On the Propagation of Thermoelastic Waves According to
the Coupled Thermoelastic Theory

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In this paper, coupled thermoelastic wave problems are treated with
approximate solutions based on the limit-value theorem and with a numeri-
cal inversion technique of the Laplace transform. The problems considered
are those of a half-space under a sudden strain with constant temperature,
or a constant velocity impact with adiabatic condition over the boundary
plane. The numerical results show that the approximate solutions are
applicable to solve these problems for short and also long times. Further,
it is shown from the analyses of two particular cases that at the wave
front of the coupled thermoelastic waves subjected to a thermoelastic damp-
ing, approach asymptotically certain values determined from the boundary
conditions. For the adiabatic boundary condition, the influence of coupled
thermoelastic effect remains at all points of the body considered, but for
the other case, it vanishes gradually at the points through which the wave
front has already passed with heat conduction from the boundary.

1. Introduction

In Fourier's heat-conduction equation, temperature fields are assumed to be inde-
dependent of the displacement fields. This assumption, however, is not valid except
for a steady state. A part of the energy supplied to an elastic body will be con-
verted into heat energy and the rest into strain one. In other words, temperature
changes in the element of an elastic body give rise to a variation of strain and stress,
while dynamic strains and stresses induce changes of temperature. Therefore,
in coupled thermoelastic problems, the modified Fourier's heat-conduction equa-
tion in which the coupling of the temperature and strain fields is taken into
account, and equations of motion should be solved as simultaneous equations such
that temperature and mechanical boundary conditions are simultaneously satisfied!

Investigations into the effect of including thermoelastic coupling have been
made by many authors, such as Boley and Tolins**, Soler and Brull**, Dillon**, Tanaka
and Kurokawa**, Muki and Breuer**, Hethareski**, and others.

In consideration of the thermoelastic coupling, this paper deals with the analy-
ysis of problems on the propagation of thermoelastic waves through a semi-infinite
medium subjected to mechanical disturbance. The problems considered here are: (a) step
time variation of strain with constant temperature; and (b) constant velocity

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Then the normal strain and the stress-displacement relation in the x-direction of Eqs. (3) reduce to
\[ \varepsilon_{xx} = \frac{\partial u}{\partial x} \]
while Eqs. (1) and (2) become
\[ \varepsilon_{xx}^T \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} + (3\lambda + 2\mu)\alpha T \]
Here, with the introduction of the following dimensionless quantities
\[ \xi = \frac{\kappa}{\kappa}, \quad \tau = \frac{c}{c} \]
where \( \tau \) is the dilatational wave velocity given by \( c = \sqrt{(\lambda + 2\mu)/\kappa} \), Eqs. (7) can be written as
\[ \varepsilon_{xx}^T \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial \Theta}{\partial x}. \]
Moreover, the boundary conditions at infinity are assumed to be
\[ u(x,t) = 0, \quad (x = -\infty) \]
In view of Eqs. (8) and (11), Eqs. (17) can be represented as follows:
\[ \left( \frac{\partial^2 \psi}{\partial \xi^2} \right)_{x=0} = \left( \frac{\partial \psi}{\partial \tau} \right)_{x=0} = 0 \]
\[ \left( \frac{\partial^2 \psi}{\partial \xi^2} \right)_{\xi=0} = \left( \frac{\partial \psi}{\partial \tau} \right)_{\xi=0} = 0 \]

A formal application of the Laplace transform to Eqs. (13) and (19) with the initial conditions Eqs. (18) yields a fourth-order ordinary differential equation for \( \psi(\xi, \tau) \) and the associated regularity conditions.

The general solution of Eq. (20), considering Eq. (21), is of the form
\[ \psi(\xi, \tau) = A \exp(-\lambda_1 \xi) + B \exp(-\lambda_2 \xi) \]
where \( A \) and \( B \) are arbitrary constants determined from boundary conditions at \( \xi = 0 \), while the quantities \( \lambda_1 \) and \( \lambda_2 \) are given by
\[ \lambda_1, \lambda_2 = \frac{1}{2} \left[ \frac{\tau^2 - 2(1-\delta) + (1+\delta)^2}{(1+\delta)^2} \right] \]

