621-522

Starting Torque of Swash Plate Type Axial Piston Motor*

By Akira HIBI**

The starting torque of a swash plate type axial piston motor was analyzed, and calculated results were compared with the experimental results.

In this paper, the starting torque was derived from the friction forces appearing at the piston, the cylinder block, the valve plate and the shaft seal.

The values of the friction coefficients at these sliding surfaces were measured experimentally. Using these measured values the starting torque was calculated and its agreement with the experimental results was relatively good.

As the result of this investigation, it has become clear that the starting torque of a swash plate type axial piston motor is can be calculated analytically.

1. Introduction

Swash plate type axial piston motors have been extensively used in hydraulic transmissions and hydraulic control systems. The friction torques of these motors become large at a low speed range. Therefore it is hard to obtain the smooth start or the smooth rotation at the low speed range and the stick-slip phenomenon⁽¹⁾ often can be observed. Practically it will be important to predict the starting torques of these motors.

The friction torques at the start of these motors are caused by the friction forces appearing at the sliding surfaces, such as piston, cylinder block, valve plate, swash plate and shaft seal. But the friction force appearing at a swash plate is negligibly small when a ball thrust bearing is used at this part. In this paper, the friction forces between a piston end and a swash plate are neglected and the starting torques are calculated from the friction forces appearing between a piston and a cylinder wall and between a valve plate and a cylinder block. The starting torques of an axial piston motor, in which hydrostatic bearings are used at piston ends, will be able to be obtained by modifying the following analyses introducing a friction force appearing at a sliding surface between a piston end and a swash plate.

Using the axial piston motor, in which a ball thrust bearing was used between the piston end and the swash plate, experimental investigations were also carried out and the results were compared with the calculated results.

2. Analysis

The starting torque of an axial piston motor is smaller than the theoretical torque which is produced by the pressure of the oil. This difference makes the friction torque. The friction torque is caused by the friction forces appearing at the sliding surfaces in the motor. In the case of the axial piston motor shown in Fig.1, the most effective parts for the starting torque are the sliding surfaces between the pistons and the cylinder walls and between the valve plate and the cylinder block.

2.1 Friction losses between the pistons and the cylinder walls

θ is the angle of the rotation of the cylinder block and at the upper dead center (where the cylinder chamber begins to expand) it is defined as $\theta=0$. In the range of $0<\theta<\pi$, the force balances at the piston become as Fig.2 and 3. In Fig.2, the piston locates near the upper dead center, and the cross point O of the axial force F_1 and the reaction force F_2 from the swash plate locates above the point M. The point M is the lower edge of the sliding surface between the piston and the

Fig.1 Swash plate type axial piston motor

* Received 15th October, 1971.

** Assistant, Faculty of Engineering, Shizuoka Univ., Hamamatsu.

Table 1 Specifications of the motor shown in Fig.1

theoretical displacement volume V_{th}	7.17 cm ³ /rad.
tilting angle of swash plate α	17 deg.
number of pistons	7
diameter of piston	2.00 cm
piston area A	3.14 cm ²
piston stroke S	2.05 cm
pitch radius of cylinder R	3.35 cm
pitch radius of cylinder port R_1	3.35 cm
spring constant k	2.55 kg/cm
compressed length of spring at $\theta = \pi/2$	2.20 cm
equivalent area λ_1	10.3 cm ²

Fig.2 Force balance at a piston

Fig.3 Force balance at a piston

cylinder wall. In Fig.3, the piston locates near the lower dead center and the cross point O locates under the point M.

In the case of Fig.2, F_1 can be obtained by discounting the friction force, which appears at the side wall of the piston, from the axial force which is given by the hydraulic pressure and the spring.

$$F_1 = AP + F_s - \lambda_1 F_2 \quad (\text{for } 0 < \theta < \pi) \quad (1)$$

where A is the piston area, P is the pressure in the cylinder, F_s is the spring force and λ_1 is the friction coefficient between the piston and the cylinder wall.

