621-522

A Study on AFH*

By Eizo URATA,** Toshio TAKENAKA,***
 Sadami AKIYAMA,**** Teruo AKASHI,*****
 Ikuro KATO,***** Morio SHOJI,*****

The subject of this paper is to make clear the basic characteristics of the AFH. To calculate the characteristics of the AFH with single conduit, the AFH containing a separator and the three-phase AFH, three matrices, namely exact matrix, approximated matrix using the Poiseuille's law and matrix approximated by lumped parameters, are considered.

When the input frequency of the AFH is lower than the fundamental characteristic frequency of the conduit, the calculations based upon the approximated matrices induce no important error.

The frictional force on the separator caused by the sealing material lowers remarkably the power transmission efficiency of the AFH.

With the three-phase AFH, the pressure variation does not become sinusoidal even if the displacement of the input pistons is sinusoidal. In this case a high input frequency can not be taken because some resonance will be induced in the system by a higher harmonics in the pressure wave.

1. Introduction

Alternating Flow Hydraulics is often abbreviated as AFH. An official Japanese translation of that word is not yet established, hence we call it AFH in this paper.

The basic principle of the AFH is shown in Fig.1 where a piston fitted at one end of a pipe filled with liquid is moved to and fro, whereby the power is transmitted to the other end of the conduit.

In the motion of liquid in a usual fluid circuit there are circulating paths. Namely the liquid delivered from a pump, transmitted through pipings, drives actuators, then returns to the sump. A specific characteristic of the AFH is that it has not such a circulation, but it transmits fluid power through the fluid filled conduit. Owing to this characteristics the AFH could be utilized in the following fields of technology.

At first, the actuating part of the circuit is exposed to the high temperature, and the handling of the hydraulic

fluid will become difficult. When the actuators are exposed to radio activity, not only the actuators but also all of the circuit involving pump unit and hydraulic fluid must be isolated.

With the AFH, hydraulic fluid near the actuator can be separated from the hydraulic fluid near the power input by a floating piston which is inserted in the connecting conduit. The AFH which was invented rather a long time ago (1)-(5), it has come to draw attention recently because it has some advantages in the special fields such as in above examples.

The most basic construction of the AFH is shown in Fig.1. The output of this case is utilized for the oscillatory motions (example, vibrating feeder), periodic motions, motion to one direction using the ratchet mechanism.

Fig.1 Basic form of AFH

In this paper we describe the results of the analysis and experiments using the oil as the actuating fluid.

2. The transmission of pressure and velocity in the oil filled conduit

With AFH the pressure and velocity of the oil in the conduit are always changing. The pressure varies around a certain mean value and the velocity changes between positive and negative.

The velocity of sound in the conduit is written as

* Received 14th January, 1972.

** Research Associate, Tokyo Institute of Technology, Tokyo.

*** Professor, Tokyo Institute of Technology, Tokyo.

**** Makino Milling Machine Co., Ltd., Tokyo.

***** Toyota Automobile Products Co., Ltd., Toyota.

***** Sony Products Co., Ltd., Tokyo.

***** Mitsubishi Electric Co., Ltd., Tokyo.

where $\alpha = \sqrt{K/\rho_0}$ is the apparent compressibility of the oil, and ρ_0 is the mean value of the density of the oil.

Let p_0 be the mean value of the pressure, Δp be the variation from the mean value, and u be the mean value of the velocity on the cross section of the conduit.

The Laplace transforms of $\Delta p/K$ and u/α are P and U respectively.

The values of P and U are, P_1 and U_1 at the input end, and P_2 and U_2 at the output end in Fig.1.

The relations between these variables are written as

$$\begin{bmatrix} P_2 \\ U_2 \end{bmatrix} = [G] \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} \quad (1)$$

$$[G] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad (2)$$

$\{G\}$ is the transfer matrix which is given by the analysis of conduit dynamics.⁽⁶⁾

The elements are

$$\left. \begin{aligned} G_{11} &= G_{22} = \cosh(s\beta L/\alpha) \\ G_{12} &= -\beta \sinh(s\beta L/\alpha) \\ G_{21} &= -(1/\beta) \sinh(s\beta L/\alpha) \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \beta &= [1 - 2J_1(z)/zJ_0(z)]^{-1/2} \\ z &= \sqrt{-sa^2/\nu} \end{aligned} \right\} \quad (4)$$

Where s is the Laplace operator, a is the radius of the tube, L is the length of the tube, ν is the kinematic viscosity of the oil.

Next we denote the transfer matrix calculated under the assumption that the effect of the viscosity of the oil is calculated by Poiseuille's law as $\{G_p\}$.

