Studies of Liquid Film Flow in Two-Phase Annular
and Annular-Mist Flow Regions*

( Part 2, Upflow in a Vertical Tube )

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Experimental data are presented on the pressure drop, mean liquid film thickness, state of gas-liquid interface and droplet entrainment for upward annular-mist two phase flow in a vertical tube. Using these data, the relations between the shear stress at gas-liquid interface, the liquid film flow rate and the mean film thickness are investigated, and the characteristics of the mean film thickness with respect to the film Reynolds number are revealed by comparing with the analytical predictions for laminar and turbulent films.

A heat transfer experiment is also performed and the results are compared with the analytical values of heat transfer coefficient for laminar and turbulent film flow. On the basis of the experimental data of heat transfer coefficient, the nondimensional thickness of the laminar sublayer of liquid film is calculated for both upflow and downflow, and correlated with the nondimensional film thickness. The result represents the difference in flow state between the upflow film and the downflow film.

1. Introduction

Two phase gas-liquid annular flow has been studied by many investigators and several analytical predictions have been presented for its liquid film flow applying the flow characteristics obtained for the wall region of single phase flow\(^{(4)}\). However, the comparison with experimental data is not necessarily satisfactory and little is known about the flow state of liquid film, especially in a case of turbulent film flow.

In the previous paper, Part 1 of this research program\(^{(9)}\), experimental investigations have been reported for the downward liquid film flow in two phase (air/water) annular and annular-mist flow regions. In the present paper, Part 2, the experimental data of liquid film flow and heat transfer are presented for the upward annular-mist two phase flow in a vertical tube of the same diameter as the downward test section, and the characteristics of the upward film and the difference in flow state from the downflow film are discussed.

The pressure drop, mean liquid film thickness, state of gas-liquid interface and droplet entrainment were measured for the developed film flow region, and a heat transfer measurement was also performed by means of an electrically heated stainless steel tube. The experiments were carried out in a range of the liquid film Reynolds numbers Re\(_L\)\(=190\sim13700\) (superficial water velocity \(0.65\sim45\) cm/s) and the gas Reynolds numbers Re\(_G\)\(=2\times10^7\sim8.7\times10^8\) (superficial air velocity \(10\sim50\) m/s).

On the basis of the experimental results, the effects of the film Reynolds number and the interfacial shear on the mean liquid film thickness and on the heat transfer coefficient were investigated and the characteristics of film flow were discussed comparing with the analytical predictions for laminar and turbulent liquid film flow. The thickness of laminar sublayer has a significant effect on the magnitude of heat transfer coefficient. Therefore, the thickness of laminar sublayer was calculated from the experimental data for both upflow and downflow based on an assumption that the film flow consists of two layers, a laminar sublayer and a turbulent layer.

2. Experimental Apparatus and Procedure

Figure 1 illustrates a schematic diagram of the experimental apparatus used in this experiment. The main section arranged vertically is made of acrylic resin pipe of 28.8 mm I.D., and consists of air entrance region, water inlet section and test section. The air from an air compressor is supplied to the lower end of the air entrance region, while the water from a pump is introduced into the test section through a quick shut-off valve and a porous sinter section. After passing
through the test section, the air-water mixture flows into a cyclone separator which is set in the upper part of the apparatus, and the separated water goes back to a water tank. The water temperature in the water tank is adjusted to 22°C during the experiment.

The four test sections shown in Table 1 were used just as in the case of downflow experiment. The test section F1 was provided with twelve pressure taps longitudinally set with 150 mm interval. Figure 1 shows a diagram showing how the test section F2 is mounted. F2 has a contact probe located at 1100 mm downstream of the upper limit of the porous section. The probe is used to investigate the state of the air-water interface. The equipment and procedure of the probe method have been explained in the previous paper.

A kind of shut-off method was used for measuring the mean liquid film thickness. Figure 2 shows a schematic diagram of the configuration of the upper part of the test section for this measurement. A hemispherical bucket just above the exit of the test section repels spraying water down to the water receiver, and repelled water flows back to the water tank. Measurement of the hold-up liquid in the test section was carried out by simultaneously switching on three solenoids, one for getting the bucket out of the way, one for switching the flexible tube at the exit of the test section, and one for working the quick shut-off valve placed near the water inlet, then by cutting water supply to the water inlet and collecting water which was left in the test section in the cyclone separator through the flexible vinyl tube. This measurement was repeated 3 to 10 times under the same flow rates of air and water for both test sections F2 (tube length L=1540 mm) and F3 (tube length L=590 mm). The hold-up water in the test region

