On the Clearance Angles of Skiving Cutter

(Part 2, Helical type cutter
for Internal Spur Gear Skiving)

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Gear skiving is a high productive method to produce internal gears. In the last papers, geometrical relationships between skiving cutter and internal gears have been analyzed. In addition to the tooth profile there are many factors which must be incorporated in the design of skiving cutter, such as the top and side clearance angles. In this paper the clearance angles of skiving cutter have been investigated numerically and the front clearance angle which ensures the whole depth of cut throughout its life has been determined.

1. Introduction

The gear skiving is a new method to produce internal gears. This gear skiving shows a higher productivity than conventional gear shaping. However the axes of skiving cutter and work are non-intersecting and non-parallel, then to achieve a high quality of work much more knowledge are necessary concerning this process.

In the previous papers [1,2], concerned with the geometrical relationships between cutter and work, the author concluded that the analytical treatment of gear skiving is in good agreement with the experimental one. Then, the complicated geometrical phenomena in gear skiving can be controlled with this analytical procedure and computer soft ware.

In this paper, in addition to the previous treatments, the clearance angles of skiving cutter are examined from the geometrical relationships. In the conventional skiving cutter, we choose a side clearance angle of 3-4°, then the front clearance angle is of order of 7-12°. Whether or not these nominal clearance angles are suitable for skiving cutter is not clear. Even in the case of gear shaping such a problem concerning this clearance angle is not yet settled.

In the conventional gear skiving there is a difference in the simulation of cutting action and there is a difference in the profile of work between the leading and trailing side. Further, as mentioned in the previous paper, there is a difference in the life of cutting edge between the leading and trailing side. The cutting edge acts as a fly cutter, then, the actual clearance angles vary with time and are not equal to the nominal ones.

In this analysis the actual clearance angles are defined and the front surface is determined with the condition of constant tooth thickness of work. In this Part 1 we are concerned with the helical type cutter for internal spur gear skiving. In the Part 2 the spur type cutter for internal helical gear skiving will be discussed.

Nomenclature

- \( m \): module
- \( r_1 \): base circle radius
- \( r_2 \): tip circle radius
- \( r_s \): root circle radius
- \( r_x \): pitch circle radius
- \( r_{bc} \): helical angle on base cylinder
- \( r_a \): reduced pitch (\( h=0 \))
- \( \alpha_c \): angle of tooth clearance
- \( Z \): number of teeth
- \( \varphi_c \): gear ratio = \( Z_1/Z_2 \)
- \( \sigma_c \): front clearance angle
- \( \sigma_a \): actual front clearance angle
- \( \beta_a \): side clearance angle
- \( \beta_e \): actual side clearance angle
- \( \xi \): top face angle
- \( \beta_c \): sharpening angle
- \( \rho \): amount of resharpening
- \( \theta \): shaft angle
- \( \zeta \): center distance
- \( \delta \): cutter feed per work rev.
- \( d_c \): offset of cutter
- \( d_g \): gauge pin radius
- \( d_m \): measurement over pins
- \( \omega_r \): angular velocity
- \( \varphi_a \): angle of rotation = \( \omega_r \times t \).

Gear 1 means a work and gear 2 means a cutter. The parameter 1 and 2 represent the quantities of gear 1 and 2 respectively. The parameter \( \xi \) and \( \varphi \) represent the quantities in leading and trailing side. According to Fig.2 the right handed rectangular coordinate system is...
Fig. 1. Helical type cutter for internal spur gear skiving

is used, setting the rotating axes $A_1$, $A_2$ equal to $a$, $b$, and $c$ axes and the common normal from $A_1$ to $A_2$ equal to $X_1$, $X_2$ axes.

In the rotating coordinate of gear, we put $X' = X + yF + ZK$ and in the rotating coordinate of cutter we put $X = X + yF + ZK$.

In the fixed coordinates we put $X$ to these expressions.

The expression $f_0$ means the derivative of $f$ with respect to $\theta$.

