac{(1+\nu)(3-2\nu)}{4} \frac{\sqrt{1+\rho^2}}{h \rho \sin \beta} \times \frac{\sqrt{1+\rho^2} \sin^2 \beta}{\lambda_T} f(Z) z \Delta z \quad (27)$$

where, $\bar{h}=h/t_0$.

In the contact process of asperities

(a)

(b)

Fig. 12. Normal view of a contact point

Fig. 13. Concentrated force on a semi-infinite body and its coordinates

which was analyzed by the authors⁽³⁾, the increment $\Delta\delta_{me}$ of mean deflection occurring between the free surface and the plane, at the depth h is

$$\begin{aligned}\Delta\delta_{me} &= \int_0^{2\pi} \int_0^\infty (\delta_{h=0} - \delta) r dr d\phi \\ &= \frac{\Delta\sigma_{cm}}{E} h(1+\nu)(1-2\nu)\end{aligned}$$

where, $\Delta\sigma_{cm}$ is the increment of the mean interface pressure. The increment of the load caused by Δz at the contact point is

$$\frac{4 \tan \theta_1 \tan \theta_2}{\sin \beta} p_m \Delta z$$

Therefore, from the contact process of the reference (3), the following equations can be obtained.

$$\Delta\sigma_{cm} = \frac{Zz}{\sigma_e^2} p_m f(Z) \Delta z$$

and

$$\begin{aligned}\Delta\delta_{me} &= K_1 \frac{\pi}{2} (1+\nu)(1-2\nu) \bar{h} \sqrt{\frac{1+\rho^2}{1+\rho^2 \cos^2 \beta}} \\ &\times \frac{\lambda_T Z z}{\sigma_e^2} \Delta z\end{aligned}\quad (28)$$

The increment $\Delta\sigma_{cm}$ of the mean interface pressure in the neighborhood of contact points should be given as $S_1 \Delta\sigma_{cm}$ considering the distribution of the distance of contact points shown in Fig. 14. Since $\Delta\sigma_{cm}$ is proportional to $1/t_s^2$, the coefficient S_1 is calculated from the probability density $R(t_s)$ as follows.

$$S_1 = \left\{ \int_0^\infty \left(\frac{t}{t_s} \right) R(t_s) dt_s \right\}^2 \quad (29)$$

where, t is the mean value of t_s . From the experimental values in Fig. 14(b),

$$S_1 = 2.5 \sim 3.5 \quad (30)$$

Therefore, the relative deflection δ_{oc} between the free surface and the plane at the depth h which is produced by the loads of the contact points around a certain

experimental range: $Z \geq 0$, material: SK3, S38C, FC15, ground surfaces

Fig. 14. Distances of projections which exist above the height Z

point can be written approximately in the form of increment $\Delta\delta_{oc}$ caused by Δz as follows.

$$\Delta\delta_{oc} = \int_0^{2\pi} \int_0^\infty (\delta_{h=0} - \delta) r dr d\phi \quad (31)$$

t_ϕ in Eq.(31) being the radius of the ellipse approximated by Eq.(17). If the values of Eq.(30) are usable for all directions of ϕ , Q in Eq.(22) can be replaced by $S_1 \Delta\sigma_{cm}$ and then Eq.(31) will be

$$\begin{aligned}\Delta\delta_{oc} &= \int_0^{2\pi} \int_0^\infty (\delta_{h=0} - \delta) r dr d\phi \\ &- \int_0^{2\pi} \int_0^{t_\phi/2} (\delta_{h=0} - \delta) r dr d\phi \\ &= S_1 \Delta\sigma_{cm} - K_1 S_1 (1+\nu) \frac{1+\rho^2}{1+\rho^2 \cos^2 \beta} \lambda_T \\ &\times \left\{ (1-\nu) K(1-\lambda^2) - \frac{3-2\nu}{16} \frac{\pi \lambda}{h} \frac{Zz}{\sigma_e^2} \right\} \Delta z\end{aligned}\quad (32)$$

where, $K(1-\lambda^2)$ is the complete elliptic integral of the first kind given by

$$K(1-\lambda^2) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1-(1-\lambda^2)\sin^2 \phi}} \quad (33)$$

The displacement δ_{ce} in the middle of contact points is important to the projections which begin to contact newly with a pressure increase. In this case, the distances between the above-mentioned projections and their neighboring ones in contact play an important role. The value of t_c in Fig. 15 (a) shows the distance between a projection which begins to contact newly at the range $Z \sim Z-\Delta Z$ and its neighboring ones which are higher than Z . The distribution of these t_c 's is shown by Fig. 15(b), where t is the mean value of t_s and the same value as t in Eq.(29).

