Heat and Mass Transfer in Countercurrent Flow of Air and Water Film in a Rectangular Vertical Duct

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Experiments were carried out concerning heat and mass transfer between falling water film and upward air flow in a rectangular vertical duct with a large aspect ratio under the various conditions of rate of flow, water temperature and humidity. The behaviour of the falling water films, e.g., the film thickness and the wave inception, was also examined. These results are represented in terms of the Nusselt and Sherwood number. The Nusselt number can be explained using the Reynolds number based on the mean air velocity relative to the water surface regardless of the water flow rate.

When the water temperature at the entrance is varied from 20 to 50°C, the Nusselt number increases, while the Sherwood number decreases in spite of an increase in the rate of evaporation. When saturated humid air is supplied, the Nusselt number is larger and the Sherwood number is slightly smaller compared with air having a relative humidity of 65 per cent.

1. Introduction

Heat and mass transfer between a falling liquid film along a vertical wall and upward flowing air contacting directly with the film is an important and interesting phenomenon in industrial apparatus such as cooling towers and wetted wall towers. A number of investigations concerning this have been made from both basic and applied viewpoints. When these experimental results are compared, however, considerable differences are found in magnitude and inclination as seen from Fig.1, for example, which shows some results in literature for mass transfer under simple conditions of air and water film action along a smooth wall.

These discrepancies are considered to arise from respective differences in experimental conditions such as the shape and dimensions of the duct, the hydrodynamic and thermodynamic conditions of the inlet air and water, the values introduced for physical properties of air and water, and also from the manner of reducing the measured data. Hence more detailed, systematic investigations are required.

In this paper the effects of inlet water and air temperatures, the inlet air humidity as well as the water flow rate and air velocity on the heat and mass transfer characteristics, are experimentally investigated using a duct with a rectangular cross section with a large aspect ratio, the use of which facilitates observation of the water film on the wall and the extension of this investigation to a rough wall application.

In correlating the measured results, air velocity relative to water surface was introduced as a reference velocity. This made for a reasonable and convenient reduction of the measured data. The hydrodynamic characteristics of the water films such as thickness, falling velocity and waviness also are discussed.

Nomenclature

A: interfacial area
dk: hydraulic diameter of duct

Fig.1 Some experimental results in literature for mass transfer characteristics.

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observation of the water film along the wall and to be able to make the wall rough easily when extending this study to the case of rough walls in the future. The duct consists of three parts as shown in Fig. 2: the test section in which the falling water film contacts the upward air flow directly; and the upper- and lower-calming sections with lengths of 1000, 800 and 2435 mm respectively. All are made of smooth bakelite plates 20 mm thick and have a rectangular cross section 250 mm by 30 mm with an aspect ratio of 18.5.

Air supplied by a blower was conveyed to the test section through the surge tank and the lower calming section after its own flow rate was measured through a standard nozzle. Here it was heated and humidified by contact with the falling water film, then passed through the upper calming section and the mixing chamber in which its mixing-cup temperature and humidity were measured. Finally, it left by way of the duct.

Water heated to a prescribed temperature by a gas heater or an electric heater with a thermostat was pumped up to the upper tanks and introduced into the test section through the slits, subsequently forming a falling film on the inner walls of the duct. Then it was collected into the lower tanks and returned to the reservoir. The bottom of the test plates was shaped as shown in Fig. 3 so as to expedite flow of water to the lower tank.

To prevent heat loss from duct walls to the surroundings, the duct was wrapped

$$f'$$: resistance coefficient of air flow

G: weight rate of flow

h: heat transfer coefficient

N: mass transfer coefficient based on absolute humidity

L: depth of wave inception

2z: distance between two parallel plates

Nu: Nusselt number

Re: Reynolds number for air flow

Re': modified Reynolds number for air flow

Sh: Sherwood number

T: temperature

$$\Delta t_{lm}$$: logarithmic mean temperature difference

u: velocity

U: air velocity at the center of duct

w: rate of evaporation of water

x: absolute humidity

$$\Delta x_{lm}$$: logarithmic mean humidity difference

y: wall distance

$$\delta$$: thickness of water film

$$\nu$$: kinematic viscosity

Subscripts

1: flowing into

2: flowing out

G: air

L: water

m: mean value

s: surface of water film

2. Experimental apparatus and procedure

In experiments of this kind, circular pipes of about 25 mm inner diameter have been often used. In the present experiment, a rectangular duct with a large aspect ratio was adopted so as to make clear

![Fig.2 Schematical arrangement of experimental apparatus.](image)

![Fig.3 Bottom end of test plate.](image)

Table 1 Rate of water flow and thickness of water film in non-wavy part.