2. The clearance angles of the conventional helical pinion type skiving cutter

The condition for generation of the conventional skiving for internal spur gear has been presented in the previous paper [1]. In this paragraph a brief information about the condition of generation of gear is given.

The cutting edge rotates about its axis $A_2$ with angular velocity $\omega_2$, and a blank rotates about its axis $A_1$ with angular velocity $\omega_1$. Then the relative velocity field of skiving can be expressed as follows:

$$\omega = \omega_2 - \omega_1$$

$$= \omega_1 + (\omega_2 - \omega_1)(aT + B_2 + S_2)$$

(1)

According to Fig. 3 the involute helicoid of cutter can be expressed as follows:

$$X = X_1 + s \eta + \beta \kappa$$

$$= \frac{b_2}{2} \left[ (a_2 - a_1 \cos \theta + b_2 \cos \theta) j + \frac{\beta_2}{2} (a_2 - a_1) \theta k \right]$$

(2)

where $n$ is a unit normal to the helicoid,

$$n = \cos \beta \cos (\pi - \theta) + \sin \beta \sin (\pi - \theta) j + \frac{\beta}{2} (\cos (\pi - \theta) - \theta) k$$

and

$$s = \frac{5 \cos \beta_2 \theta^2}{\dot{\theta}^2}$$

Fig. 2. Coordinate system

$$\chi = \chi_0 + \frac{5}{4} \alpha (\xi_0 \cos \beta_0)$$

$$\theta = \frac{\theta_0}{\alpha}$$

(3)

Fig. 3. Involute helicoid
$S_{0a} > 0$ for leading side, $S_{0a} < 0$ for trailing side. The quantities on trailing side can be represented in the same manner.

The cutting edge is defined by the intersection of this helicoid and plane of sharpening.

The plane of sharpening can be expressed by top face angle $\gamma$ and angle of sharpening $\delta$, i.e., the unit normal on this plane of sharpening is,

$$\mathbf{e} = \cos \gamma \mathbf{a} + \sin \gamma \mathbf{c}$$

Then the condition of the cutting edge is,

$$\mathbf{e} \cdot \mathbf{r} = \rho_{am} \cos \delta$$

i.e.,

$$s = \frac{\gamma}{\tan \delta} \left( \rho_{am} \cos \delta \right) + \cos \delta \cos \gamma \sin \delta + \sin \delta \sin \gamma$$

where

$$\rho = \rho_{am} \cos \delta$$

The condition for generation of gear can be expressed by Eqs. (2) and (6) as follows;

$$C = D$$

The above condition can be solved numerically by Newton-Raphson method. The time of generation $t_{g}$ is calculated with respect to the point on cutting edge $\theta$

$$t_{g} = \frac{1}{\rho_{am}} \left( \frac{w_{2} x_{2} f_{2} - f_{2}}{w_{2} x_{2} f_{2} - f_{2}} \right)$$

After the calculation of eq. (8), The expression of work can be obtained as follows;

$$X = X_{1} + Y_{1} + Z_{1}$$

where

$$X = \cos \phi_{1} \sin \phi_{1} + Y \cos \phi_{1}$$

$$Y = -X \sin \phi_{1} + Y \cos \phi_{1}$$

$$Z = Z_{0}$$

Then the profile of the axial section of work can be expressed by,

$$X = X_{1} + \frac{X}{\rho_{am}} \cos \delta$$

where

$$X_{1} = \frac{X_{1}}{\rho_{am}}$$

$$Z_{1} = \frac{Z_{1}}{\rho_{am}}$$

After resharping of the top surface, the quantities in the above equations must be changed:

$$t_{b} = t_{b} - \frac{1}{\rho_{am}} \cos \delta$$

$$t_{b} = t_{b} \cos \left( \frac{X_{1}}{\rho_{am}} + \frac{Y_{1}}{\rho_{am}} \right)$$

where

$$t_{b} = \left( X_{1} + Y_{1} \right) / 2$$

2.1 The side clearance angle

In the conventional skiving the nominal side clearance angle $\alpha_{sc}$ is not zero. As suggested in the introduction the actual side clearance angle $\varepsilon$ during the skiving is not equal to this nominal one.

