Fatigue Crack Growth in Polycarbonate *

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The effects of orientation, stress ratio, frequency and temperature on the fatigue crack growth rate are investigated in polycarbonate. A stress intensity factor is used to normalize the crack growth rate data.

It is found that the crack growth rate can be expressed by the power law of the range of stress intensity factors and the constants of the power law are functions of the factors mentioned above. A crack growth law to explain the observed results is proposed. The growth rate law obtained may be applicable to the case of metals and their alloys, although it is introduced by the experiment of the fatigue crack growth in high polymer.

1. Introduction

The fatigue crack growth rate of the region II shown in Fig.1 may be expressed as a function of the range of stress intensity factors ΔK as follows⁽¹⁾

$$dc/dN = A_1 (\Delta K)^m \quad (1)$$

where A_1 and m are constants dependent on materials.

In order to clarify the fatigue crack growth mechanism, the functional form of A_1 and m must be investigated. These constants may be influenced by several governing factors, which are stress ratio, frequency, temperature, environment and so on.

Unfortunately, not so many studies have hitherto been made on the relation of the constants to the factors mentioned above, except for the theoretical study made by Yokobori⁽²⁾⁽³⁾ and the experimental study made by Wei⁽⁴⁾⁽⁵⁾. The fatigue crack growth mechanism, therefore, has thus been left in the dark.

Recently, the studies on the fatigue crack growth in high polymers have been made⁽⁶⁾⁽⁷⁾. The results show that the similarity of the crack growth behaviour exists between metals and high polymers on the macroscopic scale⁽⁹⁾. However, many uncertainties are left, since the crack growth study of high polymers is in the early stage for comparison with that of metals.

The present experiment was carried out to investigate the effects of stress ratio, frequency and temperature on the constants A_1 and m of Eq.(1) by use of high polymer, namely polycarbonate. It may be expected that the results are applicable to the study of metals because of the general approach considering the fatigue crack growth as rate controlling process.

2. Material and experimental procedure

The material used was commercially available polycarbonate(PC) plate with thickness of 1 mm. Rectangular specimens shown in Fig.2 were cut out of this plate. In all specimens, one single side notch was made by a fret saw and a razor blade. From the observation through a polarized microscope, the region, in which residual strain was induced by notch making, was found to be limited to the vicinity of a crack tip. The direction of the artificial crack is perpendicular to the rolling direction of the plate except in the orientation dependence experiment.

The testing machine used in this experiment was a 100 kg Instron type tensile testing machine.

The length of a fatigue crack was measured by a travelling microscope with a magnification of approximately x40 which

Fig.1 Schematic illustration of fatigue crack growth rate characteristic

Fig.2 Test piece

* Received 10th July, 1972.

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was equipped with a dial indicator of one division 0.01mm. The crack length was measured at reasonable intervals without stopping the test. The rate of fatigue crack growth was obtained by counting the number of cycles required to extend a crack to a specified length.

For the frequency dependence experiment, a spring was attached to the testing machine for controlling the cross head speed. Then three kinds of frequencies (about 30 cpm, 2cpm, 0.2cpm) were obtained. For the frequency of about 30cpm, a specimen shown in Fig.2(a) was used and for the relatively low frequencies of about 2cpm and 0.2cpm, a specimen shown in Fig.2(b) was used. In the case of the frequencies of about 2cpm and 0.2cpm, the growth rate was measured from mean striation spacing, which was found to correspond to the macroscopic growth rate. Usually the test was carried out at a frequency of about 30 cpm.

For the experiment of the temperature dependence, a constant temperature furnace was constructed and used. The fluctuation of temperature in the furnace was within $\pm 2^\circ\text{C}$ at 100°C . The fluctuation to this extent might be expected to give only a little effect on the result below. Furthermore, the temperature increase of a specimen due to cyclic loading was able to be neglected because of the relatively low frequency.⁽¹⁰⁾

The range of stress intensity factor

Fig.3 Effect of orientation on fatigue crack growth rate

Fig.4 Details of making a specimen with necking part

ΔK is generally given as $Y \Delta \sigma \sqrt{c}$ ⁽¹¹⁾ where Y is a correction factor dependent on the ratio of a crack length c to a specimen width, $\Delta \sigma$ is the range of applied gross section stresses. But the critical stress intensity factor of the material used is nearly constant without considering the correction factor.⁽¹²⁾ In this paper, therefore, the value of $\Delta \sigma \sqrt{c}$ was adopted as that of ΔK .

