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Stress Analysis of a Bolt-Nut Joint by the Finite Element
Method and the Copper-Electroplating Method*

(2nd Report, Stress at the Root of a Bolt Thread under a Tensile Load)

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This paper presents a stress analysis at the root of a bolt thread by the finite element method and the copper-electroplating method which were used for the stress analysis of V-groove in the 1st report. Firstly, as a three dimensional axially symmetrical problem, the load distribution at the bolt thread which is screwed in a nut thread, is computed by the finite element method combined with the point matching method. Then, the stress concentration factor under this load distribution is computed. Secondly, a measuring method of the stress concentration factor by the copper-electroplating method presented in the 1st report, is applied to the bolt threads with three root radii. The obtained results are as follows: (1) The numerical solution of the stress concentration factor by the finite element method shows a fairly good agreement with the experimental value by the copper-electroplating method; (2) the deviation of the maximum stress point at the root of a thread toward the loaded flank becomes larger with an increase of the root radius; (3) even if the root radius is increased over the range which is specified in ISO Standard, the stress concentration factor is scarcely improved.

1. Introduction

One of the important factors for the rational design of a bolt-nut joint is the stress distribution at the root of bolt threads. As the first step to analyze the stress distribution, it was confirmed in the first report⁽¹⁾ that the stress distribution at the root of V-grooved rods could be computed by the finite element method and that then the stress concentration factors derived from them showed a very good agreement with the experimental values by the copper-electroplating method. In computing the stress at the root of bolt threads, the first question to be studied is the load distribution at the bolt thread which is screwed in a nut thread. As a method to analyze the load distribution, Maduschka's⁽²⁾ method has been used generally. But his solution is an approximate one for a two-dimensional problem, and its strict solution has not been found. Therefore, firstly, as the three-dimensional axially symmetrical problem of a contact between two elastic bodies, the load distribution at the bolt threads is computed by the finite element method⁽⁵⁾ combined with the point matching method^{(3),(4)} and its value is compared with Maduschka's one. Secondly, the stress at the root of the bolt thread un-

der this load distribution is computed, and the stress concentration factor derived from it is compared with the experimental value by the copper-electroplating method⁽⁶⁾.

2. Computation of stress by the finite element method

Firstly, the load distribution at the contact surfaces between the bolt and the nut, namely the flanks of screw threads, is computed by the finite element method combined with the point matching method which is an approximate analytical method for an elastic contact problem. Then, the load distribution at every screw thread is computed. Secondly, the stress at the root of screw thread under this load distribution is computed. The method to solve an elastic problem of a contact between two bodies by the finite element method combined with the point matching method has been reported in the literature⁽⁷⁾, therefore omitting the detailed explanation of this method, the form of the stress analysis model, contact condition and so on are explained in this report.

2.1 Model of stress analysis

Supposing the case that a plate or a cylinder is fastened by a bolt and a nut as shown in Fig. 1, the stresses in the portion where the bolt is screwed in the nut, and in its neighborhood are analyzed. The dimensions of the bolt and nut threads correspond to the metric coarse thread M24. It is supposed that the form of the nut is circular and its cross-sectional

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area is about equal to that of a hexagon nut, and that their threads are connected not as a spiral, but as annular multi-grooves, in order to simply analyze the problem as an axially symmetrical three-dimensional one. Therefore, taking a symmetrical form into consideration, an analysis is carried out with a hatched section shown in Fig. 1. In addition, it is supposed that the bearing surface of the nut is fixed in the axial direction in order to simplify the analysis.

2.2 Element division and loading condition

Figure 2 shows the element division of the threaded portion of bolt and the nut. In the bolt, the number of nodes is 496 and the number of elements is 825. In the nut, the number of nodes is 374 and the number of elements is 626. At the lower end of the bolt, a constant dis-

Fig. 1 Model of stress analysis

Fig. 2 Element division

placement in the axial direction is given and the bearing surface of the nut is fixed in the axial direction. Table 1 shows the dimension of the screw threads. The load distribution at the screw threads was computed only in the case that the root radius at the bolt thread is 0.4 mm.