The actual side clearance angle can be defined as the angle between the relative velocity of skiving and the unit normal on the helicoid of cutter:

$$\varepsilon = \frac{\sqrt{(w_{s} x_{s} + w_{s} y_{s})^{2}}}{w_{s}}$$

The conditions of skiving which have effect on this actual side clearance are picked up as follows:

1. parameters which are determined in the design of cutter: $\gamma_{1}, \beta_{1}, \Delta \beta_{0}$
2. parameters which are set at the sharpening of cutter: $\gamma, \beta_{c}$
3. parameters which are set at skiving: $\beta, \varepsilon, B_{1}, S_{0}$

In this paragraph we are concerned with the second and third terms.

Numerical calculations are executed and shown in Figs. 4 and 5 with respect to the skiving condition in Tab. 1. This condition of skiving is same as in the example in the previous paper. The vertical axis of this figure represents the clearance angle and the horizontal axis represents the radius of work in which the cutting edge locates. The point $D$ represents the point of generation. The cutting edge travels in the space of work along the line $A-B-C-D-E$. From the point $B$, which is above $D$ by $S_{0}/2$, the cutting edge begins to remove chips.

At the point of generation of work, the side clearance angle of this cutter is about 3.1-3.7° in the leading side and 2.6-3.4° in the trailing side. The actual values are greater than the clearance angle of conventional shaping cutter, but lower than the nominal value of 3.5°.

Fig. 6 represents the tooth profile of work in this proper condition.

Figs. 7-9 represent the effect of $\Delta \beta = 2°$, $\Delta \beta_{e} = 5°$ on the side clearance angle relative to the aforementioned proper condition. The effect is little. Fig. 9 represents the tooth profile of work.

Fig. 10-12 represent the effect of $\Delta \beta_{e} = 0°$, $\Delta \varepsilon = -0.5mm$ on the side clearance angle. With this small change of cutting condition, the side clearance angles in the leading and trailing side are balanced. However the actual value is lower than the nominal one by 0.6°, and a great profile error occurs.

Among these parameters the change of shaft angle is more suitable to control the side clearance angle.
Fig. 4 The side clearance angle (leading side)

Fig. 5 The side clearance angle (trailing side)

Fig. 6 The tooth profile of work (normal setting)

Δe: deviation of tooth profile of work from involute
L: roll arc
K: tip of work
F: root of work

Tab. 1 Specifications of calculation

Specifications of work
- \( m = 3 \) mm
- \( z_1 = 72 \)
- \( r_{11} = 101.487 \) mm
- \( \beta_{11} = 114.632 \) mm
- \( \delta_{1} = 107.882 \) mm
- \( \delta_{2} = 215.090 \) mm
- \( r_{2} = 2.52 \) mm
- Cutting condition
  - \( \alpha = 0.5 \)
  - \( \rho = 30^\circ \)
  - \( e = 45.640 \) mm
  - \( S_{h} = 0 \) mm
  - \( \delta_{4} = 0.4 \) mm

Specifications of cutter
- \( z_{2} = 36 \)
- \( r_{11} = -94.206 \) mm
- \( t_{11} = -125.062 \) mm
- \( \beta_{11} = 30^\circ \)
- \( \delta_{11} = 56.609 \) mm
- \( \delta_{12} = 57.654 \) mm
- \( \delta_{13} = 69.205 \) mm
- \( \gamma = 5^\circ \)
- \( \sigma = 9^\circ \)
- \( \beta_{c} = 30^\circ \)
2.2 The front clearance angle

In the conventional gear skiving cutter, the front clearance angle is determined with respect to the side clearance angle by the following formula,

$$\alpha = \alpha_{c} - \left( \tan \beta_{c} / \tan \alpha_{c} \right)$$  \hspace{1cm} (14)

where $\alpha_{c}$ is a pressure angle of cutter. This is a relationship in the pinion type shaper cutter. In the case of skiving cutter, the side clearance angle is set at 3-4° in an ordinary case, then the front clearance angle is of order of 9°.

