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A Study on the Flows of Dilute Polymer Solutions*

(3rd Report, Flow Field and Heat Transfer around a Circular Cylinder)

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Viscoelastic fluids such as polymer solutions show anomalous phenomena that are not observed in purely viscous fluids. In such situations, relating with heat transfer and possibility of using a hot-wire anemometer method in such fluids, the problem of the flow around a circular cylinder is a basic one to be solved.

In this report, the problem of the flow field and heat transfer of a two-dimensional steady laminar flow past a cylinder located perpendicular to the uniform flow of a dilute polymer solution is treated analytically. From the results of the analysis the following are obtained: (1) the drag coefficient is larger than that of the solvent, (2) the separation point moves backward compared with that of the solvent because of the visco-elastic behaviour, (3) the heat transfer coefficient is small compared with that of the solvent when Reynolds number is relatively small such as for a flow past a hot-wire anemometer.

1. Introduction

Viscoelastic fluids such as high polymer solutions exhibit various anomalous phenomena that are not observed in purely viscous fluids. From a viewpoint of application and of engineering of fluid mechanics great attention is paid to these phenomena and it has become necessary to make clear such fluid flow behaviour.

Of these fluid flows, two-dimensional flow around a circular cylinder is a basic one to be analyzed with relation to various heat transfer problem and to the possibility of using a hot-wire anemometer for velocity measurement. In spite of these circumstances, such researches are few both experimentally and analytically.

On the other hand, such experimental and analytical researches on Newtonian fluid are accumulated from early times⁽¹⁾, some of which are experimental ones on drag coefficients of a circular cylinder located in a uniform flow by Wieselsberger⁽¹⁾, by Tritton in detail⁽¹²⁾, and experimental one on stream lines etc. by F.Homan⁽⁵⁾. Besides analytical researches on flow fields around a circular cylinder were reported by Lamb for slow flow⁽³⁾, by Imai for slow and moderate speed flow⁽¹¹⁾ and by the theory with boundary-layer approximation for high speed flow⁽¹⁰⁾. In recent years numerical analyses with the aid of computer are reported on intermediate speed flow⁽¹⁵⁾.

Experimental researches on heat transfer from a circular cylinder located

in a uniform flow are reported by many authors using air flow. Mean heat-transfer coefficients were reported by R.Hilpert⁽⁴⁾ and others. Researches on local heat-transfer coefficients along a cylinder surface were reported by Eckert et al. for relatively slow flow⁽⁹⁾ and by Schmidt et al. for fast flow⁽⁶⁾. Nevertheless results in water or oil flow are few and were reported experimentally by Davis⁽²⁾ and by Piret⁽⁷⁾.

This report treats analytically two-dimensional steady laminar flow- and temperature-fields around, and forced heat transfer from, a circular cylinder located in a uniform flow of a dilute polymer solution.

Nomenclature (those defined in the text may not follow the one defined below.)

- a : radius of a circular cylinder
- $B_{(1)}^{ij}$: first kind of Rivlin-Ericksen acceleration tensor
- $B_{(2)}^{ij}$: second kind of Rivlin-Ericksen acceleration tensor
- c : specific heat at constant pressure
- c_D : drag coefficient
- g_{ij} : metric tensor
- h : mesh difference
- n : index which indicates non-Newtonian viscosity
- N_u : local Nusselt number
- $\bar{N}_u$: mean Nusselt number
- P : pressure
- p : dimensionless pressure $= P/\rho U_\infty^2$
- Pr : Prandtl number
- q : heat flux vector
- r : dimensionless radius
- Re : Reynolds number $= 2aU_\infty/\nu$
- s : index which indicates elastic behaviour
- T_{xx} etc. : physical component of a stress