2.3 Contact condition at the thread flank

Disregarding the friction at the flank of thread, a pressure is applied perpendicularly on the flank. In Fig. 3, expressing the displacement of the node i at the flank of the bolt thread which is screwed in the nut thread by δ and the components of δ in the directions of r and z by δ_{ir} and δ_{iz} , respectively, the component δ_i in the normal direction of the flank is expressed as follows:

$$\begin{aligned}\delta_i &= \delta \cos(\alpha - \gamma) = \delta \cos \alpha \cos \gamma + \delta \sin \alpha \sin \gamma \\ &= \delta_{iz} \cos \gamma + \delta_{ir} \sin \gamma\end{aligned}\quad (1)$$

Similarly, expressing the component of the displacement of the node at the flank of the nut thread corresponding to the bolt thread by δ'_{ir} and δ'_{iz} , the component δ'_i in the normal direction of the flank is expressed as follows:

$$\delta'_i = \delta'_{iz} \cos \gamma + \delta'_{ir} \sin \gamma \quad (2)$$

Expressing the number of nodes on the contact surface by m , the following expression must be satisfied in order that the nodes may be closely in contact with each other after the deformation:

$$\delta_i + \delta'_i = C \quad (i = 1, 2, 3, \dots, m) \quad (3)$$

Table 1 Dimensions of thread (unit: mm)

	Root radius	Major diameter	Pitch diameter	Minor diameter
Bolt thread	0.2mm	24.0	21.898	19.7
	0.4	24.0	21.898	20.1
	0.6	24.0	21.898	20.5
Nut thread	-	Major diameter	Pitch diameter	Minor diameter
	-	24.2	22.122	21.2

Fig. 3 Contact condition

where C is a constant.

Then, expressing the nodal forces applied on the nodes of the contact surface by P_i and P_i' , respectively and the influential coefficient by c_{ij} and c_{ij}' , respectively, δ_i and δ_i' are expressed as follows:

$$\delta_i = c_{i1}P_1 + c_{i2}P_2 + \dots + c_{ij}P_j + \dots + c_{im}P_m = \sum_{j=1}^m c_{ij}P_j \quad (4)$$

$$\delta_i' = \sum_{j=1}^m c_{ij}'P_j' \quad (5)$$

The magnitude of the nodal force P_i applied on the bolt is equal to that of P_i' applied on the nut, but the working direction is in the opposite direction. Therefore:

$$P_i = -P_i' \quad (6)$$

The component of the sum of the nodal forces P_i in the direction of axis is equal to the axial force T of the bolt. Therefore the following expression is obtained:

$$T = \sum_{i=1}^m P_i \cos \gamma \quad (7)$$

If the axial force T is given, P_i is obtained from Eqs. (3)~(7). By summing up this P_i in every thread, the rate of the load distribution is obtained.

2.4 Computation of the stress at the root of bolt thread

The computation is carried out with the element division such that the portion of the root in the element division shown in Fig. 2 is divided finer; namely the computation is carried out by the unit

Fig. 4 Element division near the root of thread (in case of $\rho=0.4$ mm)

Fig. 5 Load distribution at the threads

(a) $\rho=0.2$ mm [$\alpha_{\max}=4.8$, condition of computation (element division at the root): number of nodal point 288, number of element 512]

(b) $\rho=0.4$ mm [$\alpha_{\max}=4.4$, number of nodal point 237, number of element 424]

(c) $\rho=0.6$ mm [$\alpha_{\max}=4.2$, number of nodal point 203, number of element 357]

Fig. 6 Principal stress near the root of threads

element division method. Figure 4 shows the case that the root radius ρ is equal to 0.4 mm. The computation is carried out by giving the displacements which are obtained under the element division as shown in Fig. 2, to the nodes on the boundary. In computing, modulus of elasticity $E = 21000 \text{ kg/mm}^2$ and Poisson's ratio $\nu = 0.3$ are used.

2.5 Results of computation

Figure 5 shows the rate of the load distribution at the screw thread in the case that the root radius ρ is equal to 0.4 mm. The rate of the load distribution is changed by the pitch error between the bolt thread and the nut thread. Therefore, here is treated the case that the pitch error can be disregarded. The figure also shows Maduschka's value for comparison. The load applied on the first thread is greater than Maduschka's value, but those on the second and third threads and so on are smaller a little. In addition, as one of the reasons why the rate of the load distribution at the sixth thread is a little greater than the fifth thread, it is considered that the upper portion of the nut is bent inward.