3. Results and discussion

3.1 Effect of orientation In Fig.3, the effect of orientation on the fatigue crack growth rate is shown. The open circles in Fig.3 correspond to the crack growth parallel to the rolling direction of the material. These points nearly fall on a solid line which corresponds to the crack growth perpendicular to the rolling direction. The difference of the orientation to this extent gives no effect on the crack growth rate. Figure 3 also shows the crack growth rate of a specimen which has suffered tensile plastic deformation by the open triangle. This was investigated with the specimens, the details of which are shown in Fig.4. At first, the material used was loaded over the yield point. And then it was deformed in the plastic range. Finally, the necking part was cut out of the deformed plate in the form of the test specimen. The thickness of the necking part was nearly constant (0.71 mm) independently of the extent of tensile elongation. The specimens made in this way may be considered to have a strong orientation, since the chain of high polymer becomes unidirectional due to the plastic deformation. From Fig.3, in the case of such a strong orientation, the crack growth rate data are found to be considerably different from the result of the as received material. Then it may be concluded that the constants A_1 and m in Eq. (1) depend on the orientation of the material. This trend is similar to the relation that as the amount of ausformed treatment increases, the fatigue crack growth rate decreases in 12 Cr stainless steel.⁽¹³⁾ This may be caused by the fact that

Fig.5 Effect of stress ratio on fatigue crack growth rate

material constants, for example surface energy, exponent of strain hardening etc., may be changed with a changing orientation.

3.2 Effect of stress ratio The effect of stress ratio (the ratio of maximum gross section stress to minimum gross section stress) is shown in Fig.5. The data points at each stress ratio constitute the lines different from the others. In taking account of the previous result, the fatigue crack growth in high polymers may be considered as strongly influenced by the

Fig.6 Schematic illustration of loading method

Fig.7 Examples of variation of striation spacing during the interval at nearly constant load

Fig.8 Delayed crack growth curves at constant stress intensity factor in air

stress ratio. The solid and dashed lines in Fig.5 show respectively Forman's semi experimental relation⁽¹⁴⁾ and Broek's experimental relation⁽¹⁵⁾. It was reported that both equations were in good agreement with the results of aluminum alloys. In the case of PC used in this experiment, both equations approximately explain the trend of the effect of stress ratio. In this paper, Eq. (2) is used for simplicity.

$$dc/dN = A_2 e^{\delta P} (\Delta K)^m \quad (2)$$

where P is stress ratio and A_2 and δ are material constants. This may be applicable by assuming δ as zero to the result of materials like steel, which are considered to be unaffected by stress ratio. The lines in the figure are drawn with $\delta = 4.4$.

3.3 Effect of frequency For materials like high polymers which are strongly influenced by testing time itself, the contribution of frequency to fatigue crack growth may not be neglected. In general, the fatigue crack growth rate may be written as a function of the load repetition effect I_N and the delayed fracture effect I_D in the following form

$$dc/dN = f(I_N, I_D) \quad (3)$$

In investigating the delayed fracture effect, an experiment was carried out as follows; Some specimens were cyclically loaded as shown in Fig.6 and then the relation between a constant load duration time and a striation spacing observed at fracture surface by an optical microscope with a magnification of $\times 100$ was studied. Previous to the constant loading, two kinds of cyclic loading (about 30cpm and 0.2cpm) were applied. In the case of 30 cpm, applied load was decreased a little during loading due to the stress relaxation effect of PC. But a decrease of the load was not recognized in the case of 0.2 cpm. An example of fracture surfaces obtained in this way is shown in Fig.7 and the results are shown in Fig.8. The length of a crack increases with an increasing time elapsed. But the growth rate decreases and the crack length approaches a constant length probably corresponding to the stress intensity factor. Such a trend was observed in a 15 hours constant load

Fig.9 Delayed crack growth mode

test. The crack growth curves may be expressed within a small time duration as follows

$$c = c_0 + \Delta c \quad (4)$$

$$\Delta c = \alpha t^n \quad (5)$$

where c_0 is the initial crack length, α and n are material constants. The solid lines in Fig.8 are drawn with $n=0.35$.

On the other hand, a crack continues to propagate without load repetition under some environmental liquids, for example, kerosene, acetanolum, etc. In this case, the crack growth rate is to be constant at a constant stress intensity factor and so n takes a value of 1 in Eq.(5).⁽¹⁶⁾

From the above discussion, we find that the delayed crack growth is classified in two types. The schematic illustration of these types is shown in Fig.9. The type A is characteristic in air and the type B is in the environment.