Fig.1 Schematic diagram of experimental apparatus


Fig.2 Schematic diagram of upper part of test section for measuring mean liquid film thickness


<table>
<thead>
<tr>
<th>Test section</th>
<th>Dimensions (mm)</th>
<th>Material</th>
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<tr>
<td>F1</td>
<td>28.8 x 1825 x 5.1</td>
<td>Acrylic resin</td>
<td>Pressure distribution</td>
</tr>
<tr>
<td>F2</td>
<td>28.8 x 1540 x 5.1</td>
<td>Acrylic resin</td>
<td>Liquid hold-up</td>
</tr>
<tr>
<td>F3</td>
<td>28.8 x 590 x 5.1</td>
<td>Acrylic resin</td>
<td>State of interface</td>
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<tr>
<td>H</td>
<td>29.9 x 1880 x 1.0 (Heating length 1300)</td>
<td>Stainless steel</td>
<td>Entrainment</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Liquid hold-up</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Entrainment</td>
</tr>
</tbody>
</table>

Table 1. Test sections
from tube length 590 mm to 1540 mm (test length \( \Delta L = 950 \) mm) in which the film flow was considered to be developed, was obtained by subtracting the hold-up measured with F3 from that measured with F2. The mean liquid film thickness was then calculated from the hold-up water in the test length.

The entrained droplet flow rate was determined from the flow rate distribution measured by traversing radially an isokinetic sampling probe of 1.6 mm equivalent diameter at the exit of the test sections F2 and F3, respectively. The incremental droplet flow rate of the test length \( \Delta L = 950 \) mm was then obtained by subtracting the droplet flow rate at the exit of F3 from that of F2.

The less the liquid flow rate is, the more becomes the minimum air flow rate which can keep the liquid film flow upward. Therefore, this experiment was performed in the air flow rate range of above 30 kg/hr which was the minimum air flow rate required to keep upward liquid film flow at the lowest liquid flow rate 0.25 l/min.

A stainless steel section H was used in an experiment on the heat transfer coefficient. Figure 3 shows the equipment for heat transfer test and the test section. The length of heating section is 1300 mm, and the stainless steel pipe is connected at its both ends with acrylic resin pipes of the same inside diameter as the stainless steel pipe. Twelve thermocouples are fitted on the outer surface of the heating section with 100 mm interval from the lower electrode, and the outside of the heating section is fully insulated with glass wool and asbestos. Two thermocouples are fitted at the water inlet and three thermocouples at the exit of the test section. As for the three at the exit, they are designed such that the liquid film can be taken out for measuring the bulk mean temperature. The wet and dry bulb temperatures of inlet and outlet air were also measured. Amount of heat rate was so adjusted that the temperature difference between the wall and bulk water was approximately \( 3 \sim 4 \) °C. The heat transfer coefficient was calculated at the point of 1270 mm downstream of the test section inlet where the liquid film was considered to be fully developed.

3. Experimental Results of Flow State

Figure 4 shows the relationship between the air flow rate \( \dot{W}_A \) and the mean liquid film thickness \( \bar{y}_L \) in the test length \( \Delta L = 950 \) mm which was calculated from the two hold-ups measured by test sections F2 and F3. In the figure, liquid phase volumetric flow rate \( \dot{V}_L \) is used as a parameter. Dotted lines in the figure represent the results for the downflow in the previous report, Part I. In the range of low \( \dot{W}_A \), the value of \( \bar{y}_L \) for the upflow is quite higher than that for the downflow for the same liquid flow rate. However, the opposite trend is observed in the range of high \( \dot{W}_A \) due to higher interfacial shear in the upflow film.

Figure 5 shows the typical plots of the droplet flow rate distribution at the exit of the test sections F2 and F3 obtained by means of the 1.6 mm isokinetic probe at the air flow rate \( \dot{W}_A = 100 \) kg/hr. Although it shows a similar trend to the results obtained in the downflow, the droplet flow rate \( \dot{N}_d \) of the upflow is approximately 1.5 times that of the downflow under the condition of same gas phase and liquid phase flow rates. An abrupt increase of the droplet flow rate near the wall is considered to be due to intake of the crest of high waves at the interface. The total flow rate of entrained droplets was obtained by integrating the distribution curve of the droplet flow rate in the gas core. In this integration, the distribution curve was extrapolated up to the position of the mean film thickness \( \bar{y}_L \). The ratio of the total flow rate of entrained droplets to that of the liquid

![Fig. 3 Equipment for heat transfer test and test section](image-url)
phase was about 25% at maximum in the experiment of the test section F2. The incremental droplet flow rate $dN_d$ in the length of $dL$ was determined from the difference between the result of F2 shown by solid line and the result of F3 shown by dotted line.