When the increment $\Delta\delta_{ce}$ of the relative displacement between the free surface and the plane at the depth h , which is caused by the increment Δz , is supposed to be represented by the displacement at the

experimental range: $Z \geq 0$, $\Delta Z = 0.2\sigma$, materials: SK3, S38C, FC15, ground surfaces

Fig. 15. Distance from the projections whose heights fall in the range $Z \sim Z-\Delta Z$ to the neighboring projections higher than Z

middle point of the neighboring ones in contact, $\Delta\delta_{ce}$ can be calculated in the same manner of Eq.(32). But the range of integration t_c in Eq.(32) should be $S_2 t_\phi$ from the relation of Fig.15(b). This coefficient S_2 is given as follows when the probability density of t_c in Fig.15(b) is $\text{Re}(t_c)$.

$$S_2 = \int_0^\infty \left(\frac{t_c}{t}\right) \text{Re}(t_c) dt_c \quad (34)$$

S_2 for the experimental values of Fig.15(b) is

$$S_2 = 1.0 \sim 1.3 \quad (35)$$

So, if this value is applicable to all directions of ϕ , the increment $\Delta\delta_{ce}$ is given as

$$\begin{aligned} \Delta\delta_{ce} &= S_1 \Delta\delta_{me} - K_1 S_1 S_2 (1+\nu) \\ &\times \sqrt{\frac{1+\rho^2}{1+\rho^2 \cos^2 \beta}} \lambda_T \{ (1-\nu) K(1-\lambda^2) \\ &- \frac{3-2\nu}{16} \frac{S_2 \pi \lambda}{h} \frac{Zz}{\sigma_e^2} \Delta z \} \quad (36) \end{aligned}$$

3.2 Determination of the depth h

The depth h is important to calculate the elastic deformation of the microscopic projections of asperities. And it is determined as the distance from the free-surface to the plane where the normal pressure is supposed to be uniform and not to have any remarkable deviation. Actually, this paper deals with the elastic deformation caused by the surface asperities in contact, so the mean deflection caused by the mean interface pressure is not included. Consequently h becomes an important value. When the concentrated load Q is applied on a semi-infinite body, the normal pressure σ_c on the surface at the depth h is

Fig. 16. Distribution of the normal stress on the plane at the depth h

Fig. 17. Relation between $\bar{h}$ and $(\sigma_{co} - \sigma_{cm}) / \sigma_{cm}$

$$\sigma_c = \frac{3Q}{2\pi} \frac{h^3}{(r^2 + h^2)^{5/2}} \quad (37)$$

where the coordinates are the same ones in Fig.13 and the compression component of stress is to be positive. When there are many loads Q 's, the normal pressure will be distributed as in Fig.16 on the plane at the depth h . Let the normal pressure right under a load be σ_{co} and the mean pressure be σ_{cm} , then

$$Q = \frac{\pi \lambda t_0^2}{4} \sigma_{cm}$$

and

$$\begin{aligned} \sigma_{co} &= \frac{3}{8} \frac{t_0^2}{h^2} \lambda \sigma_{cm} \\ &+ \frac{3\sigma_{cm}}{2\pi} \int_0^\infty \int_{t_\phi/2}^\infty \frac{h^3}{(r^2 + h^2)^{5/2}} \gamma d\gamma d\phi \\ &\doteq \sigma_{cm} \left\{ 1 + \frac{15}{256} \frac{t_0^4}{h^4} \lambda (1 + \lambda^2) \right\} \end{aligned}$$

Therefore, the deviation of σ_{co} from the mean pressure σ_{cm} is

$$\frac{\sigma_{co} - \sigma_{cm}}{\sigma_{cm}} \doteq \frac{15}{256} \frac{1}{h^4} \lambda (1 + \lambda^2) \quad (38)$$

The values of Eq.(38) for $\lambda=1$ are shown by Fig.17. Let h of this case be replaced by $\bar{h}_0$. Then the normal pressure distribution is considered to be uniform in the following range.