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tensor
 T^{ij} : stress tensor
 U : X-component of velocity
 u : dimensionless velocity= U/U_∞
 U_∞ : free stream velocity
 V : Y-component of velocity
 v : dimensionless velocity= V/U_∞
 v^i : velocity vector
 X : component of orthogonal Cartesian coordinates' system
 x : dimensionless coordinate= X/a
 Y : component of orthogonal Cartesian coordinates' system
 y : dimensionless coordinate= Y/a
 γ : Euler's constant
 Δ : Laplacian operator (in dimensionless coordinates)
 Δ^* : Laplacian operator
 δ : thickness of the velocity boundary-layer
 δ_T : thickness of the thermal boundary-layer
 ε : dimensionless quantity= $\lambda/\rho a^2$
 Z : vorticity
 ζ : dimensionless vorticity= aZ/U_∞
 Θ : temperature
 θ : dimensionless temperature = $(\Theta - \Theta_\infty)/(\Theta_w - \Theta_\infty)$
 κ : thermal conductivity
 λ : coefficient which indicates elastic behaviour
 μ : coefficient which indicates viscous behaviour
 ν : kinematic viscosity
 ρ : density
 Ψ : stream function
 ψ : dimensionless stream function = Ψ/aU_∞
 ω : acceleration coefficient
Suffixes
 w : surface of the circular cylinder
 ∞ : infinity
Others
 $,$: covariant derivative

2. Basic Equations

In this report, a two-dimensional steady laminar flow around a circular cylinder located in a uniform flow of a dilute polymer solution is considered. The fluid is assumed to be incompressible, and moreover characteristic values of the fluid are assumed not to depend on temperature variation.

Let the axis of the cylinder be at the origin, the X-axis being parallel to a uniform flow, the Y-axis being perpendicular to the X-axis, Fig.1. Equations of motion in X- and Y-directions are respectively

Fig.1

$$\rho \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \frac{\partial T_{xx}}{\partial X} + \frac{\partial T_{xy}}{\partial Y} \quad (1)$$

$$\rho \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \frac{\partial T_{xy}}{\partial X} + \frac{\partial T_{yy}}{\partial Y} \quad (2)$$

According to the Denn model (13), which represents the rheological behaviour of viscoelastic fluids rather well, deviatoric stress tensor T^{ij} is expressed as

$$T^{ij} = \bar{\mu}(\Pi) B_{(1)}^{ij} - \bar{\lambda}(\Pi) B_{(2)}^{ij} \quad (3)$$

where $B_{(1)}^{ij}$ and $B_{(2)}^{ij}$ are the first and second kind of Rivlin-Ericksen acceleration tensors respectively. For incompressible fluids, $B_{(1)}^{ij}$ and $B_{(2)}^{ij}$ become

$$B_{(1)}^{ij} = g^{im} g^{jn} + g^{im} v^j_{,n} + g^{jn} v^i_{,m} \quad (4)$$

$$B_{(2)}^{ij} = \frac{\partial}{\partial t} B_{(1)}^{ij} + v^m B_{(1)}^{ij}_{,m} - v^i_{,m} B_{(1)}^{mj} - v^j_{,m} B_{(1)}^{im} \quad (5)$$

Π is a second invariant of $B_{(1)}^{ij}$ and is defined as

$$\Pi = g_{im} g_{jn} B_{(1)}^{ij} B_{(1)}^{mn} \quad (6)$$

$\bar{\mu}(\Pi)$, $\bar{\lambda}(\Pi)$ are functions of Π and are defined as

$$\bar{\mu}(\Pi) = \mu (0.5 \Pi)^{(n-1)/2} \quad (7)$$

$$\bar{\lambda}(\Pi) = \lambda (0.5 \Pi)^{(s-2)/2} \quad (8)$$

Equations (7) and (8), hence the Denn model (3) contains four material constants μ , λ , n and s . Of these constants, n and s take values in general in the following ranges:

$$0 < n \leq 1, \quad 0 < s \leq 2$$

In the case of a dilute polymer solution it is appropriate to assume $n=1$ (moreover when $\lambda=0$, the Denn model represents Newtonian fluids) and $s \approx 2$ (over an appropriate shear rate). In this report calculation is limited to the case $s=2$, considering the capacity of computer and computation time. In the case $n=1$ and $s=2$, Eq.(3) becomes

$$T^{ij} = \mu B_{(1)}^{ij} - \lambda B_{(2)}^{ij} \quad (9)$$

Equation of continuity is

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (10)$$

And an energy equation for a steady flow becomes

$$\rho c \left(U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} \right) = -\text{div } q + \frac{1}{2} B_{(1)ij} T^{ij} \quad (11)$$