$G_{p,ij}$ is written as follows;

$$\left. \begin{aligned} G_{p11} &= G_{p22} = \cosh(s\beta_p L/\alpha) \\ G_{p12} &= -\beta_p \sinh(s\beta_p L/\alpha) \\ G_{p21} &= (-1/\beta_p) \sinh(s\beta_p L/\alpha) \end{aligned} \right\} \quad (5)$$

$$\beta_p = \sqrt{1 + (8\nu/sa^2)} \quad (6)$$

The reciprocating motion of the cam at the input end is usually sinusoidal. When the angular frequency ω (rad/s) is low, the motion of the oil in the conduit can be approximated by the following lumped parameter model.

We estimated the frequency as low or high in comparison with the characteristic frequency of the conduit, namely when $\omega L/\alpha$ is smaller than $\pi/2$, we consider the angular frequency is low.

By the dynamic model assuming that the mass of oil is concentrated at the middle of the conduit, we have

$$P_1 - P = \frac{\rho_0 L \alpha}{2K} \left(s + \frac{8\nu}{a^2} \right) U_1 \quad (7)$$

$$P = \frac{\alpha}{Ls} (U_1 - U_2) \quad (8)$$

$$P - P_2 = \frac{\rho_0 L \alpha}{2K} \left(s + \frac{8\nu}{a^2} \right) U_2 \quad (9)$$

From above relations we have the following transfer matrix:

$$\left. \begin{aligned} G_{11} &= G_{22} = 1 + \frac{1}{2} \left(\frac{sL}{\alpha} \right)^2 \left(1 - \frac{8}{z^2} \right) \\ G_{12} &= -\frac{sL}{\alpha} \left(1 - \frac{8}{z^2} \right) - \frac{1}{4} \left(\frac{sL}{\alpha} \right)^3 \left(1 - \frac{8}{z^2} \right)^2 \\ G_{21} &= -\frac{sL}{\alpha} \end{aligned} \right\} \quad (10)$$

One of the purposes of the lumped parameter approximation is, of course, to simplify the calculation. At that time

we must know how the accuracy is decreased by the simplification.

The transfer matrix is calculated by one of the formulae (3), (5) or (10). The results are compared in Figs.2, 3 and 4. We can see that the lumped parameter approximation is invalid for frequencies higher than the fundamental one of the characteristic frequencies of the conduit.

Fig.2 Comparison of transfer matrix element G_{21}

Fig.3 Comparison of transfer matrix element G_{12}

Fig.4 Comparison of transfer matrix element G_{11}

Both approximations are acceptable for frequencies lower than the fundamental

characteristic frequency of the conduit. Only G_{21} decreases the accuracy by the approximation in comparison with the other elements.

3. Energy or power transmission

When the displacement of the input piston is sinusoidal, and when the boundary condition of the output end is linear, then $\Delta P/K$ and u/α at any point of the conduit become sinusoidal. Let us consider the variables P_1^* , P_2^* , U_1^* and U_2^* defined by the following expressions:

$$\left. \begin{aligned} U_1^* &= A_{11} e^{j\omega t}, \quad U_2^* = A_{12} e^{j\omega t} \\ P_1^* &= A_{21} e^{j\omega t}, \quad P_2^* = A_{22} e^{j\omega t} \end{aligned} \right\} \quad (11)$$

$$\left. \begin{aligned} u_1/\alpha &= R_1(U_1^*), \quad u_2/\alpha = R_2(U_2^*) \\ \Delta p_1/K &= R_1(P_1^*), \quad \Delta p_2/K = R_2(P_2^*) \end{aligned} \right\} \quad (12)$$

where A_{ij} is complex number, $R_c(P_i^*)$ and the similar expressions represent the real parts.

Substituting $j\omega$ in place of s in G_{ji} , and writing it as G_{ji}^* , we have

$$\begin{bmatrix} P_2^* \\ U_2^* \end{bmatrix} = \begin{bmatrix} G_{11}^* & G_{12}^* \\ G_{21}^* & G_{22}^* \end{bmatrix} \begin{bmatrix} P_1^* \\ U_1^* \end{bmatrix} \quad (13)$$

In general the power transmission efficiency is

$$\eta = \frac{\int_0^T \Delta p_2 u_2 dt}{\int_0^T \Delta p_1 u_1 dt} \quad (14)$$

where T represents the period of the input sinusoidal wave, which is written as

$$T = 2\pi/\omega$$

From Eqs. (11)-(14) we have

$$\eta = \frac{R_2(A_{22}\bar{A}_{12})}{R_1(A_{21}\bar{A}_{11})} \quad (15)$$

where $\bar{A}_{12}$ expresses the conjugate of A_{12} .