Now let us consider the load repetition effect. When the crack growth occurs also due to the load repetition effect only, Eq.(3) may be written as the additional form of $I_N + I_D$. On one hand, when the load repetition effect only increases the amount of damage at the crack tip, the effect may be involved in Δc of Eq.(4). In this case, the crack growth formula may be expressed in the form of the product of I_N and I_D . When the testing time t in Eq.(5) is substituted by W^{-1} and Δc is expressed as a power function of K , the growth rate dc/dN may be expressed as follows

$$dc/dN = A_3 W^{-n} (\Delta K)^m \quad (6)$$

where A_3 is a material constant and W is a frequency. This is formally identical to Barsom's relation.⁽¹⁷⁾

The effect of frequency on the fatigue crack growth rate is shown in Fig. 10. The growth rate increases with a decreasing frequency. But the slopes of the growth rate on a log-log diagram are nearly constant independently of frequency. Then it may be concluded that the exponent m is invariable with frequency. The solid lines in Fig.10 show Eq.(6) taking the value n equal to that of the case of delayed crack growth, namely $n=0.35$. Equation(6) is in good agreement with the experimental result. Mukherjee and Burns⁽¹⁸⁾ obtained $n=0.43$ by using polymethyl methacrylate. This is nearly equal to the present result. On the other hand, n is almost equal to 1 under kerosene.⁽¹⁶⁾ Therefore, we may conclude that the fatigue crack growth is closely related to the delayed crack growth. According to this assumption, the value n in Eq.(6) may take a value smaller than 1 in the delayed crack growth of type A and a value equal to 1 in type B.

In future, the fatigue crack growth mechanism must be treated in taking account of the effect of delayed crack growth. And furthermore, the temperature dependence of n must be also investigated.

3.4 Effect of temperature Figure 11 shows the effect of temperature on a log-log diagram. In general, the power law relationship is maintained at any temperature lower than glass transition temperature (about 150°C for PC). However, the slope at each temperature in Fig.11 is slightly different from each other. The exponent m decreases with an increasing temperature. Figure 12 shows the value of m obtained from the lines in Fig.11 as a function of the reciprocal of absolute temperature T . For PC used in this experiment, the value of m is nearly proportional to $1/T$. This is in good agreement with the nucleation theory proposed by Yokobori.⁽²⁾ Then the relation(1) may be written as

$$dc/dN = A_1 \left[(\Delta K)^{T_0/T} \right]^\eta \quad (7)$$

where T_0 is the normalized temperature and η is a constant. Equation(7) indicates that the growth rate data at different temperature fall on parallel lines on a $(T_0/T) \log(\Delta K)$ vs. $\log(dc/dN)$ diagram. These plots are shown in Fig.13. As may be seen, the data are in good agreement with the relation(7). The data for the constant modified stress intensity factor range $(\Delta K)^{T_0/T}$ are presented in the Arrhenius

Fig.10 Effect of frequency on fatigue crack growth rate

Fig.11 Effect of temperature on fatigue crack growth rate

Fig.12 Temperature dependence of the exponent m

Fig.14 Fatigue crack growth rate vs. reciprocal of temperature

Fig.13 Relation between fatigue crack growth rate and modified stress intensity factor

diagram in Fig.14. A linear relation between $\log(dc/dN)$ and $1/T$ can be obtained. Hence, the coefficient A_1 can be described as

$$A_1 \propto \exp(-Q/RT) \quad (8)$$

where Q is the apparent activation energy and R is a gas constant. As shown in Fig. 12, the other results of James⁽²¹⁾ and Ryder⁽²²⁾ show that m is a more complex function of T . Then we had better write the exponent as $m(T)$ in place of m . Equation(1) may be expressed as

$$dc/dN = A_4 (\Delta K)^{m(T)} \exp(-Q/RT) \quad (9)$$

where A_4 is a material constant. Equation (9) is formally identical to the creep law proposed by Dorn and the failure rate law of reliability engineering⁽¹⁴⁾ when a stress term is substituted into the term ΔK .