When temperature difference between the fluid and the cylinder surface is appreciable as in the case of using a hot-wire velocimeter and the free stream velocity is not so large (e.g. below 5m/s in water), it is considered that

$$\text{Eckert number} = U_\infty^2 / c(\Theta_w - \Theta_\infty) \ll 1$$

For this reason in a dilute polymer solution with relatively weak viscoelastic behaviour the following relation holds:

$$B_{(1)ij} T^{ij} \ll \rho c U_\infty (\Theta_w - \Theta_\infty) / a$$

and the second term of right hand side of Eq.(11) can be neglected. Fourier's law

$$q = -\kappa \text{ grad } \Theta \quad (12)$$

is adopted for heat flux and if dependence on temperature variation for κ is neg-

lected, then Eq.(11) becomes

$$\rho c \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \kappa \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \theta \dots (13)$$

Eliminating P from Eqs.(1) and (2) and using Eqs.(9) and (10), we obtain as equations of motion

$$\rho \left(U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) \left(Z + \frac{\lambda}{\rho} \Delta^* Z \right) = \mu \Delta^* Z \dots (14)$$

$$Z = - \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) \Psi \dots (15)$$

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = - \frac{\partial \Psi}{\partial X} \dots (16)$$

Dimensionless forms of Eqs.(14)~(16) are

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) (\zeta + \varepsilon \Delta \zeta) = \frac{2}{R_*} \Delta \zeta \dots (17)$$

$$\zeta = - \Delta \psi \dots (18)$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = - \frac{\partial \psi}{\partial x} \dots (19)$$

where $\varepsilon = \lambda / \rho a^2$, $Re = 2U_\infty a / \nu$, $\nu = \mu / \rho$. Similarly dimensionless form of Eq.(13) is (if surface temperature is not constant, suitable value may be substituted for θ_w)

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{2}{R_* P_*} \Delta \theta \dots (20)$$

The following analyses are divided into three parts, i.e. in the case of low Reynolds number, moderate Reynolds number and high Reynolds number. For such purpose, the flow behaviour far from the cylinder must be known. When transformation

$$x + iy = \exp(\xi + i\eta), \quad r = \exp \xi \dots (21)$$

is introduced, asymptotic expansions of ψ and ζ for large r become⁽⁸⁾

$$\psi = y + \frac{c_D}{2} \left(\frac{\eta}{\pi} - \operatorname{erf} Q \right) - \frac{c_D R_*}{16 \sqrt{\pi}} \left(\frac{1}{P_0} [\sqrt{2} (\operatorname{erf}(\sqrt{2} Q) - 1) - e^{-Q^2} \operatorname{erf} Q] + \frac{\sqrt{2} Q}{P_0^2 + Q^2} \right) \dots (22)$$

$$\zeta = - \frac{c_D R_*}{4 \sqrt{\pi}} \frac{Q}{r} e^{-Q^2} + \frac{c_D R_*^2}{64 \sqrt{\pi}} \frac{1}{r} \left(- \frac{1}{P_0} e^{-Q^2} \left[\frac{2}{\sqrt{\pi}} Q e^{-Q^2} + (2Q^2 - 1) \operatorname{erf} Q \right] + \frac{1}{P_0^3} [\sqrt{2} (\operatorname{erf}(\sqrt{2} Q) - 1) - e^{-Q^2} \operatorname{erf} Q] \right) \dots (23)$$

where

$$P_0 = \sqrt{0.5 R_* r} \cos(0.5 \eta), \quad Q = \sqrt{0.5 R_* r} \sin(0.5 \eta)$$

Although these expressions correspond to Newtonian fluid flow, in the case $\varepsilon \neq 0$ these formulae are similar at least up to the second term of the right hand side of Eq.(22) and the first of Eq.(23). For this reason these expressions may be introduced in the case of flow of a dilute polymer solution. (The effect of ε is included in c_D implicitly.)