The relation between P_2 and U_2 is determined by the load at the output end. We write the relation as

$$P_2 = GU_2 \quad (16)$$

In general, G is a function of s .

When we substitute $j\omega$ in place of s in G , it is written as G^* .

$$P_2^* = G^* U_2^* \quad (17)$$

As seen from the construction, the input of the AFH is regarded as U_1^* . Let us express the other variables as functions of U_1^* .

To simplify the calculation we assume that A_{11} is real which does not lose the generality of the theory.

From Eqs. (13) and (17) we have

$$\left. \begin{aligned} P_1^* &= \frac{G_{12}^* - G^* G_{22}^*}{G^* G_{21}^* - G_{11}^*} U_1^* \\ U_2^* &= \frac{1}{G_{11}^* - G^* G_{21}^*} U_1^* \\ P_2^* &= \frac{G^*}{G_{11}^* - G^* G_{21}^*} U_1^* \end{aligned} \right\} \quad (18)$$

From Eqs. (11), (15) and (18) we have

$$\eta = \frac{R_2 \left\{ \frac{G^*}{G_{11}^* - G^* G_{21}^*} \right\}}{R_1 \left\{ \frac{G^* G_{11}^* - G_{13}^*}{G_{11}^* - G^* G_{21}^*} \right\}} \quad (19)$$

The efficiency varies with the boundary condition at the output end of the conduit. For example, if G^* is purely imaginary, namely the load is an ideal spring or mass, it is easy to see $\eta = 0$.

When an ideal accumulator is set at the output end, then the pressure at the

point is kept constant. Efficiency in this case is also zero.

The maximum efficiency is sustained when the wave reflection at the output end does not exist. In this case all of the energy supplied from the input end is absorbed at the output end, hence no energy is reflected at the input end. The case corresponds to infinitely long tubing. The maximum efficiency can be obtained when the frequency characteristic of the load becomes the same as that of the infinitely long tubing.

Because it is analogous to the characteristic impedance of the distributed parameter line in the electric circuit, we call in this paper the above mentioned load characteristics the characteristic impedance Z_c .

The characteristic impedance is calculated as follows. Since the length of the tubing is infinitely long, we have

$$Z_c = P_1^*/U_1^* = P_2^*/U_2^* \quad (20)$$

From Eqs. (13) and (20) we have

$$(P_2^*/U_2^*)^2 = G_{11}^*/G_{21}^* \quad (21)$$

Using Eqs. (3) and (4) we have

$$P_2^*/U_2^* = \beta^* \quad P_2/U_2 = \beta \quad (22)$$

Thus, the characteristic impedance is equal to β . Figure 5 shows Z_c .

Fig. 5 Characteristic impedance

Fig. 6 Maximum efficiency

With similar calculation, we have

$$\frac{P_2^*}{P_1^*} = \frac{U_2^*}{U_1^*} = G_{11}^* - \sqrt{G_{11}^* G_{21}^*} = \exp(-j\omega\beta^* L/\alpha) \quad (23)$$

Denoting the maximum efficiency as $\eta_{\max}$, we obtain its value by substitution of β^* in place of G^* in Eq. (19). The result is

$$\eta_{\max} = \exp(2R_1(-j\omega\beta^* L/\alpha)) \quad (24)$$

Figure 6 is a general chart for $\eta_{\max}$.

4. Experiment of the AFH with single conduit

The experimental apparatus for the single conduit AFH is shown schematically in Fig. 7. Input and output devices are constructed of precisely manufactured piston and cylinder.

Difference between the cylinder diameter and the piston diameter is smaller than 2×10^{-3} mm. The piston moves in the cylinder smoothly but leakage across the piston can be considered practically zero. Figure 8 shows the input device. The output device is similar to this. The input piston is driven sinusoidally by an eccentric circular cam. The eccentricity of the circular cam can be varied between 0 to 0.6 cm. Since the diameter of the cam is 20 cm, the motion of the piston becomes a sufficiently exact sine wave. The eccentricity of the circular cam is equal to the amplitude of the input sine wave.

Fig. 7 Experimental setup of single conduit AFH

Fig. 8 Input piston

Experiments were performed under the following three loads; output end is fixed, output piston is loaded by an accumulator, output piston is loaded by a spring.

We show how to find the transfer function, as an example, of the mass and damper load.

The equation of the motion of the piston is as follows;

$$A_1 \Delta p_2 = m \frac{d^2 x}{dt^2} + c_f \frac{dx}{dt} + kx \quad (25)$$

Where k is the spring constant, c_f is the resistance coefficient of the damper, x is the displacement of the piston, A_1 is the cross sectional area of the piston, and Δp_2 is the pressure acting on the piston.