As mentioned in the previous paper, the wall shear stress $\tau_0$ and the interfacial shear stress $\tau_1$ are obtained by the following equations:

$$\tau_0 = -\frac{\Delta p}{2} \left( \frac{dA_p}{dL} \right) - \frac{\rho g}{2\pi r_0} \left( \frac{dM_d}{dL} \right)$$  \hspace{1cm} (1)

$$\tau_1 = -\frac{\Delta p}{2} \left( \frac{dA_p}{dL} \right) - \frac{1}{2\pi r_0} \left( \frac{dM_d}{dL} \right)$$  \hspace{1cm} (2)

where, $r_0$: inside radius of the tube wall
$r_1$: $r_0 - y_i$: radius of the gas-liquid interface
$ho$: density of liquid
$g$: pressure
$M_d$: momentum of entrained droplets in the axial direction

Therefore, the values of $\tau_0$ and $\tau_1$ can be calculated from the measured pressure drop and mean film thickness $y_i$, if the increment of momentum of entrained droplets in the test length ($dM_d/dL$) were known. Denoting the flow rate of entrained droplets as $\dot{N}_d$ and mean velocity of droplets in the axial direction as $\dot{u}_{dm}$, $(dM_d/dL)$ may be expressed as,

$$\frac{dM_d}{dL} = \frac{1}{g} \left( \dot{W}_d \frac{d\dot{u}_{em}}{dL} + \dot{u}_{em} \frac{d\dot{W}_d}{dL} \right)$$  \hspace{1cm} (3)

According to the measured results of $d\dot{N}_d$ and $dM_d$ in the downflow experiment, $\dot{u}_{dm}$ may be assumed to be equal to the mean gas velocity $\dot{u}_{em}$. So we may put $(d\dot{u}_{em}/dL) = 0$. Therefore, $(dM_d/dL)$ is calculated by the following equation using the measured $d\dot{N}_d$:

$$\frac{dM_d}{dL} = \frac{1}{g} \left( \dot{W}_d \frac{d\dot{u}_{em}}{dL} \right)$$  \hspace{1cm} (4)

Fig.4 Mean liquid film thickness

Fig.5 Droplet flow rate distribution (measured by a 1.6 mm isokinetic probe)

Fig.6 Shear stresses at wall and interface
Figure 6 shows examples of the calculated τc, τb, and their components in Eqs. (1) and (2). The dimensionless parameter of the interfacial shear τc is defined as,
\[ \tau_c^* = \mu_b \left( \frac{\partial \gamma}{\partial y} \right)^{1/3} \tag{4} \]
where, \( \nu_b \) is kinematic viscosity of liquid. The chain lines written in Fig. 4 represent the loci of constant dimensionless interfacial shear.

Figure 7 shows the thickness of continuous liquid sublayer \( \delta_b \) and the height of liquid film crest \( \delta_c \) measured by a contact probe equipment. The values of \( \delta_b \) and \( \delta_c \) are defined, the same as in the case of downflow, as the distances from the wall surface to the points in film where the readings of an electronic counter are 95% and 5% respectively to the frequency signal of the oscillator incorporated in the contact probe circuit. The value of \( \delta_b \) is relatively insensitive to the liquid flow rate, but decreases as the gas velocity increases. \( \delta_b \) is somewhat lower than that of the downflow in the range of low gas velocities, but has approximately the same value in the range of high gas velocities. It also shows approximately the same value as that obtained by Nishikawa and Sekoguchii in the experiment of upflow in 25 mm I.D. tube. The value of \( \delta_c \) is considerably larger than that of the downward annular flow when the gas velocity is low. However, with high gas velocity it has a value of 1.0~1.5 times that of the downward annular-mist flow.

The gas phase flowing in the core is accompanied with a large frictional pressure drop due to the waves of liquid film.

The frictional pressure drop gradient of the gas flow \(-\frac{\partial P_g}{\partial L} = \frac{\gamma}{2\tau_c} \) may be expressed in terms of a friction factor \( \lambda_f \) and the relative gas velocity to the interface as,
\[ \left( \frac{\partial P_g}{\partial L} \right) = \frac{\rho_g}{2\tau_c} \left( \frac{\nu_g}{\nu_b} \right)^{1/2} \tag{5} \]
where \( \rho_g \) is the density of gas phase (air), \( \nu_g \) is the mean gas velocity, and \( \nu_b \) is the velocity of liquid at the gas-liquid interface. Here, \( \nu_b \) is assumed to be 1.5 times as much as the mean liquid film velocity \( \nu_{lm} \).