The side force F_2 can be obtained by

$$F_2 = F_1 \tan \alpha \quad (2)$$

where α is the tilting angle of the swash plate.

From Eqs.(1) and (2) one can obtain

$$F_2 = \frac{(AP + F_s) \tan \alpha}{1 + \lambda_1 \tan \alpha} \quad (3)$$

(for $0 < \theta < \pi$, $l_1 < l_2$)

In the case of Fig.3, the point O locates under the point M, therefore the bending moment acts on the piston. At the point M, the reaction force F_3 appears, which has the opposite direction to F_2 . And at the point N, F_4 appears which has same direction as F_2 . Then the axial force F_1 in this case is given by

$$F_1 = AP + F_s - \lambda_1 (F_2 + F_4) \quad (4)$$

F_4 is the sum of F_2 and F_3 , then

$$F_4 = F_2 + F_3 \quad (5)$$

From the balance of the bending moment around the point N one can obtain

$$l_1 F_3 = l_2 F_2 \quad (6)$$

where l_1 is the axial distance between N and O and l_2 is that between N and M.

From Eqs.(4)-(6) and (2) one can obtain

$$F_2 = \frac{(AP + F_s) \tan \alpha}{1 + \lambda_1 \tan \alpha \left\{ \frac{l_1}{l_2} + \left(\frac{l_1}{l_2} - 1 \right) \right\}} \quad (7)$$

(for $0 < \theta < \pi$, $l_1 > l_2$)

Equations (3) and (7) can be expressed by

$$F_2 = \frac{(AP + F_s) \tan \alpha}{1 + \lambda_1 \tan \alpha \left\{ \frac{l_1}{l_2} + \left| \frac{l_1 - l_2}{l_2} \right| \left(\frac{l_1}{l_2} - 1 \right) \right\}} \quad (8)$$

(for $0 < \theta < \pi$)

In the range of $\pi < \theta < 2\pi$, the volume in the cylinder chamber shrinks, therefore the directions of the friction forces are reversed compared to Eqs.(1) and (4). Then, in this range, the sign of λ_1 in Eq.(8) should be changed. That is

$$F_2 = \frac{(AP + F_s) \tan \alpha}{1 - \lambda_1 \tan \alpha \left\{ \frac{l_1}{l_2} + \left| \frac{l_1 - l_2}{l_2} \right| \left(\frac{l_1}{l_2} - 1 \right) \right\}} \quad (9)$$

(for $\pi < \theta < 2\pi$)

Equations (8) and (9) can be expressed by

$$F_2 = \xi (AP + F_s) \tan \alpha \quad (10)$$

where

$$\xi = \frac{1}{1 + \frac{\sin \theta}{|\sin \theta|} \lambda_1 \tan \alpha \left\{ \frac{l_1}{l_2} + \left| \frac{l_1 - l_2}{l_2} \right| \left(\frac{l_1}{l_2} - 1 \right) \right\}} \quad (11)$$

The torque T_1 which is produced by one piston is given by $F_2 R \sin \theta$, then from Eq.(10) one can obtain

$$T_1 = \xi (AP + F_s) R \tan \alpha \sin \theta \quad (12)$$

where R is the pitch radius of the cylinder bore.

The spring force F_s changes according to the motion of the piston as

$$F_s = k(x_0 + 0.5S \cos \theta) \quad (13)$$

where k is the spring constant, x_0 is the compressed length of the spring at $\theta = \pi/2$ and S is the piston stroke that is $2R \tan \alpha$

From Eqs.(12),(13) one can obtain

$$T_1 = \xi \{ 0.5(AP + kx_0) S \sin \theta + 0.125 k S^2 \sin 2\theta \} \quad (14)$$

The total torque T , which is produced by all pistons can be obtained by summing up T_1 with the phase of $(2\pi/n)$ shifted from

each other. When T_1 , which is a function of θ , is replaced by $T_1(\theta)$, one can obtain

$$T_z = \sum_{i=1}^z T_1(\theta + 2\pi i/z) \quad (15)$$

where z is the number of pistons.