Let A_p be the cross sectional area of the conduit, then we have

$$x = \frac{1}{A_1} \int A_p u_2 dt \quad (26)$$

From Eqs. (25) and (26) we have

$$\Delta p_2 = \frac{m A_p}{A_1^2} \frac{du_2}{dt} + \frac{c_f A_p}{A_1^2} u_2 + \frac{k A_p}{A_1^2} \int u_2 dt \quad (27)$$

By the definition of Eq. (16) we have

$$G = \frac{\alpha A_p}{K A_1^2} \left(ms + c_f + \frac{k}{s} \right) \quad (28)$$

In our experimental apparatus, the loads are designed to represent an ideal case, for example, in the inertial load the other forces are minimized. The numerical data of the experimental apparatus are shown in the table 1.

Table 1 Data of single conduit

a	$= 0.4 \text{ cm}$
L	$= 1.00 \times 10^4 \text{ cm}$
α	$= 9.98 \times 10^4 \text{ cm/s}$
ρ_0	$= 0.881 \times 10^{-6} \text{ kg.s}^2/\text{cm}$
ν	$= 0.58 \text{ cm}^2/\text{s}$ (oil temperature 22°C)
k	$= 100 \text{ kg/cm}$

The pressure at both ends of the conduit are detected by piezo-resistor pressure transducers. The displacements of both pistons are detected by differential transformers.

These variables are recorded using a directly visible electro-magnetic oscillograph. Figure 9 shows examples of oscillograms

for the spring load. In general the pressure wave deviates from sine wave with an increasing frequency.

To make clear the ripples in the oscillogram is difficult because many different causes exist, for example some parts of the experimental device will resonate with the higher harmonics involved in the input.

Figures 10 - 12 compare the transfer matrices described by Eqs. (3), (5) and (10) and the experimental results. When the input frequency of the AFH is increased, a problem other than formerly explained resonance will occur. That is, when the input frequency comes close to near the resonance frequency of the

Fig. 9 (a) Oscillogram

Fig. 9 (b) Oscillogram

Fig. 9 (c) Oscillogram

conduit, the pressure amplitude grows. As the result bubbles can be easily generated when the pressure becomes low. Generally, as seen from Fig. 6, the efficiency decreases with an increase of the frequency. Henceforth in our experiment ω is less than the fundamental resonance frequency of the conduit. We can see from Figs. 10 - 12 that the two approximated calculations involve small errors which are the order of the scattering in the experimental values. Hence we can use a simpler expression described as Eq. (10) which is based on the lumped parameter approximation.

When the relation between input piston amplitude and pressure at the input end of the conduit is given as

$$P_1 = Z_i U_1 \quad (29)$$

then, using Eq. (18) we have

$$Z_i^* = \frac{G_{12}^* - G^* G_{21}^*}{G^* G_{21}^* - G_{11}^*} \quad (30)$$

We call it the input impedance Z_i^* analogous to the electric circuit. When Z_i^* , input amplitude and frequency are given, we can find easily the force to drive the piston, the input power, etc. Figures 13 - 15 show the input impedances of the cases with fixed end, spring load and accumulator load respectively.

The approximated expression coincides well with experiments except one with fixed end. The output impedances ($= G^*$) of accumulator load and spring load used in the calculations are measured experimentally. Figures 16 and 17 show the experimental results of efficiency.

From these calculations and experiments we can see that the lumped parameter approximation gives sufficient result for the usual calculation of AFH.

Figure 16 shows two cases, namely $x_{10} = 0.204 \text{ cm}$, $P_0 = 15 \text{ kg/cm}^2$ and $x_{10} = 0.100 \text{ cm}$, $P_0 = 10 \text{ kg/cm}^2$. When the amplitude of the input displacement increases, the corresponding pressure amplitude also increases. Hence if we want larger input

Fig. 10 Experimental value of G_{11}^* (Spring load) Fig. 11 Experimental value of G_{12}^* (Spring load) Fig. 12 Experimental value of G_{21}^* (Spring load)

Fig.13 Input impedance (Fixed end)

Fig.14 Input impedance with spring load ($x_{10}=0.2\text{cm}$, spring constant 100kg/cm)

Fig.15 Input impedance with accumulator load

Fig.16 Efficiency with spring load

Fig.17 Efficiency with accumulator load

displacement, then P_0 must be increased, otherwise vacuum pressure will be generated in the conduit.

In Fig.16 the experimental value is less than the calculated one when the amplitude of the input displacement is large, one cause of this result being the increase of the leakage flow from the conduit to the tank through the direction control valve. Another cause of such a result would be the rigid wall assumption in the theory.

When the pressure variation increases, the expansion and contraction of the tube also increase. Hence the gain of both pressure transmission and flow rate transmission are supposed to decrease.