In Fig. 8, the value of \( \lambda_f \) obtained from Eq. (5) is plotted against the relative Reynolds number of the gas phase. The parameter \( Re_f \) denotes the liquid film Reynolds number. The dotted lines in Fig. 8 show the results of the downflow in the

Fig. 8 Friction factor of gas phase flow

Fig. 9 Comparison of shears at interface and wall with Lockhart-Martinelli correlation
previous paper, and they are very close to the value of upflow in the range of downward dispersed flow. Incidentally, though Anderson et al. and Collier et al. plotted $x_0^+$ with a parameter $Re_	au/\sqrt{\nu}$ where $y^+ = \frac{\sqrt{\nu} \tau}{\partial_x y}$, no clear correlation was obtained with this parameter.

Figure 9 shows the ratios of interfacial shear $\tau_i$ and wall shear $\tau_w$ to $\tau_0$, plotted against Lockhart-Martineilli parameter $X_\mu^\tau$, where $\tau_0$ is the wall shear which would exist if the gas phase were assumed to flow alone. The experimental results are represented as separated curves depending on the liquid flow rate, and the magnitude of deviation from the curve given by Lockhart-Martineilli becomes large with a decrease of the gas phase flow rate.

4. Mean Liquid Film Thickness

As for the mean film thickness of the upward liquid film flow, analytical predictions by Calbert et al. and Anderson et al. and Hewitt have been presented for turbulent liquid film. In these analyses, the relationship between the liquid film thickness and the liquid flow rate was derived by applying the flow characteristics determined for single phase flow.

Anderson et al. applied von Kármán’s universal velocity profile in the single phase flow to the liquid film flow. By introducing the friction velocity $\sqrt{\nu \tau_0}$ and non-dimensional parameters of liquid velocity and distance from the wall surface $u^+ = u/\sqrt{\nu \tau_0}$, $y^+ = y/\sqrt{\nu \tau_0}$, the liquid film Reynolds number is expressed as follows:

$$Re_\tau = 4 \int_0^1 u^+ dy^+$$

Assuming the mean film thickness is comparatively small against the pipe radius, that is $y^+ \ll \tau_0^+$, the wall shear $\tau_0$ may be simply related to the interfacial shear $\tau_i$ from Eqs. (1) and (2) as follows:

$$\tau_0 = \frac{1}{\tau_0^+} \left( \frac{\tau_i}{\tau_0^+} \right)$$

Therefore, in terms of the following non-dimensional mean film thickness

$$y^+ = y^+ / \sqrt{\nu \tau_0}$$

and $\tau_0^+$, $y^+$ can be expressed as follows,

$$y^+ = \frac{\tau_0^+}{\tau_0^+} - \frac{\tau_i^+}{\tau_0^+}$$

Hence, the relationship between $Re_\tau$, $y^+$ and $\tau_0^+$ can be obtained from Eqs. (6) and (8) by assuming existence of the universal velocity profile in the liquid film.

Applying Dukler’s analysis for the downflow to the upflow, Hewitt has presented the relationship between $Re_\tau$, $y^+$ and $\tau_0^+$ by putting $\frac{d\tau_0^+}{\tau_0^+} (\nu \tau_0) dy$ and substituting the eddy diffusivity of single phase flow into $c$ of this equation.

When the liquid film flow is totally laminar, the relationship for the upflow is expressed, similarly to the previous paper, as

$$Re_\tau = y^+ + 2 \tau_0^+ y^+ \frac{\partial y^+}{\partial \tau_0^+}$$

The experimental data are compared with the results of the predictions described above in Fig. 10. Since the liquid droplets are entrained in the gas core of upflow, the value of droplet flow rate should be considered. Therefore, experimental data are plotted against Reynolds number $Re_\tau$ calculated from the real liquid film flow rate which is obtained by subtracting the droplet flow rate at the middle of the test section from the total flow rate of liquid phase. In the range of small $y^+$, the experimental value of $y^+$ is slightly less than the predicted value for laminar flow. In the range of high $Re_\tau$, $y^+$ is in fairly good agreement with Hewitt’s prediction, but as in the case of downflow, the gradient of film thickness with an increase of $Re_\tau$ is a little steeper than that of the prediction for turbulent flow. Incidentally, the dotted part of $\tau_0^+ = 10$ line represents the state where $\tau_0$ becomes negative and pulsation occurs in the flow.