The mean value of T_z is given by

$$\bar{T}_z = \frac{z}{2\pi} \int_0^{2\pi} \xi [0.5(AP + kx_0)S \sin \theta + 0.125kS^2 \sin 2\theta] d\theta \quad (16)$$

where $\bar{T}_z$ is the mean value of T_z .

In the case of an ordinary swash plate type axial piston motor, the terms that are related to k in Eq.(16) are sufficiently small compared with the terms that are related to P . Then the terms that are related to k in Eq.(16) can be eliminated. Then Eq.(16) becomes

$$\bar{T}_z = \frac{V_{th}}{2} \int_0^{2\pi} \xi P \sin \theta d\theta \quad (17)$$

where V_{th} is the theoretical displacement per radian ($=ASz/2\pi$).

The mean value of the friction torque

T_{fp} , occurring at this part is given by

$$T_{fp} = T_{th} - \bar{T}_z \quad (18)$$

where T_{th} is the theoretical torque ($= (P_1 - P_2)V_{th}$), P_1 and P_2 are the inlet and outlet pressures respectively.

When the pressure in the cylinder changes ideally and the outlet pressure is zero, that is $P=P_1$ for $0 < \theta < \pi$ and $P=P_2=0$ for $\pi < \theta < 2\pi$, Eqs.(17) and (18) become

$$\bar{T}_z = \phi T_{th} = \phi P_1 V_{th} \quad (19)$$

$$T_{fp} = (1 - \phi) T_{th} = (1 - \phi) P_1 V_{th} \quad (20)$$

where

$$\phi = \frac{1}{2} \int_0^\pi \xi \sin \theta d\theta \quad (21)$$

Fig.4 Calculated example of the relationship between ϕ and λ_1

Figure 4 shows a calculated example of Eq.(21) for the axial piston motor shown in Fig.1 and Table 1. In the calculation of Fig.4, the values of λ_1 and λ_2 were obtained every 10 degrees of θ geometrically on the plan and using these values of λ_1 and λ_2 , ξ was calculated, and then the integration of Eq.(21) was performed numerically. Figure 4 indicates that the larger the friction coefficient λ_1 , ϕ becomes the smaller. In the case of Fig.4, when λ_1 is 0.3, ϕ reduces to about 0.9, that is 10 % of the theoretical torque is wasted by the friction between the pistons and the cylinder walls.

The resultant torque that can be obtained by subtracting the friction

torque between the valve plate and the cylinder block from the torque which is given by Eq.(16) or approximately by Eq.(17) is transmitted to the drive shaft.

2.2 Friction torque between the valve plate and the cylinder block

The valve plate and the cylinder block form a kind of a hydrostatic thrust bearing, which sustains the axial thrust force of the cylinder block against the valve plate. The major part of the thrust force is sustained hydrostatically and the rest is sustained by the metal contacts at the sliding surface between the valve plate and the cylinder block. These metal contacts produce the friction torque of this part.

F denotes the axial force acting on the cylinder block by one piston. F is the sum of the hydrostatic force and the spring force. That is F is given by

$$F = AP + F_s \quad (22)$$

Because of the friction between the piston and the cylinder wall, another axial force acts on the cylinder block. But this axial force is sustained by the friction between the cylinder block and the center shaft. Then this axial force need not be considered in Eq.(22).

From Eqs.(22) and (13) one can obtain

$$F = (AP + k(x_0 + 0.5S \cos \theta)) \quad (23)$$

F_s denotes the total axial force acting on the cylinder block by all pistons. F_s can be obtained by summing up F with the phase of $2\pi/z$ shifted from each other. When F , which is a function of θ , is replaced by $F(\theta)$, F_s is given by

$$F_s = \sum_{i=1}^z F(\theta + 2\pi i/z) \quad (24)$$

When the motor starts, the pushing force of the cylinder block to the valve plate is larger than the separating force. Then the clearance between the valve plate and the cylinder block disappears. At the start of the motor the oil film does not build up completely and because of the roughness of the sliding surface the metal contacts appear at the convex parts and the oil pressure acts in the dents. W denotes the load which is sustained by the metal contacts. W is the difference between the pushing force F_s and the hydrostatically sustained force. Then one can obtain

$$W = F_s - (A_{s1}P_1 + A_{s2}P_2) \quad (25)$$

where A_{s1} and A_{s2} are the equivalent areas of hydrostatic thrust bearing, P_1 and P_2 are the inlet and outlet pressures respectively.