By Figs.16 and 17, we may see the efficiency of the AFH is low, but in fact it depends on the characteristics of the loads.

In the cases shown in the pictures, efficiency becomes zero if the load is an ideal spring or accumulator.

Actual devices are not ideal, namely the frictional force acting on the output piston is not zero. Measured values of Q^* involve the friction, hence the results deviate from zero.

5. Theory of AFH including a separator

An important feature of the AFH is that it can transport the power between two different fluids. This is achieved by separating the liquid columns by a floating piston.

We call this floating piston a separator. The separator must have characteristics such as transmit the pressure and mean velocity but the

separated liquids do not mix with each other. Hence a diaphragm or a bellow is also applicable as the separator. But if the separation is needed on account of high temperature fluid or radioactivity, the diaphragm or bellow is not suitable. Hence our study about the separator is limited to the floating piston type which is illustrated in Fig. 18.

Fig. 18 Separator

Although the composite of diaphragm and floating piston is also available, the motion of the fluid is not different from that of a simple floating piston, even if the structural devices is varied. Hence we only consider the simple floating piston.

The equation of the motion of the separator is as follows;

$$m_s \frac{d^2 x_s}{dt^2} + c_s \frac{dx_s}{dt} + F \frac{dx_s}{dt} \bigg/ \frac{dx_s}{dt} = A_s (p_{s1} - p_{s2}) \quad (31)$$

Where m_s is the mass of the separator piston, c_s is the resistance coefficient,

F is the Coulomb friction, A_s is the cross sectional area of the piston, and p_{s1} and p_{s2} are the pressures on both sides of the piston. The Coulomb friction is replaced by the following equivalent viscous friction. Assuming the motion of the piston is sinusoidal, the energy dissipation in a period becomes $4X_{so}F$. The viscous friction dissipating the same energy is

$$c_s = 4F/\pi\omega x_{so}$$

Then Eq. (31) becomes

(32)

$$m_s \frac{d^2 x_s}{dt^2} + (c_s + c_e) \frac{dx_s}{dt} = A_s (p_{s1} - p_{s2}) \quad (33)$$

Let the flow velocities at both sides of the separator be U_{s1} and U_{s2} respectively. Both of them are equal to $(A_s/A_p) dx_s/dt$.

The relation between the velocities and the pressures at both sides of the separator is as follows;

$$\begin{bmatrix} P_{s1} \\ U_{s1} \end{bmatrix} = \begin{bmatrix} 1 & -M_{s1} - C_{s1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_{s1} \\ U_{s1} \end{bmatrix} \quad (34)$$

where

$$M_{s1} = \frac{m_s \alpha}{A_p K} \left(\frac{A_p}{A_s} \right)^2, \quad C_{s1} = \frac{(c_s + c_e) \alpha}{A_p K} \left(\frac{A_p}{A_s} \right)^2 \quad (35)$$

$$U_{s1} = U_{s1}/\alpha, \quad P_{s1} = \Delta p_{s1}/K \quad (i=1, 2)$$

We consider the composite system of a separator and two conduits. The length of the upstream conduit is L_1 and the length of the downstream conduit is L_2 . The transfer matrices of the conduits are $G(L_1)$ and $G(L_2)$ respectively. The transfer matrix of the separator is S . The input to this composite system is given at the upstream end of the conduit with length L_1 . The pressure and the flow velocity at the end of the downstream conduit are given by;

$$\begin{bmatrix} P_2 \\ U_2 \end{bmatrix} = G(L_2) S G(L_1) \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} \quad (36)$$

Let us consider the following matrix;

$$H = G(L_2) S G(L_1) \quad (37)$$

To see the outline of the characteristic of matrix H , the components H_{11} , H_{12} and H_{21} are calculated using the data shown in Table 2.

Table 2 Data for the calculation of H matrix

$\alpha = 0.4 \text{ cm}$
$L_1 = L_2 = 5 \times 10^2 \text{ cm}$
$\alpha = 9.98 \times 10^4 \text{ cm/s}$
$\nu = 0.65 \text{ cm}^2/\text{s}$ (oil temperature 20°C)
$Me = 2 \times 10^{-4} \text{ s}$
$C_{e1} = 6, 15, 60, 100$

The results are shown in Figs. 19-21. As explained in Chap. 3, lumped parameter

Fig. 19 Matrix element with separator, H_{11}

Fig. 20 Matrix element with separator, H_{12}

Fig. 21 Matrix element with separator, H_{21}

approximation gives good result in the low frequency range. Figures 19, 20 and 21 have been calculated by that approximation. In this case it holds $L_1 = L_2$, hence it also $H_{11} = H_{22}$ holds.