$$\bar{h}_0 \geq 3 \quad (39)$$

Consequently, for any λ

$$\bar{h} = \left\{ \frac{\lambda(1+\lambda^2)}{2} \right\}^{1/4} \bar{h}_0 \quad (40)$$

3.3 Deformations of the contact regions

Fig.18 shows the deformation process. This figure implies that the rigid flat surface touches the peak a_2 after it has contacted a_1 , where a_2 is Δz lower than a_1 . The value of $(\Delta\delta_0 + \Delta\delta_{oc} - \Delta\delta_{oh})$ is the increment of the elastic deformation caused by the plastic deformation Δz of the projection a_1 . The value of $\Delta\delta_{ce}$ is an increment of the elastic deformation of a_2 caused by the increments of contact loads around the point a_2 . From Fig.18, the following equation is obtained.

$$\Delta z - \Delta z = \Delta\delta_0 + \Delta\delta_{oc} - \Delta\delta_{oh} - \Delta\delta_{ce} \quad (41)$$

Fig. 18. Deformations produced by the contact between a rigid flat surface and an equivalent rough one

If $\Delta z/\Delta Z$ is replaced by dz/dZ , Eq.(41) becomes the following differential equation.

$$\frac{dz}{dZ} = \frac{1}{1+K_1 A(Z, z)} \quad (42)$$

$$\begin{aligned} A(Z, z) = & \frac{(1-\nu^2)\sqrt{1+\rho^2}}{4\rho \sin \beta} S(\beta, \rho) \\ & + S_1(1+\nu)\lambda_T \sqrt{\frac{1+\rho^2}{1+\rho^2 \cos^2 \beta}} \\ & \times \left\{ (1-\nu)(S_2-1)K(1-\lambda^2) - \frac{3-2\nu}{16} \right. \\ & \times \frac{\pi\lambda}{h}(S_2^2-1) \left. \right\} \frac{Zz}{\sigma_e^2} - \frac{(1+\nu)(3-2\nu)}{4} \\ & \times \frac{\sqrt{(1+\rho^2)(1+\rho^2 \sin^2 \beta)}}{h\rho \sin \beta \lambda_T} f(Z)z \end{aligned} \quad (43)$$

From the last two equations, the relation between the plastic deformation z of projections and the asperity height Z is established.

The elastic deformation δ_E of asperities is given as follows in the form of the increment $\Delta\delta_E$ caused by the increment Δz .

$$\Delta\delta_E = \Delta\delta_0 + \Delta\delta_{oc} - \Delta\delta_{oh} - \Delta\delta_{me} \quad (44)$$

Equations (42), (44) are integrated over the range of $Z \sim k\sigma_e$ by giving the initial condition $z=0$ for any Z . Consequently, the relation between δ_E and Z is obtained.

The increment of contact area of a projection caused by the increment Δz is

$$4 \tan \theta_1 \tan \theta_2 z \Delta z / \sin \beta$$

Therefore, the total normal load P on the nominal area A is calculated as follows according to Eq.(5) and the contact process derived by the authors in the previous paper. (3)

$$\frac{P}{P_m} = A \int_0^z \frac{Zz}{\sigma_e^2} f(Z) dz \quad (45)$$

The value z is the deflection caused by the plastic deformation, which occurs until the rigid flat surface touches the projections at the height Z after it has contacted the initial contact position $k\sigma_e$. This z can be obtained by integrating Eq.(42) over the range $Z \sim k\sigma_e$. So, the plastic deformation δ_S is

$$\delta_S = z \quad (46)$$

The above relations are derived from the assumption that there is no change of $f(Z)$ before and after contact. This as-

sumption will be discussed in the next report.

4. Conclusions

This report presents an analysis on the elastic and plastic deformations of surface asperities in contact, where the directional asperities of two surfaces are laid at an arbitrary cross angle. In this analysis, the following items have been considered.

(i) The elastic and plastic deformations of a projection.

(ii) The elastic deformation caused by the loads on its neighboring projections.

(iii) The elastic deformation of the projections which begin to contact newly. The following results were obtained in this report.

(1) The contact of two surfaces having directional asperities can be made equivalent to that between a rigid flat surface and a rough surface with pyramid-shaped asperities.

(2) The elastic and plastic deformations in the contact region can be calculated from Eqs.(44)~(46).

(3) These deformations can be classified according to the contact surface coefficient K_1 given in Eq.(25).

The comparison between the computed results and the experimental data will be shown in the next report.

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