The sliding surface between the cylinder block and the valve plate of the axial piston motor shown in Fig.1 is illustrated in Fig.5. In this case A_{s1} is the sum of the port area A_1 and one-half of the seal lip area A_2 (see appendix). A_{s1} of the sliding surface shown in Fig.5 is illustrated in Fig.6. Figure 6 indicates that A_{s1} changes with every $\theta = \pi/z$, then the separating force changes with every $\theta = \pi/z$. But the fluctuation of the separating force is similar to that of the

Fig.5 Sliding surface between the valve plate and the cylinder block of the axial piston motor shown in Fig.1

Fig.6 Calculated example of A_{e1} of the valve plate shown in Fig.5

Fig.7 Calculated example of W , $A_{e1} P_1$ and F_z

pushing force F_z . Therefore the fluctuation of W is more placid than that of F_z . The calculated results of W , $A_{e1} P_1$ and F_z are shown in Fig.7.

The friction torque between the valve plate and the cylinder block is given by

$$T_f = \lambda_2 R_1 W \quad (26)$$

where λ_2 is the friction coefficient and R_1 is the pitch radius of the cylinder ports. From Eqs.(23)-(26) one can obtain

$$T_{fd} = \lambda_2 R_1 \left\{ \frac{AZ}{2\pi} \int_0^{2\pi} P d\theta + kx_0 z - (\bar{A}_{e1} P_1 + \bar{A}_{e2} P_2) \right\} \quad (27)$$

where T_{fd} , $\bar{A}_{e1}$ and $\bar{A}_{e2}$ are the mean values of T_f , A_{e1} and A_{e2} respectively.

When $P=P_1$ for $0 < \theta < \pi$ and $P=P_2=0$ for $\pi < \theta < 2\pi$, Eq.(27) becomes

$$T_{fd} = \lambda_2 R_1 (0.5 A P_1 z + kx_0 z - \bar{A}_{e1} P_1) \quad (28)$$

2.3 Starting torque

When the motor is at a standstill, the output torque of the drive shaft of the motor can take a value freely within the range of

$$(T_{th} - T_{loss}) < T < (T_{th} + T_{loss}) \quad (29)$$

where T is the torque of the drive shaft, T_{th} is the theoretical torque and T_{loss} is the friction torque.

In Fig.8 the rotary system of the oil motor is replaced by the linear motion system. The mass M is equivalent to the rotative inertia and the spring K is equivalent to the twist elasticity of the shaft assembly. The torque of the drive shaft when the load begins to move is the starting torque T_s . Then T_s is the lower limit of Eq.(29). That is

$$T_s = T_{th} - T_{loss} \quad (30)$$

where T_{loss} is the sum of T_{fp} , T_{fd} , and T_{fs} . T_{fs} is the friction torque of the shaft seal. Then

$$T_{loss} = T_{fp} + T_{fd} + T_{fs} \quad (31)$$

From Eqs.(20) and (28) one can obtain

$$T_s = \left\{ \phi V_{th} - \lambda_2 R_1 (0.5 A z - \bar{A}_{e1}) \right\} P_1 - \lambda_2 k R_1 x_0 z - T_{fs} \quad (32)$$

(for $P=P_1$ in $0 < \theta < \pi$, $P=P_2=0$ in $\pi < \theta < 2\pi$)

3. Experiments

Using the swash plate type axial piston motor which is shown in Fig.1, λ_1 , λ_2 , T_{fs} and the starting torque are measured. And the experimental results of the starting torque are compared with the calculated results. The measurements of the starting torque are performed by two methods. In the first method, referring to Fig.8, L moves in right hand direction with constant value of T_{th} and in the second method T_{th} changes when L is fixed.