6. Experiment of the composite system including a separator

The separator must have such a characteristic that there is no leakage flow across it in addition to smooth movement in a cylinder. Hence it must be manufactured precisely as the input and output pistons. The experiments as mentioned in Chap. 4 are performed inserting a separator schematically shown in Fig.18. In this experiment the fluid filling the conduit is the same oil. A spring load is used. The data of the experimental apparatus are shown in Table 3.

Table 3 Data of separator and conduits

$a = 0.4\text{cm}$
$L_1 = L_2 = 5 \times 10^2\text{cm}$
$\alpha = 9.98 \times 10^4\text{cm/s}$
$\rho_o = 0.881 \times 10^{-6}\text{kg}\cdot\text{s}^2/\text{cm}^4$
$\nu = 0.65\text{cm}^2/\text{s}$ (oil temperature 20°C)
$m_s = 3.5 \times 10^{-4}\text{kg}\cdot\text{s}^2/\text{cm}$
$F = 0.65\text{kg}$, when $x_{io} = 0.5\text{cm}$
10kg , when $x_{io} = 0.6\text{cm}$
$A_1 = 3.14\text{cm}^2$
$A_s = 3.14\text{cm}^2$
$\bar{p} = 100\text{kg}/\text{cm}$

From Eq.(34) we have $-M_e S - C_{eg} = S_{12}$. Figures 22 - 25 show the theoretical values and the experimental values of S_{12} , $H_{11} = H_{22}$, H_{21} and H_{12} . Figure 26 shows the efficiency. In the Fig.26, $C_{eg} = 0$ and $M_e = 0$ correspond to the case without a separator. The case $C_{eg} = 100$ represents a

Fig.22 Matrix element of separator ($C_{eg} = 6.0$)

Fig.23 Comparison of experimental and theoretical values of H_{11}

Fig.24 Comparison of experimental and theoretical values of H_{12}

Fig.25 Comparison of experimental and theoretical values of H_{21}

Fig.26 Efficiency of AFH including separator (spring loaded, lines indicate theoretical value by lumped parameter approximation)

separator using the O-ring seal. The case $C_{eq} = 6.0$ represents the separator shown in Fig.18.

It is not easy to manufacture the precise piston and cylinder as shown in Fig.18. But use of a simple sealing, like O-ring, is not advisable because the increase of the Coulomb friction lowers the efficiency of the power transmission. The theory and experiment coincide well as observed from Figs.22-26.

The friction of the separator affects the non-dimensional quantity C_{eq} . As the result, H_{12} widely varies, but H_{11} and H_{21} are scarcely affected. These facts are observed also from Figs.19-21.

In the above example where $L_1 = L_2$ holds, diameters of the conduits at both sides of the separator take the same value, the fluid is the same in both sides, hence $H_{11} = H_{22}$ holds. Generally we cannot expect this relation to hold.

The input impedance Z_{ij}^* and efficiency η can be determined as shown in Chap. 3.

We take H_{ij}^* instead of G_{ij}^* :

$$Z_{ij}^* = \frac{H_{12}^* - G^* H_{22}^*}{G^* H_{11}^* - H_{11}^*} \quad (38)$$

$$\eta = \frac{R_e \left\{ \frac{G^*}{H_{11}^* - G^* H_{11}^*} \left(\frac{1}{H_{11}^* - G^* H_{11}^*} \right) \right\}}{R_e \left\{ \frac{G^* H_{11}^* - H_{11}^*}{H_{11}^* - G^* H_{11}^*} \right\}} \quad (39)$$

7. The three phase AFH

One of the methods to take more uniform flow and pressure by the AFH is the following three-phase alternative fluid flow circuit.

Figure 27 shows the experimental setup of the three phase circuit. The cam in the picture is an eccentric circular plate. Three input pistons are arranged making 120 degrees to each other round the circular plate cam. Since the circular plate cam is eccentric, when the cam is rotated each piston is driven sinusoidally each with 120 degrees phase difference.

Fig.27 Experimental configuration of three-phase AFH

When the diameter of the circle is sufficiently large in comparison with the eccentricity, the motion of the piston become sufficiently exact sine waves. The maximum eccentricity of the setup is 0.6 cm whereas the diameter of the circular plate is 20 cm. Since the motions of the pistons have 120 degrees phase differences, the velocities of the oil columns driven by the pistons also have 120 degrees phase differences. The reaction unbalance of the plate cam is less than the case of the single conduit.