Figure 6 shows the distribution of the principal stresses in r - z plane near the root of the bolt thread. The point plotted in the figure shows the ratio of the stress at the node to the nominal stress at the cross-section of the root perpendicular to the axis. The stress at a node is computed as the mean value of the stresses in the elements around the node. At the first root from the bearing surface, namely expressing by the number of threads or the multiple of pitch from the bearing surface, at the $0.5 \times$ pitch, the maximum tensile stress occurs and at the second root the value of stress is reduced to about $1/3$. The angular position of the point where the maximum stress occurs at the first root tends to approach a loaded flank with an increase of the root radius. It is considered as the reason for this that the influence of the bending stress due to the bend of the screw thread becomes greater than that of the stress concentration by the tensile stress occurring at the notched root, with an increase of the root radius. This tendency is remarkable at the root on and after the second thread.

3. Experiment by the copper-electroplating method

The stress distribution at the root of the bolt thread is measured by the copper-electroplating method. The bolt used in the experiment consists of a tapered part and a threaded part so that the principle of measurement presented in the 1st report can be applied. Here, the principle of measurement is explained briefly, again; A cyclic load P is applied between the bearing surfaces of a copper-

electroplated bolt and a nut as shown in Fig. 7. If the flecks spread as far as the diameter d_t in the tapered part of the bolt, and the flecks in the root of bolt thread are recognized for the first time, when the number of repetitions is N , it is considered that the stress in the tapered part is equal to that at the root of screw thread. Therefore, if it is investigated how far the flecks spread in the axial direction of the tapered part when they are recognized at the root for the first time, stopping the fatigue testing machine often, the stress concentration factor at that point is obtained as the square of the ratio of the diameter of the tapered part where the flecks are recognized, to the minor diameter d of the screw thread.

3.1 Specimens

Figure 8 shows the dimensions of test pieces used in the experiment. Figure 7 shows a photograph of test pieces. Test pieces consist of a bolt and a nut with screw thread of M24 (pitch: 3 mm). The bolt has a tapered part at the shank in order to calibrate the stress. The allowable root radius of the bolt thread in ISO Standard, is from 0.3 mm to 0.43 mm for M24. In this experiment, the wider range of root radius is adopted, namely their radii are 0.2, 0.4 and 0.6 mm. The number of test pieces for the above three kinds of root radius is five respectively. The bolt thread is finished by grinding after turning. It is considered that the ma-

Fig. 7 Photograph of test piece

(a) Bolt

(b) Nut

Fig. 8 Dimensions of test piece

chining accuracy of the root has an important effect upon the stress concentration. Therefore special attention was paid to grinding. Figure 9 shows magnified photographs of the roots under the toolmaker's microscope with the radii ρ of 0.2, 0.4 and 0.6 mm. It is seen in these photographs that arcs are smoothly connected to straight lines of flanks. The accuracy of the root radii was within ± 0.02 mm, overlapping the magnified circles drawn on a transparent plate concentrically on them.

The accuracy of the screw thread was measured by the SIP universal measuring apparatus MU-214B. Figure 10 shows pitch accuracy on the loaded flanks of bolt threads. The abscissa shows the number of threads and the ordinate shows the cumulative pitch error. Though the pitch of bolt thread is somewhat larger than the specified one, its accuracy is very good as a bolt for fastening.

The nut thread was finished by fluteless tapping after rough turning. Figure 11 shows pitch accuracy at the thread-groove center of nut thread. Though it is desirable that the accuracy of pitch at the loaded flanks be measured for the nut, the pitch of the thread-groove center was measured because of the difficulty for measuring. In addition, the machining accuracy of the screw threads of the bolts and the nuts was within the range of tolerances in ISO 6H/6g for the major, pitch, minor diameters and so on.

Table 2 shows the materials of the test pieces. In measuring the stresses by the copper-electroplating method, the basic metal often fails by fatigue before

flecks appear. Therefore, a chromium-molybdenum steel is used as the material of the bolts. The bolt is turned roughly, heat-treated, and finally finished by grinding. The material of the nut is a carbon steel for machine structural use. These materials are generally used as those of a bolt-nut joint on which a high tensile load acts. In addition, special attention was paid to the machining of the bearing surfaces of the bolts and nuts to minimize the bending load. In finishing the bearing surface of the nut, a special male screw was used as a jig. As a result, the squareness of the bearing surface to the axis was within 0.02 mm.