5. Heat Transfer Coefficient

In this experiment, the heat transfer coefficient is defined as,

$$h = q_0 / (t_w - t_m)$$

where, $q_0$ is the wall heat flux, $t_w$ and $t_m$ denote the temperature at the inner surface of the pipe and the bulk mean temperature of liquid film, respectively. All values were measured or interpolated at the cross section of 1270 mm downstream of the inlet of the test section, that is 950 mm downstream of the lower end of the heating section.
Temperature $t_0$ was determined from the measured temperature at the outer surface of the pipe, taking into account the temperature drop across the pipe wall of a uniform heat source. Since the heat rate distribution was uniform in the flow direction, the bulk mean temperature $t_{bm}$ was determined by assuming that the bulk mean temperature rose linearly from the inlet to the exit of the heating section.

Figure 11 shows examples of the heat transfer data. In this figure, the non-dimensional heat transfer coefficient $(h/k_x)(v_f^2/\rho g)$ is plotted against the liquid film Reynolds number $Re_f$, where $k_x$ is the thermal conductivity of liquid. The dotted line shows, for reference, the results of a falling film with no interfacial shear by Wilke [2].

The heat transfer coefficient of fully developed laminar film flow with a uniform heat rate in the flow direction can be calculated as follows.

In this case,

$$\frac{\partial h}{\partial x} = \text{const.}$$

$$\frac{dq}{dy} = -\rho_i g c_i \left( \frac{\partial t}{\partial x} \right)$$

Therefore,

$$q = q_0 - \rho_i g c_i \left( \frac{\partial t}{\partial x} \right) \int_0^y w \, dy$$

As was discussed in the previous paper, the velocity profile in the laminar liquid film flow is

$$u_i = \frac{\tau_i}{\mu_i} \frac{v_f}{\mu_i} \left( y_i - y^* \right)$$

where, negative sign of the second term of the right hand side is for the upflow and positive sign for the downflow.

Substitution of Eq.(12) into Eq.(11) gives the distribution of the heat flux.

Hence, the temperature distribution can be obtained by the following equation:

$$dt = -\frac{q}{h} \, dy \quad \therefore \quad t_0 - t = \int_0^{y_f} \frac{q}{h} \, dy$$

Therefore, the bulk mean temperature $t_{bm}$ and the heat transfer coefficient $h$ can be determined as,

$$t_0 - t_{bm} = \int_0^{y_f} u_i(t_0 - t) \, dy / \int_0^{y_f} u_i \, dy$$

$$h = \frac{q_0}{t_0 - t_{bm}}$$

In the case of no heat flux at the gas-liquid interface $y_i$, that is $q_i = 0$,

$$\frac{\partial t}{\partial x} = \frac{q_0}{\rho_i g c_i (\Gamma / \rho_i)}$$

Then, the result is expressed in a dimensionless form as,

$$h \frac{\left( \frac{u_i}{g} \right)^{1/3}}{k_i} = Re_f \left( \frac{4}{3} Re_f \left( \frac{\tau_i}{y_i} + \frac{\tau_i}{y_i} \right) \left( y_i^* \right) \right)^{1/3}$$

$$\pm \frac{Re_f}{2} \left( \frac{\tau_i}{y_i} + \frac{\tau_i}{y_i} \right) \left( y_i^* \right)$$

$$\mp \frac{1}{3} \left( \frac{\tau_i}{y_i} + \frac{\tau_i}{y_i} \right) \left( y_i^* \right)$$

In the case of constant heat flux across the film, that is $q = q_0$, ($\partial t / \partial x = 0$), and the result becomes

$$h \frac{\left( \frac{u_i}{g} \right)^{1/3}}{k_i} = Re_f \left( \frac{4}{3} Re_f \left( \frac{\tau_i}{y_i} + \frac{\tau_i}{y_i} \right) \left( y_i^* \right) \right)^{1/3}$$

$$\pm \frac{1}{2} \left( y_i^* \right)^{3/2}$$

On the other hand, as discussed in the previous paper, there is the following relationship between $Re_f$, $y_i$ and $\tau_i$ in the laminar flow:

$$Re_f = \frac{4}{3} \left( y_i^* \right)^{3/2} + 2 \tau_i \left( y_i^* \right)^{3/2}$$

Therefore, the dimensionless heat transfer coefficient of laminar film for both upflow and downflow may be expressed in terms of $\tau_i$ and $Re_f$ from Eqs.(15) and (17) in the case of $q_i = 0$ and from Eqs.(15) and (17) in the case of $q = q_0$. In the terms with both positive and negative signs in Eqs.(15) to (17), the upper sign is for upflow and the lower one is for downflow. The result for downflow was shown in Fig.19 in the previous paper.