The flows induced in the three conduits are rectified by the six check valves shown in Fig.27. When the pressure of the output end of the conduit is higher than the pressure of the high pressure accumulator, the oil in the conduits will flow into the high pressure accumulator. At that time a flow to the low pressure accumulator will not occur because the check valves prevent the flow. When the pressure of the high pressure accumulator is higher than the pressure of the ends of the conduits, the check valves prevent the flow from the accumulator to the conduits. At that time the flows of oil from the low pressure accumulator to the conduits are not prevented. Hence we can see that the oil flows from the low pressure accumulator, through the conduits, to the high pressure accumulator.

Let us now consider the case where the load, as shown in Fig.27, is inserted between the two accumulators. Assume that the inertia and friction of the load cylinder are small enough compared with the orifice load made up of a four way valve.

When the cam is driven with constant angular velocity, the pressure difference between two accumulators is determined as follows.

$$Q = cf \sqrt{(p_h - p_l) / \rho_0} \quad (40)$$

Where C is the discharge coefficient of the valve, ρ_0 is the density of the oil, $p_h - p_l$ is the pressure difference, and f is the valve opening.

The time average of the summation of the flows through the three conduits is denoted as Q_i . We have

$$Q_i = 3A_1 z_{10} \omega \int_0^{2\pi/\omega} |\sin \omega t| dt / \left(\frac{2\pi}{\omega} \right) = 6A_1 z_{10} \omega / \pi \quad (41)$$

where A_1 is the cross sectional area of an input piston.

When $Q_i = Q$ holds the pressure difference of the two accumulators balances. Hence we have

$$p_h - p_l = \left(\frac{6A_1 z_{10} \omega}{\pi c f} \right)^2 \rho_0 \quad (42)$$

In the device shown in Fig.27, a pressure difference develops between the accumulators as the cam rotates, even if there is no difference initially. Finally the pressure difference becomes the value expressed by Eq.(42). Then the mean pressure in the conduit which is denoted

by P_0 becomes

$$P_0 = (P_h + P_l)/2$$

The mean pressure in the single phase AFH studied in Chaps. 3 and 4 was built up by the pump. In the three-phase AFH, the mean pressure is determined by the gas pressure of gas charged in the accumulators.

When the fluid flows through the check valve, though it is small, a pressure drop occurs. The relation between flow Q_c and pressure drop across the check valve ΔP_c used in the experiment is shown in Fig. 28. After this pressure loss the amplitude of the pressure variation at the end of the conduit is greater than the pressure difference between accumulators by twice the pressure drop across the check valve.

Fig. 28 Characteristics of check valve

8. Experiment of the three-phase AFH

The experimental setup for the three-phase AFH is shown in Fig. 27. The arrangement is seen from Fig. 29.

Fig. 29 Experimental setup of three-phase AFH

The pressures and the displacements of the pistons are measured for one of the three conduits. This is sufficient for our purposes. Figure 30 shows the oscillograms. Data of the experimental apparatus are given in Table 4.

Table 4. Data of three-phase AFH

$a = 0.4$ cm
$L = 1.00 \times 10^3$ cm
$\alpha = 9.98 \times 10^4$ cm/s
$\nu = 0.65$ cm ² /s
$\Delta P_{ch} = 0.2$ kg/cm ²
$\rho = 0.881 \times 10^{-6}$ kg·s ² /cm ⁴
$x_0 = 0.5$ cm
$A_1 = 3.14$ cm ²

The rectifier is set at the end of the conduit. This does not constitute a linear boundary condition. Hence the pressure wave does not become sine wave. But we can calculate the input power and other values rather easily.

The power output of the three-phase AFH is given as the product of the pressure difference between the accumulators and the flow through the load. Therefore the input power is given by $Q(P_h - P_l)/\eta$.

At the output ends of the conduits, the oil flows into the high pressure accumulator through the check valves 1, 2 and 3 in Fig. 27 when the pressures in the conduits grow higher than $P_h + \Delta P_c$. The oil flows into the conduits through the check valves 4, 5 and 6 from the low pressure accumulator when the pressures in the conduits becomes less than $P_l - \Delta P_c$.

Although ΔP_c varies with Q_c , the variation is sufficiently small as seen from Fig. 28. Therefore we approximate ΔP_c by a constant ΔP_{ch} .

The value of ΔP_{ch} is found from Fig. 28, as the value corresponding to $Q_c = A_1 \omega x_0 / 2$.

When the pressure at the end of the conduit is lower than $P_h + \Delta P_c$ but higher than $P_l - \Delta P_c$ the flow velocity at that point is zero.

If the response of the check valve is sufficiently fast, the pressure does not increase above $P_h + \Delta P_c$ and does not decrease below $P_l - \Delta P_c$, regardless of flow velocity. We denote the pressure

Fig. 30 Oscillogram of three-phase AFH

deviation from p_0 as Δp and the flow through a conduit as Q_d , then the relation between Δp and Q_d is, from the fact explained above, drawn as Fig.31.