Eliminating \( \Theta \) from Eqs. (12), we find the fourth-order partial differential equation for \( \psi \) as
\[ \left( \frac{\partial^4 \psi}{\partial \xi^4} + \frac{\partial^4 \psi}{\partial \xi^4} \right)_{x=0} = 0 \]
while in terms of Eqs. (8) and (11), Eqs. (5) and (6) become
\[ \varepsilon_{xx} = \frac{\partial u}{\partial x} \quad \varepsilon_{xx} = \frac{\partial u}{\partial x} \]

2. General solutions of the Laplace transform domain

In this paper, the Laplace transform will be used for solving problems. We write \( \tilde{\psi}(\xi, \tau) \) for the Laplace transform with respect to \( \tau \) of \( \psi(\xi, \tau) \); thus
\[ \tilde{\psi}(\xi, \tau) = \int_0^\infty \psi(\xi, x; \tau) \exp(-\tau \xi) \, d\tau \]

Similarly, let the Laplace transforms of \( \tilde{\Theta}(\xi, \tau) \) and \( \tilde{\varepsilon}_{xx}(\xi, \tau) \) be \( \tilde{\Theta}(\xi, \tau) \) and \( \tilde{\varepsilon}_{xx}(\xi, \tau) \), respectively. At this stage, assuming that the semi-infinite medium is free from thermal and mechanical disturbances at \( t = 0 \), initial conditions are given as follows:
\[ u(x, t) = \left( \frac{\partial u}{\partial t} \right)_{t=0} = 0 \]

3. Analysis

3.1 Boundary condition

The integral constants \( A \) and \( B \) in Eqs. (22) and (24) determined from the thermal and mechanical conditions at \( x = 0 \) will be discussed in this section. For some basic combinations of mechanical and thermal conditions described by the Heaviside unit step function and the Dirac delta function, Boley and Hetnarski\[7\] have discussed the discontinuity at the wave front. Under the boundary condition of time variations of temperature being given over the free surface, the problem is reduced to Danilovskaya's thermal shock problem\[9\], while when the boundary is subjected to a sudden change of strain with the constant temperature, the problem is reduced to, so-called, Boley and Tolins' one\[8\].
In this paper, we shall consider the following problems of 1 and 2. A constant velocity impact is applied to the surface of a semi-infinite medium and the surface is thermally insulated.

\[
\left. \frac{\partial u}{\partial t} \right|_{t=0} = \alpha H(t), \quad \left. \frac{\partial T}{\partial t} \right|_{t=0} = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ld...
where $n = (i - \delta) \rho / \sqrt{\lambda}$, and $J_{-\lambda}$ denotes Bessel function of first kind, and of order $(n-1)$;

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{\sqrt{\lambda + \tau}} \exp(-\xi \sqrt{\lambda + \tau}) \exp(i\rho \tau) d\tau = (4\pi)^{n-1} \text{erfc}\left(\frac{\xi}{2\sqrt{\tau}}\right)$$

(35)

where $\text{erfc}(x) = \int_{x}^{\infty} e^{-t^2} dt$, $\int_{0}^{\infty} e^{ipt} dt = \frac{\sqrt{\pi}}{2} \text{erfc}\left(\frac{ip}{2\sqrt{\tau}}\right)$, and $\text{erf}(x)$ is the error function. Using the transform relations of Eqs. (35) and (36), we can obtain series solutions corresponding to short times as follows:

$$\epsilon(t, \xi) / V_0 = \exp\left[-\frac{\xi}{2\tau}\right] \sum_{n=0}^{\infty} \lambda_{J_{-\lambda}}(\xi / \sqrt{\lambda + \tau}) J_{-\lambda}(2\sqrt{\lambda + \tau}) |H(\tau - \xi)|$$

(37)

$$\theta(t, \xi) / V_0 = \exp\left[-\frac{\xi}{2\tau}\right] \sum_{n=0}^{\infty} \lambda_{L_{-\lambda}}(\xi / \sqrt{\lambda + \tau}) L_{-\lambda}(2\sqrt{\lambda + \tau}) |H(\tau - \xi)|$$

(38)

where $J_{-\lambda}(\xi / \sqrt{\lambda + \tau}) = -J_{\lambda}(\xi / \sqrt{\lambda + \tau})$, $L_{-\lambda}(\xi / \sqrt{\lambda + \tau}) = -L_{\lambda}(\xi / \sqrt{\lambda + \tau})$, provided that when subscripts of $J$ and $L$ in the right hand side are not integers, those coefficients $J$ and $L$ are zero.