As for the heat transfer coefficient for upward turbulent film flow, based on the expression for heat flux,
\[ q = -\rho \varphi \left( \frac{\nu_1}{P_{Ri}} + \varepsilon_1 \right) \frac{dT}{dy} \]

Hewitt introduced the relationship between nondimensional heat transfer coefficient \( \tau^* \) and \( \text{Re}_L \) in the case of constant heat flux across the film, that is \( q = q_1 \), by assuming the eddy diffusivity \( \varepsilon_1 \) to be equal to that for momentum in single phase flow.

Figure 13 shows the experimental data and the results calculated by the procedure mentioned above. Experimental data fall below Hewitt's prediction for turbulent film in the range of low film Reynolds numbers, and fall above the prediction in the range of high film Reynolds numbers. This fact seems to suggest that the flow state of liquid film flow, especially the thickness of the laminar sublayer which has a great influence on heat transfer coefficient, is different from that of the wall region in the single phase flow.

The ratio of the heat flux at gas-liquid interface \( q_1 \) to \( q_0 \) varied with the gas flow rate in this experiment. Its value was approximately 10% in the range of high liquid film Reynolds numbers, and increased up to 35% at maximum with a decrease of the liquid film Reynolds number. In Hewitt's prediction \( q = q_2 \) was assumed, but anyway the effect of the distribution of heat flux across the film on heat transfer coefficient is considered to be comparatively small in turbulent liquid film flow.

6. Laminar Sublayer Thickness of Liquid Film

This paragraph discusses about the thickness of laminar sublayer with the aid of the measured heat transfer coefficient assuming that the turbulent liquid film flow consists of a laminar sublayer and a turbulent layer. Although the approximate values of the thickness and the nondimensional thickness of laminar sublayer may be obtained by the following equations, as was discussed in the previous paper,

\[ \delta_h = \frac{h_1}{h}, \quad \delta_n^* = \frac{h_1}{h^*} \frac{T_0}{\nu_1} \quad \text{.........(18)} \]

The thickness of laminar sublayer \( \delta_s \) will be calculated here taking into account the thermal resistance of the turbulent layer.

The shear stress \( \tau \) and the heat flux \( q \) at an arbitrary point \( y \) in the liquid film are expressed as,

\[ \tau = \rho (u + \varepsilon) \frac{\partial u}{\partial y} \]

\[ q = -\rho \varphi \left( \frac{\nu_1}{P_{Ri}} + \varepsilon_1 \right) \frac{dT}{dy} \quad \text{.........(19)} \]

To continue the calculation, the distributions of shear stress \( \tau \), heat flux \( q \), and eddy diffusivities for momentum \( \varepsilon \) and for heat \( \varepsilon_1 \) in the liquid film should be known in advance. Hence, the following assumptions are made:

1. Neglecting the minor terms, the shear stress \( \tau \) at point \( y \) in the film is expressed as,

\[ \tau = \frac{r}{2} \frac{\partial p}{\partial y} + \frac{r^2 - r_s^2}{2r} \rho \varphi \frac{1}{\pi r^2} \frac{(\Delta M_0)}{\Delta L} \]

Substituting Eq. (9) in the previous paper into \( (\partial p/\partial L) \),

\[ \tau = \frac{r}{2} \frac{\partial p}{\partial y} + \frac{1}{\pi r^2} \frac{(\Delta M_0)}{\Delta L} \]

In the range of this experiment,

\[ \frac{(r_s)}{r_o} \rho \varphi \frac{1}{\pi r^2} \frac{(\Delta M_0)}{\Delta L} \]

Therefore, the shear stress distribution is expressed approximately as follows:

For upflow,

\[ \tau = \tau_0 + \varphi \psi, \quad \psi = (r_i/r_o)^2 \rho \varphi - \tau_0/r_o \]

For downflow,

\[ \tau = \tau_0 - \varphi \psi, \quad \psi = (r_i/r_o)^2 \rho \varphi + \tau_0/r_o \]

(2) It is possible to obtain the heat flux distribution from Eq. (11), since the heat rate is uniform in flow direction. But in this case the following assumptions are made:

In laminar sublayer: \( q = q_0 \)