<table>
<thead>
<tr>
<th>Rate of water G_L kg/h</th>
<th>1800</th>
<th>1000</th>
<th>750</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of water per unit width of wall G_L kg/h-n</td>
<td>1635</td>
<td>910</td>
<td>680</td>
<td>455</td>
</tr>
<tr>
<td>Mean film thickness $$\delta$$ mm</td>
<td>0.59</td>
<td>0.48</td>
<td>0.43</td>
<td>0.37</td>
</tr>
<tr>
<td>Mean velocity of water film $$u_w$$ m/s</td>
<td>0.77</td>
<td>0.52</td>
<td>0.44</td>
<td>0.34</td>
</tr>
<tr>
<td>Film Reynolds number $$Re_w = 4u_L\rho L/\mu_w$$ (corresponding to 20-50°C of water temperature)</td>
<td>1800</td>
<td>990</td>
<td>750</td>
<td>500</td>
</tr>
</tbody>
</table>

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with a blanket, and as a result, the heat loss was reduced to less than two per cent of the total heat flow discharged from the falling water film in the test section.

Temperature of water and air flow was measured by platinum resistance thermometers and thermocouples consisting of copper-constantan of 0.3 mm diameter. Humidity was measured by Assmann type psychrometers in the surge tank and mixing chamber.

The measurements were conducted for four kinds of water flow rate as noted in Table 1. The temperature of inlet water was fixed at about 20, 30, 40 and 50°C. The air velocity was so varied as not to exceed the range within which splash would occur. Moreover, in order to examine the effect of inlet air humidity, moist air saturated through a humidifying device was also used in addition to unsaturated room air with a relative humidity of about 65 per cent.

3. Experimental results and discussion

3.1 Velocity profiles of air flow

In the first place measurement of air velocity distribution ascertained that the air flow in the test section was developed to a fully turbulent and satisfactorily two-dimensional state throughout almost the whole range of this experiment. Some of the distributions are shown in Figs. 4 and 5. The profiles for dry wall agree well with the 1/7-th-power law of $u_i/U = (y/L)^{1/7}$ as seen from Fig. 4. In the case of wetted wall, on the other hand, when $u_i$ is plotted against the wall distance $y$ similarly to Fig. 4, systematic change in the profiles with the flow rate of water is found as shown in Fig. 5(a). However, when the air velocity relative to the water surface velocity ($u_i + u_w$) is plotted against the distance from the water surface ($y - d_w$), the points fall rather regularly into a curve regardless of the water flow rate, and the profiles can be approximately represented by the 1/6 or 1/5-th-power law (see Fig.5(b)). Thus the water film surface can be considered a kind of roughness.

3.2 Flow behavior of water film

Hydrodynamic characteristics and behavior of the water film are very important for heat and mass transfer between air flow and film. They have been studied both theoretically and experimentally by a number of investigators from various viewpoints. However, it seems the details are not so clear yet, even aside from air flow. Some available results are herewith described.

3.2.1 Photographic survey of the water film

Water films flowing down one of the test plates were observed through the opposite plate which was replaced by a transparent one. The plates of Fig. 6 show the phenomenon when both the flow rate of water and the air velocity were varied. From such photographs it may be inferred that, although appearance of the film surfaces depends both on the flow rate of water and the air velocity, every descending water film generally has three parts in common: (1) a non-wavy smooth part formed near the top of the test plate; (2) a region where transverse ripples with shorter wave-length prevail; and (3) a more downstream part composed of longitudinal waves or large waves.

Length of the smooth part, i.e., the distance from the top of the film to the wave inception, is considered significant in the hydrodynamics of a falling liquid.
film. In this connection comprehensive results are reported by Taihy-Portalski.(3) As to the effect of air flow, however, little is mentioned. Figure 7 shows the depth of the wave inception \( L \) obtained from the photographs against the mean air velocity \( u_{am} \). It is seen from this that \( L \) increases with the increasing flow rate of water and the decreasing air velocity. The influence of the inlet water temperature on \( L \) is weaker than that of the above two factors in the range from 5 to 50°C, but a tendency for \( L \) to be minimum at about 20°C was perceived.