3.2.2 Long-time solution

Next, expanding $J_{-\lambda}$ and $L_{-\lambda}$ of Eqs. (23) for small values of $p$ in Taylor's series

$$J_{-\lambda} = (1 + \delta)^{-1/2} + 0(p^3), \quad L_{-\lambda} = (1 + \delta)^{-1/2} + 0(p^3)$$

substituting Eqs. (39) into (40), we finally obtain the following expressions:

$$\epsilon(t, \xi) / V_0 = \exp\left[-(1 + \delta)^{-1/2} \xi / \sqrt{\lambda + \tau}\right]$$

(39)

$$\theta(t, \xi) / V_0 = \exp\left[-(1 + \delta)^{-1/2} \xi / \sqrt{\lambda + \tau}\right]$$

Inversions of Eqs. (40) yield long-time solutions which show a behaviour of strain and temperature at $\tau \to \infty$

$$\epsilon(t, \xi) / V_0 = \frac{1}{(1 + \delta)^{1/2}} \exp\left[-\frac{(1 + \delta)^{1/2}}{4\tau} \xi^2\right]$$

(40)

$$\theta(t, \xi) / V_0 = \frac{1}{(1 + \delta)^{1/2}} \exp\left[-\frac{(1 + \delta)^{1/2}}{4\tau} \xi^2\right]$$

from the examination of Eqs. (41), it will be seen that the strain and the temperature approach asymptotically $-1/(1 + \delta)^{1/2}$, and $1/(1 + \delta)^{1/2}$, respectively. This agrees with the results of Wilh.8a and Tasaka and Kurokawa.9b

3.2.3 Numerical exact solution

We seek also numerical exact solutions by means of a numerical inversion technique for the Laplace transform developed in authors previous paper. An inversion formula of the Laplace transform, by replacing $p$ by $s$, is a real number, 1, is represented in the form

$$F(t, \tau) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} f(\xi, \tau) e^{-s\tau} ds = 2\pi \int_{0}^{\infty} R_{\tau}(\xi, \tau + i\pi) \cos(\pi\xi) d\xi$$

(42)

where $f(\xi, \tau)$ denotes the Laplace transform of $f(\xi, \tau)$ of time $\tau$, and $\pi$ is an arbitrary real constant such that all singularities of $f(\xi, \tau)$ are in $R(\xi) < \pi$. Therefore, by making a transform $\eta = \pi / r^2$, the inverse Laplace transforms of Eqs. (29) can be represented as follows:

$$\epsilon(t, \xi) / V_0 = 2\pi \int_{0}^{\infty} -\frac{\xi}{\sqrt{\lambda + \tau}} \cos(\pi\xi) |R_{\epsilon}(\eta, \tau + i\pi)\eta| d\eta$$

(43)

$$\theta(t, \xi) / V_0 = 2\pi \int_{0}^{\infty} -\frac{\xi}{\sqrt{\lambda + \tau}} \cos(\pi\xi) |R_{\theta}(\eta, \tau + i\pi)\eta| d\eta$$

where $\tau > 1 / \delta$. In the above equations $R_{\epsilon}(\eta)$ and $R_{\theta}(\eta)$ denote the following equations, respectively:

$$R_{\epsilon}(\eta) = (-1)^{\eta} \frac{\sqrt{\pi}}{2\pi} \cos\left(\theta - \frac{\eta}{2}\right) \exp\left(-\xi \sqrt{\eta} \cos\left(\frac{\eta}{2}\right)\right)$$

(44)

$$R_{\theta}(\eta) = (-1)^{\eta} \frac{n\eta}{2\pi} \cos\left(\theta - \frac{\eta}{2}\right) \exp\left(-\xi \sqrt{\eta} \cos\left(\frac{\eta}{2}\right)\right)$$