In turbulent layer, in the case where the heat transfer due to evaporation at the
gas-liquid interface is neglected, that is \( q_i = 0 \),
\[
q = q_0 (y_1 - y)/(y_1 - \delta_i)
\]
in the case where the heat transfer at the interface is taken into account, \( q_i \neq 0 \),
\[
q = q_0 - (q_0 - \delta_i)/(y_1 - \delta_i)
\]
(3) The eddy diffusivities are assumed as follows:
In laminar sublayer: \( \varepsilon = \varepsilon_0 = 0 \)
In turbulent layer, the temperature drop across this layer is generally small comparing with the total temperature drop \( \delta_i \) across the liquid film. Then, it is considered to have little effect on the final result to make a rough approximation of this value. Therefore, the following value which is determined for the single phase flow is assumed.
\[
\varepsilon = \varepsilon_0 = 0.4 \bigg( \frac{\nu_1 \varepsilon_{11}}{\nu_1} \bigg)
\]
Substituting these assumptions into Eq.(19), the distributions of velocity and temperature in the liquid film may be obtained as follows.
In laminar sublayer,
\[
u = \frac{1}{\rho \nu_1} \bigg( \frac{\nu_1}{\nu} \bigg)^{\frac{1}{2}}
\]
(20)
\[

In turbulent layer, the temperature transfer at the interface were neglected,
\[
u = \nu_0 + 2.5 \bigg( \frac{\nu_0}{\nu} \bigg)^{\frac{1}{2}} \bigg( \frac{\nu_1}{\nu} \bigg)^{\frac{1}{2}}
\]
(22)
\[
t = t_0 - \frac{2}{\rho_0 g \nu_1 \delta_i / \rho_0} \bigg( \frac{\nu_1}{\nu} \bigg)^{\frac{1}{2}} \bigg( \frac{\nu_1}{\nu} \bigg)^{\frac{1}{2}}
\]
(23)
where, \( \nu_0 \) and \( t_0 \) denote the velocity and temperature at \( y = \delta_i \), respectively.
Therefore, the bulk mean temperature of liquid film can be obtained by substituting Eqs.(20)\(\sim\)(23) into the following equation:
\[
t_{mean} = \int y_1 u(\nu_1 - y)dy \int y_1 u(\nu_1 - y)dy
\]
(24)
Wall shear stress \( \tau_0 \), wall heat flux \( q_0 \), wall temperature \( \delta_i \), mean film thickness \( \delta_i \) and bulk mean temperature of the liquid film \( t_{mean} \) have been already known in the experiment. Therefore, the thickness of laminar sublayer \( \delta_i \) can be determined by a try and error method in which the assumptions of value of \( \delta_i \) are repeated until \( t_{mean} \) and \( t_{mean} \) obtained from the above equation match each other.
Figures 15 and 16 show plots of \( \delta_i^* \), the nondimensional parameter of \( \delta_i \), against \( y_1^* \), the nondimensional parameter of the mean film thickness. There,
\[
\delta_i^* = \frac{\delta_i}{\nu_1 \sqrt{\nu_1}}
\]
(25)
Figure 15 shows the results obtained from the data of the previous downflow experiment. A dotted line in the figure obtained by Eq.(18) is the same one which has been shown in the previous paper. Figure 16 shows the results of this upflow experiment. The dotted line is the result of \( q_i = 0 \), while the dotted line is the one obtained taking into account the evaporative heat transfer at the interface, and the difference between the both lines is recognized to be very little. The diameters of test sections H and F were not quite same. So the values of \( y_1 \) and \( \tau_0 \) in Figs.15 and 16 were determined from the data obtained by the test section F under the same conditions of \( Re_y \) and air flow rate.
The thickness of laminar sublayer \( \delta_i \) depends on the restraint by wall and the turbulence in main flow. In case of single phase flow, it is known that the value of \( \delta_i \) decreases with an increase of \( \tau_0 \) and its nondimensional parameter \( \delta_i^* \) has a constant value. On the contrary, in case of two phase liquid film flow, as was shown in Figs.15 and 16, \( \delta_i^* \) decreases with an increase of \( y_1^* \) for both upflow and downflow. The turbulence in the liquid film flow considered to be caused mainly by wavy disturbance at the gas-liquid interface. Therefore, taking into account the results shown in Figs.15 and 16, it is presumed that the turbulence in liquid film flow has different character from that in
single phase flow, and that in the range of small \( y^* \) the film flow is of low turbulence and preserves considerably laminar character, but it tends to have greater turbulence with an increase of \( y^* \). The reason why \( \delta^* \) obtained in upflow is less than that in downflow may be explained by the greater disturbance at the interface of upflow than that of downflow.