The appearance of the film surface changes considerably with the air velocity. Up to \( u_{am} = 1.5 \) m/s, comparatively little change appears as opposed to the case of \( u_{am} = 0 \); with subsequent increase of air velocity, longitudinal waves on the surface change to transverse waves of shorter wave length. In the case of larger water flow rate this change to transverse waves occurs at a greater air velocity and the transverse waves become large waves which seem to be gliding downward just like lapping waves. By further increasing the air velocity the crests of the large waves gliding downward became clear, and at \( u_{am} = 7-8 \) m/s, water splashes shorn off from the water film began to disperse in the duct. Beyond about 11 m/s, flooding occurred. This experiment excluded cases accompanied by heavy splash.

3.2.2 Thickness and velocity of the water film

Thickness of the liquid film is very significant in studying the hydrodynamic characteristics themselves and the transport phenomena existing between a liquid film and its surroundings. Its measurement is very difficult, especially in the wavy region. Hence, various methods and techniques have been devised and tried, but the results seem rather scattered, owing to the complex devices and procedures involved. Present results measured by using a needle probe to detect contact with water film were also scattered. Thus the film thickness in the upstream smooth part, which can be correctly measured by this contact method, was made to serve as the approximate measure of film thickness, as will be covered subsequently in detail.

The film thickness within the non-wavy smooth part was measured by the contact method under the various conditions of water flow rate, inlet water temperature, air velocity and distance from the top of the plate. The results indicate that the film thickness is little affected by these conditions except the water flow rate in
so far as the film surface at the measuring point remains smooth; with the increasing flow rate of water, thickness increases as shown in Fig. 8. Although the tendency for the water film to become thicker with the air velocity can be noted in the same figure, this increase is less than 10 per cent even in the case of $G_L = 500$ kg/h as far as the non-wave part is concerned.

As a result of these measurements, assuming that the thickness of a film is a function of the water flow rate alone, a film thickness measured at the smooth part and under $u_{w0} = 0$, is used in this paper as the mean thickness of the water film. Mean thicknesses of the films obtained from a number of measurements and the corresponding mean falling velocities calculated by $u_{lm} = G_l/2\nu L \delta m A$ are listed in Table 1 together with the film Reynolds number defined by $Re_f = 4u_{lm} \delta m /\nu$.

The above approximation is certainly crude. However, the application of the values thus obtained is not entirely inappropriate here, when taking account of the fact that $\delta m$ and $u_{lm}$ are used only as a reference values in nondimensional numbers such as Reynolds, Nusselt and Sherwood numbers. Moreover, one can point to a supportive description found in a recent paper that a liquid film on a vertical smooth wall may be said to consist of a bottom layer and a wavy layer covering it, the thickness of the bottom layer remaining constant along the thickness of a film and being nearly equal to that of the non-wave part.

The surface velocity of the water film is also significant, especially when air flow contacts it. According to Nusselt's theory for a falling film in laminar and non-wave flow, the surface velocity of the film is 1.5 times its mean velocity. In the wavy region, $u_{lm}/u_{lm} = 2$, proposed by Jackson based on his own experiments, has been accepted broadly. The experimental results of $u_{lm}/u_{lm} = 1.0 - 1.5$ for wavy regions are also advanced in a recent paper.

Since surface velocity was not measured in the present experiment, a trial was conducted to find appropriate values of $u_{lm}$ by which the data concerning the resistance coefficient, the Nusselt and Sherwood numbers could be reduced to convenient forms. After trial and error procedures, $u_{lm}/u_{lm} = 1.0$ was adopted as a reasonable ratio in so far as the present experiment was concerned.

### 3.3 Resistance coefficient of air flow

Resistance coefficient of the air flow through the test section was measured to investigate its connection with the heat and mass transfer coefficients and to grasp the degree of waviness of the water film as a kind of surface roughness. In Fig. 9 the resistance coefficient defined by

$$ f = \frac{d}{L} \frac{d\rho (u_{w0}+u_{lm})^2}{2} \tag{1} $$

is plotted against the Reynolds number of

$$ \text{Re}_{aw}' = \frac{(u_{w0}+u_{lm})d_{m}}{\nu} \tag{2} $$

In these equations, $(u_{w0}+u_{lm})$ is the mean velocity of air flow to the surface velocity of the water film, is assumed as a reference velocity, and $d_{m}$ is pressure drop over the length $L$ of the duct. In the estimation of $d_{m}$ and $u_{lm}$ the mean thickness of the film was taken into account and the properties at the mean temperature of the inlet and outlet air were used.