(45)

where

$$r = \left[(\gamma - 1 + \delta)^2 + (\gamma - \delta)^2 - 2\gamma + 1\right] / 2\gamma$$

$$\omega = \frac{\omega_{0}}{2} + \omega_{0}^{*}$$

$$\omega_{0}^{*} = \tan^{-1} \frac{2(\gamma - 1 + \delta)}{\omega_{0} - (\gamma - \delta)^2}$$

$$\omega_{0}^{*} = \tan^{-1} \frac{\omega_{0}}{2}$$
\[ q_1 = \frac{1}{2} \left[ \left( (2 \tau + 1 + \delta) y \pm \sqrt{\tau} \left( \frac{y \sin \frac{\omega}{2} + \frac{y \cos \frac{\omega}{2}}{2} \right) \right) + \left( \frac{y^2 + (1 + \delta) \tau - y^2 \pm \sqrt{\tau} \left( \frac{y \sin \frac{\omega}{2} + \frac{y \cos \frac{\omega}{2}}{2} \right) \right) \right]^{1/2} \]

\[ \theta_1 = \tan^{-1} \left( \frac{(2 \tau + 1 + \delta) y \pm \sqrt{\tau} \left( \frac{y \sin \frac{\omega}{2} + \frac{y \cos \frac{\omega}{2}}{2} \right)}{y^2 + (1 + \delta) \tau - y^2 \pm \sqrt{\tau} \left( \frac{y \sin \frac{\omega}{2} + \frac{y \cos \frac{\omega}{2}}{2} \right)} \right) \]

In the same manner as the foregoing problem, analyses will be carried out for the problem in which boundary conditions are given by Eqs. (26). The integral constants \( A_1 \) and \( B_1 \) for this problem are given by Eqs. (28), hence from Eqs. (24), the transformed strain and temperature are obtained as follows:

\[ \xi_0, \psi_0, \theta_0 \]

3.3 Step in strain with constant temperature

3.3.1 Short-time solution

By using Eqs. (30) and (31), and representing Eqs. (45) in the series (1/p), we can find the following solutions equivalent to Eqs. (32) and (33).

\[ \xi_0, \psi_0, \theta_0 \]

3.3.2 Long-time solution

In the same manner as Problem 1, long-time solutions are obtained as follows:

\[ \xi_0, \psi_0, \theta_0 \]

3.3.3 Numerical exact solution

By the numerical inversion technique exact solutions are obtained in the same forms as those for the problem 1, but \( R_1(\xi_0) \) and \( R_1(\theta_0) \) are the following equations.
where $\Gamma(x)$ denotes the Gamma function. On the other hand, for the uncoupled case $\delta = 0$, Eqs. (37) and (38) with the nonvanishing coefficient $I_4 = 1$ become the well-known solutions of a one-dimensional elastic wave.

$$
\text{sett}(\xi, t)/Vs = -H(t-\xi), \ \theta(\xi, t)/V_s = 0
$$

For the problem 2, the short-time solutions of Eqs. (46) and (49) with $\delta = 0$ become

$$
\text{sett}(0, t)/Vs = -1, \ \theta(0, t)/Vs = 0
$$

which always satisfy the boundary conditions, while in the uncoupled case those equations should become identical to Eqs. (53).

$$
\text{sett}(\xi, t)/Vs = -H(t-\xi), \ \theta(\xi, t)/Vs = 0
$$

It will be seen from Eqs. (53) and (55) that, in the absence of thermoelastic coupling, the strain waves of both problems are transmitted with the square shape through the medium where the portion ahead of the wave front remains undisturbed. In contrast to this phenomenon, when the thermoelastic coupling is taken into account, the strain waves of both problems propagate in different shapes accompanied with heat flow.

Figures 1 and 2 show the time dependence of strain and temperature in the problem 1 with the coupling parameter $\delta = 0.1$ at different distances $\xi = 0, 0.5, 1$ and 2. They also illustrate a comparison of the
numerical results of the short-time series solutions given by Eqs. (37) and (38) with those of the exact solutions given by Eqs. (43). From Figs. 1 and 2, the short-time series solutions are reasonable for the initial wave shape nearly within $r<1.5$, even though some differences in range of time are present depending on the values of the coupling parameter.

Figures 3 and 4 show the initial shapes of strain waves and temperature distributions calculated from the short-time series solutions for the problem 1 with $\delta=0.1$. Similarly, for the problem 2 numerical results of the short-time series solutions given by Eqs. (48) and (49) are shown in Figs. 5 and 6. It is found from Figs. 3-6 that under the thermoelastic coupling effect the propagation of strain waves is accompanied with heat flow, so that the strain wave ahead of the wave front is attenuated with the precursor strain derived from the heat flow, hence it is propagated as a damping wave. Moreover, it is obvious that the discontinuity at the wave front decreases with its propagation while the precursor strain gradually increases.