The value of \( \delta^* \) means the nondimensional thickness of laminar sublayer for the temperature distribution obtained under the assumption that the liquid film consists of a laminar sublayer and a turbulent layer. For the purpose of comparison, let us calculate the value of \( \delta^* \) in single phase flow under the same assumption. Assuming von Karman's universal velocity profile, consider only the flow close to the wall in which it may be assumed \( \tau = \tau_0 \) and \( q = q_0 \). \( \tau \) and \( q \) are expressed as Eq. (19). Therefore, with generally accepted assumptions that \( \tau = \tau_0 \) and in the turbulent region \( \varepsilon = \varepsilon' \) and \( \varepsilon_0 = \varepsilon_0' \), \( \varepsilon (\varepsilon_0') \) is determined from the equation for \( \tau \), and then the temperature distribution is easily obtained from the equation for \( q \). Let us introduce \( \delta^* \), a nondimensional parameter of the temperature difference between wall \( T_0 \) and fluid \( T \), which is expressed as,

\[
\delta^* = \frac{\rho c_p}{\rho_0} \sqrt{\frac{g c}{k_0 (T_0-T)}} \tag{26}
\]

Then, the nondimensional temperature differences in laminar sublayer, buffer layer and turbulent layer are expressed respectively as follows,

\[
\delta^* = \frac{\delta^*}{P} \tag{27}
\]

\[
\delta^* = 5P_0 + 5\ln \left(1 + \frac{y^*}{5} P - P_0\right) \tag{28}
\]

\[
\delta^* = 5P_0 + 5\ln \left(1 + 5P_0\right) + 2.5 \ln \left(\frac{y^*}{30}\right) \tag{29}
\]

Therefore, if single phase flow is assumed to consist of two layers, a laminar sublayer and a turbulent layer, \( \delta^* \) is determined as the intersection of Eqs. (27) and (29), that is the value of \( y^* \) which makes \( \delta^*_u = \delta^*_t \). Since \( P_r \) is not canceled in the procedure of calculation above, \( \delta^* \) in single phase flow is a function of \( P_r \), and is nearly equal to 7.1 in the range of \( P_r = 6.5-7 \) in this experiment. Therefore, in the range of large \( y^* \), \( \delta^* \) of downflow film takes approximately the same value as that in single phase flow. On the contrary, the lower value of \( \delta^* \) is obtained in upflow film.

Figure 17 shows an example of experimental data of thickness of laminar sublayer \( \delta^* \), mean liquid film thickness \( y^* \), and thickness of continuous sublayer \( \delta^c \). While in the range of small \( y^* \), \( \delta^* \) and \( \delta^c \) have about the same magnitude, \( \delta^* \) is less than \( \delta^c \) in the range of large \( y^* \).

7. Conclusions

An experiment was performed for the upward annular-mist two phase flow in a vertical tube of the same inside diameter as that used in Part 1, and the state of gas-liquid interface, mean liquid film thickness, heat transfer coefficient and thickness of laminar sublayer of the liquid film flow were discussed.

From the results, the following conclusions are obtained.

1. Comparing with the results obtained in the downflow experiment under the same gas phase and liquid phase flow rates, while the thickness of continuous sublayer \( \delta^c \) of upflow has approximately the same value, the thickness of wavy layer \( (\delta^c - \delta^u) \) of upflow is generally greater. Moreover, the flow rate of entrained droplets is approximately 1.5 times that of downflow.

2. By comparing the mean film thickness obtained in the experiment with analytical predictions, it is revealed that in the range of film Reynolds numbers \( Re_f \), less than approximately 1000, measured value of \( y^* \) is close to the predicted value for laminar flow but slightly less than that, and that in the range of \( Re_f \), higher than 2000, measured value of \( y^* \) is in fairly good agreement with the prediction for turbulent liquid film flow by Hewitt (Fig. 10).

3. The data of heat transfer coefficient obtained in the experiment were compared with the results of analytical predictions. Comparing with the prediction for turbulent film flow by Hewitt, experimental data show considerably lower value in the range of low \( Re_f \), and considerably higher value in the range of high \( Re_f \) (Fig. 13).

4. Assuming the turbulent film flow consisting of a laminar sublayer and a turbulent layer, the thickness of laminar sublayer \( \delta^c \) was calculated from the data obtained in the heat transfer experiment.
The results show that the nondimensional parameter of the thickness of laminar sublayer $\delta^*$, which has a constant value in single phase flow, decreases with an increase of the nondimensional parameter of the mean film thickness $y_1^*$ in two phase liquid film flow, and that $\delta^*$ is considerably less in upflow than in downflow (Fig. 16).

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References

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