Since the surface temperature is frequently constant as for the hot-wire velocimeter, the assumption that the surface temperature is constant (i.e. $\theta_w = \text{const.}$) is adopted. For temperature behaviour far from the cylinder first approximate solution is obtained with $u=1$ and $v=0$ and the second is obtained with the perturbation method (infinite series are approximated by erf- and tanh-functions). Then asymptotic solution θ is

$$\theta = A_0 \frac{1}{\sqrt{r}} \cos \frac{\eta}{2} \exp \left\{ - \frac{P_* R_*}{4} r (1 - \cos \eta) \right\} + c_D A_0 \frac{1}{r} \sqrt{\frac{R_*}{8\pi}} \left[\left(R^2 + \frac{5\delta_0 - 4}{6} R^4 \right) e^{-\delta_0 R^2} + \frac{(11\delta_0^2 - 18\delta_0 + 8)\pi \sqrt{\pi}}{10(4 - \pi) \delta_0 \sqrt{\delta_0}} R^3 \left\{ \operatorname{erf}(\sqrt{\delta_0} R) - \tanh \left(\frac{2\sqrt{\delta_0}}{\sqrt{\pi}} \right) \right\} \right] + c_D A_0 \frac{1}{r \sqrt{r}} \left[\left(- \frac{R^2}{2\pi} - \frac{R^4}{6\pi} \right) e^{-R^2} - \frac{\sqrt{\pi}}{30(4 - \pi)} R^3 \left\{ \operatorname{erf}(R) - \tanh \left(\frac{2R}{\sqrt{\pi}} \right) \right\} \right] \dots (24)$$

where

$$R = \sqrt{0.5 P_* R_* r} \sin(0.5 \eta), \quad \delta_0 = 1 + P_*^{-1}, \quad A_0 = (2\pi)^{0.5} (P_* R_*)^{-1.5} \bar{N}_*$$

$\bar{N}_*$ indicates mean Nusselt number and Nusselt number based on a radius a is defined as

$$N_* = - \left[\frac{\partial \theta}{\partial r} \right]_{r=1}$$

Boundary conditions are for the velocity field

$$u = v = 0 \quad (r=1); \quad u=1, \quad v=0 \quad (r=\infty)$$

and for the temperature field

$$\theta = 1 \quad (r=1); \quad \theta = 0 \quad (r=\infty)$$

At the surface the following relations hold:

$$\left[\frac{\partial u_r}{\partial r} \right]_{r=1} = 0, \quad [\zeta]_{r=1} = \left[\frac{\partial u_\eta}{\partial r} \right]_{r=1}, \quad [T_{r\theta}]_{r=1} = \frac{2}{R_*} \rho U_\infty^2 [\zeta]_{r=1}$$

$$\left[\frac{1}{\rho U_\infty^2} \frac{\partial P}{\partial \eta} \right]_{r=1} = \frac{2}{R_*} \left[\frac{\partial^2 u_\eta}{\partial r^2} + \frac{\partial u_\eta}{\partial r} \right]_{r=1} + \frac{\varepsilon}{2} \left[\frac{\partial}{\partial \eta} \left(\frac{\partial u_\eta}{\partial r} \right)^2 \right]_{r=1}$$

$$[T_{rr}]_{r=1} = 0$$

Hence the coefficients of pressure drag c_{Dp} and of friction drag c_{Df} become