Figures 7 and 8 show a comparison of numerical results of the long-time solutions for the problem 1 with those of exact solutions. These figures are illustrated only for $\delta=0.3$, also for $\delta<0.3$ the long-time solutions have agreed well with the exact solutions in the time-range $t>5$. Consequently, for the portion of the medium through which the wave front has already passed, the long-time approximate solutions will give sufficiently exact values of strain and temperature. In the problem 1 where a constant velocity impact is applied to the thermally insulated boundary, it is clear from Eqs. (41) that the strain near the wave front gradually approaches $-1/(1+\delta)^{1/4}$ which is less than that for the uncoupled case, while the temperature approaches $\delta/(1+\delta)^{1/4}$. On the other hand, from Eqs. (50) which are the long-time solutions for the problem 2, the following results will be obtained. That is, in the same way as for the problem 1, the strain wave for the problem 2 where the boundary is kept at a constant temperature, is accompanied with heat flow and gradually approaches a certain value with its propagation. However, since the influence of the thermoelastic coupling at the points through which the wave front has already passed vanishes gradually due to the heat conduction from the boundary, the strain becomes equal to the applied strain while the temperature becomes equal to that at the boundary.

Figures 9 and 10 show the strain wave and the temperature distributions for the problem 1, while those for the problem 2...
are shown in Figs. 11 and 12. These figures are obtained from the results of numerical exact solutions given by Eqs. (14) and (51). The coupling parameter in Figs. 9 - 12 is taken as \( \delta = 0.3 \), which is larger than those for most of the real materials. In the case of the reference temperature \( T^* = 293K \), the following values for each material are found from Eq. (10): steel (0.008), copper (0.015), aluminum (0.025), lead (0.062).

In the foregoing calculations, the value of \( \gamma \) in the numerical exact solutions of Eqs. (29) and (45) is chosen as 1.5, because those solutions are valid so long as \( \gamma > 1 - \delta \).

Now, if we rewrite the dimensionless variables \( \xi \) and \( r \) into real dimensional ones \( x \) and \( t \), the relations \( x = 285 \times 10^{-4} \) and \( t = 0.12 \times 10^{-4} \) are found for copper. Consequently, for instance, the time needed for the wave front to arrive at the distance \( x = 1 \) cm corresponds approximately to \( t = 0.351 \times 10^4 \), then in the real dimension the influence of the thermoelastic coupling should be predicted only from the long-time solutions. To be more specific, after the wave front has passed through the point above mentioned, the strain and the temperature at that point become for the problem 1

\[
\frac{\xi_{te}}{V_0} = \frac{1}{(1+\delta)^{1.5}}, \quad \frac{\delta_0}{V_0} = \frac{\delta}{(1+\delta)^{1.5}}
\]

and for the problem 2

\[
\frac{\xi_{te}}{\xi_0} = \frac{1}{1+\delta}, \quad \frac{\delta_0}{\xi_0} = \frac{\delta}{1+\delta}
\]

But for the problem 2, as mentioned above, after sufficiently long time, the temperature tends to zero and the strain becomes equal to the applied strain.

5. Conclusions

In the present paper, we have treated the coupled thermoelastic wave problems for two particular cases. And the analysis has been developed by means of the Laplace transform. As a result of this investigation, the following conclusions can be obtained.

1. A method of series expansion
based on the limit-value theorem of the Laplace transform can be useful to examine the effect of the thermoelastic coupling on the propagation of elastic waves.

(2) It is well known that the one-dimensional elastic wave according to the classical theory of elasticity is transmitted with the square shape through the medium without regard to thermal conditions. However, considering the thermoelastic coupling, the strain wave is propagated with different shapes determined from the thermal and mechanical boundary conditions while accompanied with heat flow and a precursor strain in front of the wave front. Namely, the coupled thermoelastic waves are subjected to a thermoelastic damping and then approach certain values determined from the boundary conditions.

(3) For the adiabatic boundary condition, the influence of the thermoelastic coupling effect remains unaltered at all points of the body, but in the other case, it vanishes gradually at the point through which the wave front has already passed with the heat conduction from the boundary.

Acknowledgement

The authors wish to thank Professor Masachika Maito of Maroran Institute of Technology and Professor Ken-ichi Naka of Hokkaido University for their encouragement and helpful advice.

References

(1) For example, Boley, B.A. and