Fig.8 Modeled illustration of an oil motor system

3.1 Measurement of λ_1

The experimental apparatus for the measurement of the friction coefficient λ_1 between the piston and the cylinder wall is shown in Fig.9. The load at the sliding surface was given by the rod and the weights. The one end of the rod was suspended by the string and the other end was sustained by the inner side of the piston. The rod end was pulled in the right hand direction slowly by hand with a spring scale. The friction forces were measured when the piston began to move. The experimental results are shown in Fig.10. The

Fig. 9 Experimental apparatus for the measurement of λ_1

Fig. 10 Relationship between the friction force and the load at the piston and the cylinder wall

Fig. 11 Experimental apparatus for the measurement of λ_2

Fig. 12 Relationship between the friction torques and the weights

mean value of the experimental results of λ_1 is 0.309.

The oil used in this research is the petroleum of paraffin series which contains variable additives, its properties being shown in Table 2.

Table 2 Properties of used oil

brand	DAPHNE 52	
viscosity kgs/cm ²	6.0×10^{-7}	(35 °C)
	4.7×10^{-7}	(40 °C)
specific gravity	0.869	(35 °C)
	0.865	(40 °C)

Fig. 13 Experimental results of T_n

3.2 Measurement of λ_2

The experimental apparatus for the measurement of the friction coefficient λ_2 between the valve plate and the cylinder block is shown in Fig. 11. The cylinder block was laid horizontally and the valve plate, the lever and the weights were placed on it. The one end of the lever was pulled slowly by hand with a spring scale. The pulling force was measured when the valve plate began to rotate and the friction torque at this part was measured. The pivot was placed at the centers of the valve plate and cylinder block so that these centers did not move from each other. The relationship between the weights and the friction torques is shown in Fig. 12. The mean value of λ_2 is 0.248.

3.3 Measurement of T_n

The friction torque which was not concerned with the piston and the cylinder block was measured. Before the measurement, the pistons, the springs and the cylinder block were removed. The torque was added by a string which was wound around the flange mounted on the drive shaft. The tensile force of the string was measured by a spring scale when the flange began to move. The experimental results are shown in Fig. 13. The mean value of T_n is 2.25 kgcm.

3.4 Starting torque when the supply pressure is constant and the torque of the drive shaft changes

The experimental apparatus for the measurement of the starting torques is shown in Fig. 14. The oil was supplied from the pump to the test motor ① through the reducing valve, the filter and the accumulator. The inlet pressure of the motor was measured by the Bourdon tube type pressure gauge ③ and the outlet pressure was atmospheric pressure. ④ is a strain gauge type pressure transducer. The output torque was measured by a strain gauge type torque transducer ② which was

Fig. 14 Experimental apparatus for the measurement of the starting torque

Fig. 15 Relationship between the output torque and the rotated angle of the worm-gear

Fig. 16 Relationship between the twisted angle of the shaft assembly and the torque

connected to the drive shaft of the motor directly. At the other end of the torque transducer the worm-wheel was mounted which engaged with the worm-gear. The speed reduction at the worm-wheel and the worm-gear was 1/50. Therefore when the torque acted on the worm-wheel, the worm-gear did not move. The worm-gear could be rotated by hand or a hydraulic drive system that is not shown in Fig. 14.

The torque of the drive shaft of a hydraulic motor is not always the starting torque, because it remains within the range of Eq. (29). The lower limit of Eq. (29) is the starting torque. This can be measured by moving the worm-gear slowly by hand. As shown in Fig. 15, according to the rotation of the worm-gear the torque of the drive shaft is reduced, but the

Fig. 17 Experimental results of the starting torque

Fig. 18 Relationship between the starting torque and the inlet pressure

drive shaft of the test motor does not move for a while. When the torque of the drive shaft reaches a certain value (point C in Fig. 15), the drive shaft rotates a little angle suddenly. The torque at the point C in Fig. 15 can be regarded as the starting torque of the motor. This method is equivalent to the measurement of the force at the spring K in Fig. 8 when L moves slowly in the right hand direction and M begins to move. The stiffness of the shaft assembly between the test motor and the worm-wheel, which is equivalent to that of the spring K in Fig. 8, is shown in Fig. 16.