3.2 Copper-electroplating

Table 3 shows the composition of

Fig.10 Pitch accuracy of the bolt threads

Fig.11 Pitch accuracy of the nut threads

Table 2 Materials of specimens

	Material	State as received	Chemical composition %									Heat treatment	Hardness
			C	Si	Mn	P	S	Cu	Ni	Cr	Mo		
Bolt	SCM 3	Bar of dia. 38 mm hot rolled	0.35	0.25	0.75	0.08	0.20	0.21	0.06	0.99	0.17	Quench and temper	H _V 300
Nut	S45C	Width across flats 35 Bar of cold finished hexagon steel	0.48	0.30	0.73	0.022	0.018	0.03	—	—	—	As received	H _V 238

Fig. 9 Magnified photograph of the root of bolt thread

electroplating solution and the conditions of electroplating used in this experiment. Firstly, the preliminary plating is carried out for the threaded part and the tapered part of the bolt, and secondly the additional plating is done.

3.3 Tension fatigue test

In order to give a tension to the bolt and the nut, two U-shaped attachments as shown in Fig. 12 are used. They receive a compressive force from the fatigue testing machine and give a tensile one to the test piece. In order to apply a cyclic load to the test pieces, Kuno's tension and compression fatigue testing machine RTC-K10 is used (Fig. 13). Table 4 shows the cyclic tensile loads acting on the test pieces. In the case that the root radius at the screw thread is large, the higher cyclic load is applied so that the number of flecks appearing should not be reduced.

Table 3 Composition of electroplating solution and conditions of electroplating

	Preliminary plating		Additional plating	
Composition	CuCN	23 g	CuSO ₄ ·5H ₂ O	250 g
	NaCN	30 g	H ₂ SO ₄	80 g
	Na ₂ CO ₃	10 g	H ₂ O	1000 cc
	H ₂ O	1000cc		
pH	11.50		0.18	
Current density	0.006 mA/mm ²		0.30 mA/mm ²	
Temperature	30°C		23°C	
Bath voltage	0.46V		0.70V	
Duration of plating	20min		15min	

Fig.12 Bolt and nut set on the U-shaped attachments

3.4 Results of experiments

Figure 14 shows an example of the measurements for the stress distribution. The root radius of the bolt thread is equal to 0.4 mm and the abscissa shows the number of threads from the bearing surface. The length of thread engagement is equal to $6.3 \times \text{pitch}$. The ordinate shows the stress concentration factor. The hatched area corresponds to the part where the flecks appear when stopping the fatigue testing machine at some number of repetitions. Most of flecks appear between the bearing surface and the first thread. It is desirable that the experiment be carried on until the flecks appear at all roots of threads. For this purpose, a cyclic load must be increased. But, if the cyclic load is increased more than the above mentioned value, the bolt may fail by fatigue. Therefore, the experiment was carried out with the cyclic load where the maximum stress concentration factor would be obtained. In obtaining the mean value between 0 and the first thread, the part where flecks do not appear yet, is treated

Table 4 Acting load

Root radius at the bolt thread mm	Mean load P_m kg	Oscillating load P_a kg
0.2	2.0×10^3	0.8×10^3
0.4	2.0	0.8
0.6	2.4	1.0

Fig.13 Kuno's tension and compression fatigue testing machine RTC-K10

as follows: By supposing that flecks appear at the whole root between 0 and 1st thread in the step which is lower than the lowest step of the stress concentration factor by the part corresponding to the mean value of the difference between adjacent steps, the mean value is obtained. Figure 15 shows the relationship between the stress concentration factor obtained in the experiment and the root radius of bolt thread. The irregularities in the stress distribution in Fig. 14 may be due to the difference of contact states at the flanks of the bolt and the nut thread.