$$c_{Dp} = \frac{-2}{\rho U_\infty^2} \int_0^\pi \cos \eta [P]_{r=1} d\eta = \frac{4}{R_*} \int_0^\pi \sin \eta \left[\frac{\partial \zeta}{\partial r} \right]_{r=1} d\eta - \varepsilon \int_0^\pi \cos \eta [\zeta^2]_{r=1} d\eta \dots (25)$$

$$c_{Df} = \frac{-2}{\rho U_\infty^2} \int_0^\pi \sin \eta [T_{r\eta}]_{r=1} d\eta = - \frac{4}{R_*} \int_0^\pi \sin \eta [\zeta]_{r=1} d\eta \dots (26)$$

respectively. And the coefficient of drag c_D becomes

$$c_D = c_{Dp} + c_{Df} = \frac{4}{R_*} \int_0^\pi \sin \eta \left(\frac{\partial \zeta}{\partial r} - \zeta \right)_{r=1} d\eta - \varepsilon \int_0^\pi \cos \eta [\zeta^2]_{r=1} d\eta \dots (27)$$

3. Analysis

3.1 Case of low Reynolds number

In this case the solution may be obtained by direct numerical computation of Eq.(17), but here the solution is obtained with Oseen's approximation. Let the difference between u and free stream velocity(=1) be u and neglect the product u and v etc. as higher order infinitesimal in the equations of motion. Then the equations of motion become

$$\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \frac{2}{R_*} \Delta u - \varepsilon \frac{\partial}{\partial x} \Delta u \quad (28)$$

$$\frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y} + \frac{2}{R_*} \Delta v - \varepsilon \frac{\partial}{\partial x} \Delta v \quad (29)$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (30)$$

The solutions of Eqs. (28) - (30) are expressed as

$$u = -\chi - \varepsilon \frac{\partial^2}{\partial x^2} \chi + \frac{2}{R_*} \frac{\partial}{\partial x} \chi + \frac{\partial}{\partial x} \phi \quad (31)$$

$$v = -\varepsilon \frac{\partial^2}{\partial x \partial y} \chi + \frac{2}{R_*} \frac{\partial}{\partial y} \chi + \frac{\partial}{\partial y} \phi \quad (32)$$

$$p = -\frac{\partial}{\partial x} \phi \quad (33)$$

where

$$\Delta \phi = 0 \quad (34)$$

$$\frac{\partial}{\partial x} \chi = -\varepsilon \frac{\partial}{\partial x} \Delta \chi + \frac{2}{R_*} \Delta \chi \quad (35)$$

With the transformation $\chi = e^{\alpha x} f$, Eq.(35) becomes

$$\begin{aligned} &(\alpha - 2\alpha^2 R_*^{-1} + \varepsilon \alpha^3) f + (2\varepsilon \alpha - 2R_*^{-1}) \Delta f \\ &+ (1 + 3\varepsilon \alpha^2 - 4\alpha R_*^{-1}) f_x + \varepsilon \Delta f_x \\ &+ \alpha \varepsilon (2f_{xx} - \Delta f) = 0 \end{aligned} \quad (36)$$

where suffix x indicates partial differentiation with respect to x . Since it is rather difficult to get an analytical solution of Eq.(36), the approximation $f_{xx} \approx 0.5 \Delta f$ may be introduced. This means that the operation is averaged with respect to the angle when f is a function of r only. Moreover the same assumption is appropriate when $\varepsilon \leq 1$. Hence Eq.(36) becomes

$$\begin{aligned} &(\alpha - 2\alpha^2 R_*^{-1} + \varepsilon \alpha^3) f + (2\varepsilon \alpha - 2R_*^{-1}) \Delta f \\ &+ (1 + 3\varepsilon \alpha^2 - 4\alpha R_*^{-1}) f_x + \varepsilon \Delta f_x = 0 \end{aligned} \quad (37)$$