Fig. 31 Characteristics of output end of conduit

Assuming the flow at the output end, Q_d , is sinusoidal, the corresponding pressure variation is approximated by a rectangular wave. The oscillograms shown in Fig.30 verify this at comparatively low frequency. Pressure at the output is, in any way, not sinusoidal. We consider the fundamental harmonic wave of the Fourier expansion of the pressure wave. That is given by

$$G^* = \frac{4}{\pi} \frac{(p_h - p_l + 2\Delta p_{ch})/2}{\omega x_{10} A_1 / A_p} \frac{\alpha}{K} \quad (43)$$

The flow rate is supposed to be a sufficiently exact sine wave because the compressibility of the oil is sufficiently large. Therefore the higher harmonics in the pressure wave does not affect the integral of the product of the pressure and the flow rate during a period. Hence the expression of Eq. (43) which only considers the fundamental, shall give good approximation in the calculation of the efficiency. The pressure variation at the input end is also complex, but we can consider the problem similarly to the above case.

When we substitute the value of G^* given by Eq. (43) into Eq.(19), the result gives the ratio of powers of the both ends of the conduits. The efficiency of the three-phase circuit is the product of this value and the loss by the check valves;

$$(p_h - p_l) / (p_h - p_l + 2\Delta p_{ch})$$

Hence we have

$$\eta = \frac{p_h - p_l}{p_h - p_l + 2\Delta p_{ch}} \times \frac{R_s \left\{ \frac{G^*}{G_{11}^* - G^* G_{21}} \left(\frac{1}{G_{11}^* - G^* G_{21}^*} \right) \right\}}{R_s \left\{ \frac{G^* G_{11}^* - G_{12}^*}{G_{11}^* - G^* G_{21}^*} \right\}} \quad (44)$$

Figure 32 compares the theoretical and experimental values of the efficiency. The theoretical value is calculated using the lumped parameter approximation. The results coincide well with experimental values.

For the other multi-phases circuits, similar method of calculation and the construction of the circuits are employed. But too much multiplicity results in weight increase whereas the obtained merits are not so great.

Fig. 32 Efficiency of three-phase AFH

9. Conclusions

The special feature of the AFH is that it can transmit the power with no or little circulating fluid flow. Where such appropriate need exists, it will prove a valuable method. In the presence of long conduits, the efficiency is lowered generally. Hence where the usual pump unit is sufficient, it is no use to replace it by the AFH.

The flow in the conduit of the AFH is usually sinusoidal, hence where the sinusoidal output motion is required, the fluid is controlled by a simple device which is considered profitable than usual pump unit and control circuits.

The mean pressure and the amplitude of the input piston will not introduce much change in the characteristics of the AFH except in special cases. In the AFH the lumped parameter approximation of the conduit dynamics gives satisfactory result.

Usually the separator will be necessary where the AFH is required. The characteristics of the AFH including the separator is also treated in this paper.

In the three-phase AFH the circulating flow of fluid occurs partially. It rather resembles the usual pump unit. The separator can be inserted also in this case. Anyway the frequency of the input sine wave must be lower than the fundamental one of the characteristic frequencies of the conduit.

Especially in the three-phase AFH, the flows are blocked by the check valves. Hence the maximum pressure becomes extremely high near the characteristic frequency of the conduit. Hence a lower input frequency is preferred. The theoretical calculation under such a low frequency can be performed with the lumped parameter approximation. The results of such a calculation coincide well with the experimental values.

Acknowledgement

The authors acknowledge Mr. M. Hosokawa who assisted in a part of the experimental work, and Yachiyoda Kogyo Inc., Ltd. which helped the Manufacturing of the experimental apparatus. The authors also acknowledge support from Science Foundation of the Japanese Ministry of Education, Encouraged Research A (year of Showa-45, grant 5053).

References

- (1) G. Constantinesco, *Proc. Conf. Oil Hyd. Power Transmission and Control*, (1962), p.69, *Instn. Mech. Engrs..*
 - (2) K. Foster, G. A. Parker, *Proc. Instn. Mech. Engrs.*, Vol.179, Pt.1, No.19, (1964-1965), p.599.
 - (3) C.-H. Weng, *Trans ASME*, Vol.88, Ser.D-2 (1966-6), P.316.
 - (4) T. Takenaka and E. Urata, *Oil Hydraulics* (in Japanese), (1970), Yokendo Co., Ltd., p.234.
 - (5) A. H. Gibson, *Hydraulics and its Applications*, 3rd ed., (1939), Constable and Comp., Ltd., p.708.
 - (6) E. Urata, *Bull. JSME*, Vol.14, No.68. (1971), p.147.
-