According to Eq.(9), the ratio of the crack growth rate at temperature T_1 to that at temperature T_2 is given as follows

$$\left(\frac{dc}{dN}\right)_{T_1} / \left(\frac{dc}{dN}\right)_{T_2} = \exp \left[-\frac{Q}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \right] \times$$

$$(\Delta K)^{m(T_1) - m(T_2)}$$

When $m(T)$ does not depend on temperature, i.e. $m(T_1) = m(T_2)$, Q can be directly obtained from the above relation. But when $m(T)$ varies with a changing temperature, the term $(\Delta K)^{m(T_1) - m(T_2)}$ takes a value dependent on the units of ΔK and then Q takes different values with the selection of the units of ΔK .⁽²⁰⁾ When we express the unit of ΔK in $\text{kg-mm}^{-3/2}$, the value of Q in this study is about 7.1 kcal/mol. In order to relate a physical phenomenon to the activation energy directly obtained in this way, the term of ΔK may be necessarily expressed as dimensionless form. Equation(9) may be understood by considering the fatigue crack growth process as an activated one.

As discussed in the previous chapter, the several governing factors, i.e., stress ratio, frequency and temperature of the fatigue crack growth in PC were investigated. The fatigue crack growth rate law may be obtained in a synthetic form as follows

$$dc/dN = A \omega^{-n} e^{Sp} (\Delta K)^{m(T)} \exp(-Q/RT) \quad (10)$$

where A is a constant dependent on materials. As the constants A , n , S , $m(T)$ and Q may be expected to depend on environmental liquids, the experiment of an environmental fatigue will be required in future.

3.4 Microscopic observation Fig.15 shows examples of fracture surface observed by an optical microscope. Clear striations peculiar to the fatigue fracture surface of PC are observed at temperature lower than 30°C. These spacings are nearly equivalent to the macroscopic crack growth rate. The measurement by use of a surface roughness meter indicates that the depth of striation is nearly proportional to its spacing.⁽¹²⁾ At higher temperatures, numerous crazes or microcracks are generated at the vicinity of a crack tip and their generation zone extends all over the specimen. Microcracks appearing at fracture surface also increase with an increasing temperature. At the fracture

surfaces of the microcracks, striations are also observed. The spacings of such striations are larger than those of the middle section of the specimen. This fact indicates that the crack growth rate at the microcracks is greater than that at the middle section of the specimen. Furthermore, the fracture surface of the middle section exhibits a complex aspect

(a) Middle section of a specimen tested at 30°C

(b) Middle section of a specimen tested at 100°C

(c) Side section of a specimen tested at 55°C

(d) Side section of a specimen tested at 100°C

Fig.15 Examples of fracture surfaces by use of optical microscope

different from that of a specimen tested at 30°C. Besides the so called striations, many stripes are observed. These are convex to the crack growth direction. According to our observation of the fracture surface at the temperature lower than 30°C, the length of a crack at the middle section of a specimen is found to be nearly equal to that of the surface. But at higher temperatures, the crack front of the middle section is observed before that of the surface.

When the testing temperature approaches the glass transition temperature, the maintenance of a constant load becomes difficult due to the great stress relaxation effect of the material and the growth rate may not be able to be measured.

4. Conclusions

The effects of some governing factors on the rate of fatigue crack growth were systematically studied in high polymer, i.e. polycarbonate. This experiment was carried out as a series of studies on the fatigue crack growth in high polymers. The results are summarized as follows;

(1) The fatigue crack growth is strongly influenced by stress ratio and the effect may be expressed as $e^{\delta P}$.

(2) In the frequency range of this experiment, the rate of fatigue crack growth is proportional to W^{-n} . The exponent n may be expected to depend on the type of delayed crack growth and take a value smaller than 1 or equal to 1 according to the type of the crack growth.

(3) In general, the power law exponent m varies with a changing temperature. The temperature dependence of fatigue crack growth is formally identical to the theory obtained from a rate controlling process.

(4) The density of microcracks or crazes appearing at a specimen surface increases with an increasing temperature. And the fracture surface has a more complex aspect and at the middle section of a specimen, many stripes convex to the crack growth direction are observed besides the so called striations.

(5) The law of fatigue crack growth rate may be obtained as a function of stress ratio, frequency and temperature in the following form

$$dc/dN = A w^{-n} e^{\delta P} (\Delta K)^m (T) \exp(-Q/RT)$$

The author wishes to acknowledge the encouragement and useful discussions of Prof. T. Yokobori, Tohoku University, in the preparation of this paper. Also thanks are expressed to S. Tozawa and K. Motomura Ibaraki University, for their cooperation in this experiment.

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