According to Fig. 9 the results for the dry wall $(G_L = 0$, and thus: $u_{lm} = 0)$ agree well with Blasius' equation for a circular pipe, $f^{*} = 0.336 \text{Re}_{aw}^{-1/4}$, and the resistance coefficient for every wetted case is larger than that for the dry wall. The points for a fixed flow rate of water distribute along a line nearly parallel to Blasius' one and, besides, the line becomes higher with the increasing flow rate of water. Therefore it can be said that although the surface of film keeps smooth partly, the interface of film for a larger flow rate of water is hydrodynamically rougher as a whole duct, hence the larger waves found on the lower part of the wall play an important role in the air flow resistance.

Degree of the mean surface roughness due to the water wave was estimated following Cohen-Hamratty although their experimental conditions differed from this. An equivalent sand roughness of $k_s = 0.7$ mm for $G_L = 1800$ kg/h and $u_{lm} = 3.5$ m/s, was obtained.

### 3.4 Heat and mass transfer

Mean heat and mass transfer coefficients between air and water film in the duct were obtained by

$$ h_{w} = q/A \Delta T_{lm} \tag{3} $$

$$ h_{w} = w/A \Delta T_{lm} \tag{4} $$

where $A$ is the interfacial area for which in this paper the increment due to waves was not taken into account; and $q$ is sensitive heat flow transferred from the water to the flowing air; while $\Delta T_{lm}$ and $\Delta T_{lm}$ are defined as follows:

$$ \Delta T_{lm} = \frac{(T_{lm}-T_{m})-(T_{lm}-T_{m})}{\ln((T_{lm}-T_{m})/(T_{lm}-T_{m}))} \tag{5} $$

$$ \Delta T_{lm} = \frac{(T_{lm}-T_{m})-(T_{lm}-T_{m})}{\ln((T_{lm}-T_{m})/(T_{lm}-T_{m}))} \tag{6} $$

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![Fig.9 Resistance coefficient of air flow with and without water film.](image-url)
The temperatures of the water film surface $t_{w}$ and $t_{x}$ were approximately replaced with the mixed ones measured in the upper and lower water tanks because of difficulty in measuring the surface temperature correctly. In connection with this, the saturated humidities at the above mixed temperatures were also used instead of those at the surface temperatures such as $x_{w}$ and $x_{x}$.

Mean Nusselt and Sherwood numbers were obtained by their own definitions:

$$N_u = \frac{h_d d}{k}$$  \hspace{1cm} (7)

$$S_h = \frac{h_d d}{\rho D}$$  \hspace{1cm} (8)

where $k$, $\rho$, and $D$ are thermal conductivity, specific weight of air and diffusion coefficient of water vapour in air. These property values were introduced at the mean temperature of the inlet and outlet air temperatures. As a reference length of this system three kinds of length can be considered, namely (a) the distance between two parallel walls of the duct, (b) the hydraulic diameter and (c) the heat transfer diameter. In the present duct (a) is just a half of (c) and (c) is nearly equal to (b) because of its large aspect ratio. Taking account of this and the works in the past, the hydraulic diameter was adopted here.

### 3.4.1 Effect of the water flow rate

Since the investigation by Gilliland-Sherwood\(^2\) was reported, air velocity to the duct wall has been often used as a reference velocity for such a flow system. Here, the air velocity relative to the water film surface was used as earlier described in the sections on velocity profile and the friction coefficient of air flow. In Fig. 10(a), in which the results for Nu are shown against the Reynolds number of $Re_d = \frac{u_d d}{v}$, systematic change of distribution with the flow rate of water is clearly visible. On the other hand, in Fig. 10(b), which shows the same results plotted against the modified Reynolds number $Re_{eq} = \frac{(u_{ave} + u_d) d}{v}$ (based on the air velocity relative to the film surface), the difference due to the flow rate of water disappears and the points readily fall into a curve. This was found for every condition of temperature and air humidity, not only for Nu but also for Sh. Therefore, it can be said that the effect of the water flow rate can be approximately absorbed on these graphs for Nu and Sh by introducing the modified Reynolds number and that this relative velocity is appropriate as a reference velocity especially for reducing the data to convenient forms.