The starting torques measured by the method mentioned above are shown in Fig. 17. The measurements were performed at twenty points every 7.2 degrees of the drive shaft rotation. The periodicity which

Figs. 19-21 Variation of the torque when the inlet pressure is changed

should appear every $1/14$ rotation of the shaft is masked in the irregularities of the measured values. In Fig. 17, the test motor was driven by the worm-gear when $P_1 = 0$, if one described this situation likened to Fig. 8, L was pulled in the right hand direction with $T_{th} = 0$. The dashed lines in Fig. 17 are the mean values of the measured values. The relationship between the mean values of the starting torque and the inlet pressure is linear as shown in Fig. 18. The calculated results of Eq. (32) using the values of $\lambda_1 = 0.309$, $\lambda_2 = 0.248$ and $T_{ts} = 2.25$ kgcm and theoretical torque are also shown in Fig. 18. The calculated results of the starting torque are slightly larger than the experimental results.

3.5 Output torque when the inlet pressure changes

The output torque of the test motor was measured when the inlet pressure P_1 changed. This method is equivalent to the measurement of the force appearing at the spring K in Fig. 8 when T_{th} changes and L is at a standstill.

In Figs. 19-20 the variations of the torque when the inlet pressure has changed with the constant rate of pressure change 0.5 or 1 kg/cm²/sec are shown. The inlet pressure was linearly increased from $P_1 \approx 5$ kg/cm² to $P_1 = 40$ kg/cm², then it was decreased. The inlet pressure and the output torque were recorded by the oscillograph. From Figs. 19-21 it is clear that the output torque is smaller than the theoretical torque when the inlet pressure increases and it is larger when the inlet pressure decreases. These hystereses can be seen because the direction of the friction force in the motor is reversed according to the sign of the gradient of the change of the inlet pressure. When the inlet pressure is increasing, the output torque can be considered the starting torque. It is obvious from Fig. 8 that according to the change of the inlet pressure the worm-wheel does not rotate but the drive shaft of the test motor slightly moves. This movement corresponds to the elastic deformation of the shaft assembly between the drive shaft and the worm-wheel. The

rotated angles of the drive shaft in Fig. 19 were about 0.8 degrees. In Fig. 19 the rotation of the drive shaft was relatively smooth but in Figs. 20 and 21 it was not smooth. Figures 19 and 20 were recorded at the same rotary position of the drive shaft but the rates of the pressure change were not the same. The rate of the pressure change was larger in Fig. 19 than in Fig. 20. Therefore the variation of the torque was more smooth in Fig. 19 than in Fig. 20. In Fig. 21, the rotary position of the drive shaft was different from that of Figs. 19 and 20.

In Figs. 19-21 the theoretical torque, the calculated result of the starting torque and mean value of the starting torque obtained in section 3.4 are also illustrated. When the inlet pressure increases the output torque is similar to the calculated result of the starting torque. When the inlet pressure decreases the output torque is similar to the sum of the theoretical torque and the friction torque which is obtained from the starting torque measured in section 3.4.

3.6 Consideration of the experimental results

It is clear from Figs. 19-21 that the starting torques measured in section 3.4 are smaller than that measured in section 3.5. These differences arose from the fact that in section 3.4 the starting torque was measured after the stationary time of 5-10 seconds but in section 3.5 the drive shaft did not stay so long time and a small slip continued intermittently. Therefore the conditions of the oil film of both cases were different and the friction coefficient of the former case is slightly larger than that of the latter case or that one obtained from the experimental apparatus shown in Fig. 11.