This front clearance angle in Eq. (14) is also the nominal one and is not equal to the actual one. At the tip of cutter:

$$\beta_{c} = 35^\circ$$ \hspace{1cm} (15)

the unit normal on the front surface is,

$$\mathbf{n} = \cos \gamma \mathbf{i} - \sin \gamma \mathbf{k}$$  \hspace{1cm} (16)
The relative velocity of skiving at the tip of cutter is,

\[ \frac{2 \dot{\alpha}}{6} = \gamma_1 \left( \frac{1}{\sin \alpha} \right) \left[ \frac{v_i}{v_f} \sin \alpha \cos \beta \right] \left[ 1 + \frac{x}{\sin \alpha \cos \beta} \right] \left( \cos \alpha - \cos \beta - \frac{y_1}{\sin \alpha \cos \beta} \right) \]

The actual front clearance angle can be determined by the angle between the normal and the relative velocity:

\[ \Sigma = \cos^{-1} \left( \frac{\sqrt{v_i^2 v_f^2} \cos \beta}{v_{rel}^2} \right) \]  

According to Tab.1 the actual front clearance angle is calculated and shown in Fig. 13. A vertical axis represents the front clearance angle and a horizontal axis represents the radius of work in which the cutting edge locates. The cutting edge travels through A-B-C and removes chips from point C. At point C the actual front clearance angle is of order of 7.5°, which is lower than the nominal one of 9°.
3. The front surface of the skiving cutter

In the case of pinion type cutter for gear shaping, the front surface is determined by the condition to assure the whole depth of cut through its life [3,4]. Under the same condition the front surface can be determined in the case of skiving cutter as represented in Fig.14. In this case the base circle radius is changed in Tab.1 as follows;

\[ R_0 = 56.664 \text{ mm} \]
\[ R_0 = 57.595 \text{ mm} \]

From the figure the front surface of this skiving cutter must be concave. If this front surface is approximated by a cone, the front clearance angle is of order of 9°48'25" and the maximum amount of convex is of order of 2\( \mu \) in the case of Fig.14. Then Eq.(14) is the design formula on safety side.

However this front clearance angle assures only a constant thickness of tooth of work, then the tooth profile of work varies with sharpening as shown in Fig.15. Such error must be eliminated by a change of cutting condition. For instance, in the case of H=5mm, the error in tooth profile of work can be eliminated by \( \phi = 30' \), permitting the error of -60\( \mu \) in the measurement over pins.

4. Conclusion

In the case of helical type cutter for internal spur gear skiving, the actual clearance angles are defined. Numerical calculations are executed on examples.

The results may be summarized as follows:

1. The actual side clearance and front clearance angles are not equal to the nominal ones and there are cases in which the actual clearance angles are lower than the nominal ones. From the numerical results, it is concluded that;
2. The actual side clearance angle is of order of 3.1°-3.7° in the leading side. This is lower than the nominal value of 3.5°. The actual side clearance angle in the trailing side is lower than in the leading side by 0.4°.
3. The actual side clearance angle is lowest at the tip of trailing side and at the root of leading side of cutter.
4. The fluctuation of the side clearance angle during the cutting is greater in trailing side than in leading side.
5. The side clearance angle is controllable by a small change of cutting condition. In this example, the side clearance angle is balanced by \( \Delta \phi = 30' \) in leading and trailing sides.
6. The front clearance angle varies from 7.7° to 27° during the cutting with respect
to the nominal one of $9^\circ$.

(7) The calculated front surface is concave. In this example the conventional front surface is on the safety side with respect to the whole depth of cut.

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References