Figure 16 shows the photographs of

Fig. 14 Distribution of stresses ($\rho=0.4$ mm)

Fig. 15 Experimental values of stress concentration factor

the flecks appearing at the root. The value of n expresses the number of threads from the bearing surface. The flecks do not appear at the center of the root, but a little at the loaded flank of thread. In order to find the angular position θ of these flecks from the center of root, the co-ordinate x_A , x_B and x_C of the point A, B and the flecks C, respectively in Fig. 17 were measured by the SIP universal measuring apparatus and θ was obtained by the following expression as shown in Fig. 18. The marks $\circ \Delta \times \square$ show the difference in the test pieces.

$$\theta = \sin^{-1} \frac{x_C - (x_A + x_B)/2}{\rho}$$

4. Consideration

In Fig. 19, the stress concentration factors which were obtained by dividing the maximum stress at the root computed by the finite element method by the nominal stress at the minor section, are compared with the experimental values by the copper-electroplating method. As the screw thread which has annular multi-grooves is supposed in the analysis by the finite element method, the mean values between 0 and 1st thread are plotted as experimental values in the figure.

Though the finite element solutions show a fairly good agreement with the experimental values, the former are lower than the latter when ρ is small. As a reason for the disagreement, it is considered that the screw thread which was not axially symmetrical was treated as the annular multi-grooves, and had too many stress concentration points to be analyzed

Fig. 17 Explanation of angular position of flecks in bolt thread

(a) $\rho=0.2$ mm, $n=0.5$ (b) $\rho=0.4$ mm, $n=0.64$ (c) $\rho=0.6$ mm, $n=0.63$

Fig. 16 Flecks at the root of bolt thread ($N=8.89 \times 10^6$)

Fig.18 Angular position of the stress concentration point from the center of root

by the finite element method. In addition, the experiment involves the following question: the contact state between the flanks of the bolt threads and the nut threads is not always uniform, and as the local stress concentration is too large, the minimum diameter at the tapered part is to be reduced, and therefore the large mean load cannot be applied on the specimen, so the experimental value seems to be influenced greatly by the accuracy of pitch. In Fig. 19, Nishida's(8) and Otaki's(9) theoretical values for a screw thread of W 1 are plotted for reference. Both Nishida's and Otaki's values are a little higher than the experimental ones.

Though the stress concentration factor becomes smaller with an increase of the root radius, the difference between 0.4 mm and 0.6 mm is smaller than that between 0.2 mm and 0.4 mm. Therefore, even if the root radius is increased over the range of 0.3 ~ 0.43 mm which is specified in ISO Standard, it is considered that the strength for fatigue becomes little larger. But if it is decreased under the lower limit, it is considered that the strength for fatigue becomes smaller, steeply.

It was confirmed by the experiment that the position of the maximum stress point at the roots was fairly changed with the number of thread from the bearing surface as shown in Fig. 18. And the deviation of the maximum stress point at the root of a thread toward the loaded flank becomes larger with an increase of the root radius. The deviations at the number of thread 0.5 are about equal to 20°, 25° and 29° for $\rho = 0.2, 0.4$ and 0.6 mm, respectively. Similarly to the results of computation as shown in Fig. 6, the deviations of the maximum stress point toward the loaded flank tend to become larger with an increase of the root radius.

5. Conclusions

The stress of the screw thread in the bolt-nut joint was analyzed by the finite element method and the copper-electroplating method. And the results are summarized as follows:

(1) As a three-dimensional two-body con-

Fig.19 Comparison between solutions of computation and experimental values

tact problem, the load distribution at the bolt thread which is screwed in a nut thread is computed by the finite element method combined with the point matching method.

- (2) The numerical solution of the stress concentration factor by the finite element method shows a fairly good agreement with the experimental value by the copper-electroplating method. But it is fairly difficult for reason of the element divisions to treat a structure which has such complicated form as a screw thread which has many stress concentration points by the finite element method.
- (3) According to the measurement by the copper-electroplating method, the deviation of the maximum stress point at the root of a thread toward the loaded flank becomes larger with an increase of the root radius. This coincides with the tendency of the result of the computation by the finite element method.
- (4) The relationship between the stress concentration factor of the thread and the root radius is clarified. As a result, even if the root radius is increased over the range which is specified in ISO Standard, the stress concentration factor is scarcely improved, but if the former is decreased under the lower limit, the latter becomes steeply larger.

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