For the expression such as Fig. 10(b), it was found that the points distributed along a broken line, and a branch of the line extending to large Reynolds number can be approximately expressed by $Nu \propto Re_{eq}^{\frac{2}{3}}$ and the other branch by $Nu \propto Re_{eq}^{0.8}$, as

![Fig.10](image)

**Fig.10** Effect of water flow rate on Nusselt number; (a) using air velocity relative to duct wall surface, and (b) using air velocity relative to water film surface as a reference velocity.

![Fig.11](image)

**Fig.11** Effect of inlet water temperature; (a) on Nusselt number, and (b) on Sherwood number.
shown by solid lines in the figure. This broken line distribution is considered to be a result of air flow changes from the transitional to the turbulent state, a part with a steeper slope corresponding to the transitional region and the others to the turbulent. The exponent of 0.8 is equal to that for heat transfer in turbulent pipe flow. The value of 1.5 is only a mean value in the present experiments, because for the transitional region no defined exponent is known even in the case of simple pipe flow. Berman(8) and Kaffešjian(9) have also explained the broken line distribution as a result of the transition of the air flow.

3.4.2 Effect of air and water temperatures
At first, measurements were performed under a constant temperature difference between the inlet water and air of \( \Delta t_i = t_{a,2} - t_{a,1} \). Then a difference due to the water temperature level was found clearly in Nu and Sh even where \( \Delta t_i \) was kept constant. Thus, to reveal the effect of the water temperature the inlet water temperature was varied from 20 to 50°C. The corresponding results are shown in Fig. 11(a) and (b) for Nu and Sh respectively.

![Image](image.png)

Figure 11(a) shows that the higher the water temperature the larger is the Nusselt number, and that Nu for every case is larger than Nu = 0.023 \( Re^{0.8} Pr^{0.8} \) which is a familiar equation of heat transfer coefficient for turbulent pipe flow with dry wall.

Concerning mass transfer, on the other hand, Sherwood number decreases with a rising water temperature as seen in Fig. 11 (b), although the rate of evaporation increases with the water temperature as shown in Fig. 12. One reason for this is considered to be the difficulty of measuring the temperature at the water film surface; the saturated humidity corresponding to the mean temperature of the water film was employed instead of \( \Delta h_m \) at the surface of the water film as mentioned before, and thus the driving force of mass transfer \( \Delta x_m \) was inaccurately estimated.

In the case of higher water temperature, properties like the saturated vapour pressure of water vary markedly with temperature, and mist appears in the test section; then the phenomenon itself becomes complex, so Nu and Sh may be affected by many factors under these conditions.

3.4.3 Effect of the inlet air humidity

![Image](image.png)

In order to examine the effect of inlet air humidity, saturated air (\( \gamma = 100 \) per cent) was supplied also. In Fig. 13(a) and (b) the results of Nu and Sh for the saturated air are compared with those for unsaturated air with a relative humidity \( \gamma \), of about 65 per cent. Nusselt number is larger for the saturated air than for the unsaturated, and inversely, Sherwood number is slightly smaller for saturated air. When the saturated air was introduced the mist was visible even with inlet water at 20°C. Hence this condition seems to be similar to the cases for higher water temperature as mentioned in 3.4.2.

4. Conclusions

The results of the present study may be summarized as follows.

1) The measured results for the resistance coefficient of air flow could be reduced well by introducing the modified Reynolds number based on the air velocity relative to the surface velocity of the water film.

2) As for Nu and Sh numbers also, by using the preceding Reynolds number the difference due to the rate of water flow disappeared and the data roughly fell into a broken line which had a breaking point at \( Re = (6 \sim 10) \times 10^3 \). The upper branch

(a)

(b)
of the line extending to a large Re number corresponds to the turbulent flow and the lower branch to the transitional flow.

(3) When changing the temperature of the inlet water from 20 to 50°C, Nu number becomes larger with the increasing water temperature. On the other hand, Sh number inversely decreases, although the rate of water evaporation increases in terms of temperature.

(4) When saturated air is introduced, Nu number is larger than that for unsaturated air, whereas Sh number is slightly smaller.

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