In other words, one can obtain the value of λ_2 from the friction torque of the motor when the inlet pressure $P_1 = 0$. This friction torque is the sum of that one at the valve plate where the cylinder block is pushed by springs against the valve plate and that one of the shaft seal T_{ts} . The value of λ_2 obtained by this

Fig.22 Comparison between the experimental result and the calculated result which is obtained using the value of λ_2 which is derived from the value of the friction torque when $P_1=0$

method is 0.332 (the friction torque of the motor when $P_1=0$ is 45.9 kgcm, $T_{1s}=2.25$ kgcm and the spring force is 39.3 kg). Introducing the values of $\lambda_2=0.332$, $\lambda_1=0.309$ and $T_{1s}=2.25$ kgcm in to Eq.(32) one can obtain Fig.22. The agreements of the calculated results and the experimental results are relatively good.

4. Conclusions

The starting torque of the swash plate type axial piston motor was analyzed and the calculated results were compared with the experimental results. The agreements of both results were relatively good. And the following informations were obtained.

At the start of the swash plate type axial piston motor the great part of the friction torque was caused by the friction forces at the pistons and the valve plate. When the tilting angle of the swash plate is 17 degrees and the friction coefficient is 0.3, 10 % of the theoretical torque is wasted by the frictions between the pistons and the cylinder walls. If the friction coefficient at this part reduces to 0.02 the wasted torque becomes 0.8 % of the theoretical torque. Therefore it is important to use a material which has a low friction coefficient for the purpose of improving the starting performances.

The friction torque at the valve plate depends vastly upon the design of the valve plate. Especially the determination of the seal lip areas is important. It is necessary to determine this area to minimize the difference between the pushing force and the separating force.

The friction torque caused by the springs that are used for the returns of the pistons becomes relatively large. Therefore no use of springs is desirable.

Acknowledgements

The author wishes to express his

great appreciation to Professor T.Ichikawa, Shizuoka University, for his encouragement and suggestion throughout this work, as well as to H.Nakagawa and the personnel in the Hydraulic Laboratory of Shizuoka University for their help in experimental setups.

Appendix

The equivalent area A_e of the circular hydrostatic thrust bearing which is shown in Fig.23 is as follows.

It is assumed that the oil film thickness in the seal lip is uniform and the flow rate of Q is flowing radially and the flow is fully developed. The pressure gradient is given by⁽²⁾

$$\frac{dP}{dr} = -\frac{1}{r} \frac{6\mu Q}{\pi h^3} \quad (33)$$

where P is the pressure in the oil film, r is the radius, μ is the viscosity and h is the film thickness.

Integrating Eq.(33) with r and introducing the boundary conditions of $P=P_1$ at $r=r_1$ and $P=0$ at $r=r_2$ one can obtain

$$P=P_1 \ln R / \ln \beta \quad (34)$$

where $R=r/r_2$ and $\beta=r_1/r_2$.

Then the thrust F produced by the seal lip area is given by

$$F = \int_{r_1}^{r_2} 2\pi r P dr = \int_{r_1}^{r_2} 2\pi r \frac{P_1 \ln(r/r_2)}{\ln \beta} dr \quad (35)$$

From Eq.(35) and the definition of $F=A_e P_1$ one can obtain

$$\epsilon = \frac{A_e}{A_0} = \frac{1}{1-\beta^2} \left[\frac{\beta^2-1}{2 \ln \beta} - \beta^2 \right] \quad (36)$$

where $A_0=\pi(r_2^2-r_1^2)$.

The numerical result of Eq.(36) is shown in Fig.24. From Fig.24 it is clear that if $\beta \approx 1$ then $\epsilon \approx 0.5$, that is the equivalent area is a half of the seal lip area A_0 .

Fig.23 Hydrostatic circular thrust bearing

Fig.24 Relationship between β and ϵ

References

- (1) Ichikawa, T. and Kosodo, H., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.33, No. 255 (1967-11), p.1800.
- (2) Takenaka, T. and Urata, E., Yurikigaku (in Japanese), (1968-1), p.55, Yokendo.