Choosing α such that

$$\left(\alpha - \frac{2}{R_*} \alpha^2 + \varepsilon \alpha^3 \right) \left(1 - \frac{4\alpha}{R_*} + 3\varepsilon \alpha^2 \right)^{-1} = \left(2\varepsilon \alpha - \frac{2}{R_*} \right) \varepsilon^{-1}$$

we obtain

$$\Delta f - \beta^2 f = 0 \quad (38)$$

where

$$\beta = (\alpha - 2\alpha^2 R_*^{-1} + \varepsilon \alpha^3)^{0.5} (2R_*^{-1} - 2\varepsilon \alpha)^{-0.5} \quad (39)$$

$$\alpha \left(\varepsilon + \frac{8}{R_*^2} \right) - \frac{2}{R_*} - 12\varepsilon \frac{\alpha^2}{R_*} + 5\varepsilon^2 \alpha^3 = 0 \quad (40)$$

Since $f \rightarrow 0$ ($r \rightarrow \infty$), the solution of Eq.(38) is expressed as

$$f = CK_0(\beta r)$$

where K_0 is a modified Bessel function of zeroth order. From the fact that $\alpha \sim O(R_*)$ and $\beta \sim O(R_*)$, Laurant expansions of u/C and v/C with respect to Re are (in ascending

power of Re and up to the zeroth)

$$\begin{aligned} \frac{u}{C} = & -\frac{\alpha}{R_*} + \left(1 - \frac{2\alpha}{R_*} \right) \left(\gamma + \ln \frac{\beta r}{2} \right) - \frac{2}{R_*} \frac{\partial}{\partial x} \ln r \\ & + \frac{\alpha r^2}{R_*} \frac{\partial^2}{\partial x^2} \ln r + \varepsilon \frac{\partial^2}{\partial x^2} \ln r + \frac{\partial}{\partial x} \left(\frac{\phi}{C} \right) \dots \dots (41) \end{aligned}$$

$$\begin{aligned} \frac{v}{C} = & -\frac{2}{R_*} \frac{\partial}{\partial y} \ln r + \frac{\alpha r^2}{R_*} \frac{\partial^2}{\partial x \partial y} \ln r \\ & + \varepsilon \frac{\partial^2}{\partial x \partial y} \ln r + \frac{\partial}{\partial y} \left(\frac{\phi}{C} \right) \dots \dots (42) \end{aligned}$$

Under the boundary condition ($r=1:u=-1$ and $v=0$), the following are sufficient in the range cited before:

$$\frac{\phi}{C} = \frac{2}{R_*} \ln r - \left(\frac{\alpha}{R_*} + \varepsilon \right) \frac{\partial}{\partial x} \ln r \quad (43)$$

$$\frac{1}{C} = \frac{\alpha}{R_*} - \left(1 - \frac{2\alpha}{R_*} \right) \left(\gamma + \ln \frac{\beta}{2} \right) \quad (44)$$

Near the cylinder

$$\begin{aligned} \frac{u}{C} = & -\frac{\alpha}{R_*} + \left(1 - \frac{2\alpha}{R_*} \right) \left(\gamma + \ln \frac{\beta r}{2} \right) + \frac{\alpha}{R_*} (r^2 - 1) \frac{\partial^2}{\partial x^2} \ln r \\ & \dots \dots (45) \end{aligned}$$

$$\frac{v}{C} = \frac{\alpha}{R_*} (r^2 - 1) \frac{\partial^2}{\partial x \partial y} \ln r \quad (46)$$

Vorticity ζ becomes

$$\zeta = \frac{\partial}{\partial y} \chi = C e^{\alpha x} \frac{\partial}{\partial y} K_0(\beta r) \quad (47)$$

And the drag coefficient c_D is

$$\begin{aligned} c_D = & \frac{4}{R_*} C \beta K_1(\beta) \int_0^\pi \sin^2 \eta e^{\alpha \cos \eta} \\ & \times \left\{ 1 - \alpha \cos \eta - \beta \frac{K_1'(\beta)}{K_1(\beta)} \right\} d\eta \\ & - \varepsilon C^2 \beta^2 K_1^2(\beta) \int_0^\pi \cos \eta \sin^2 \eta e^{2\alpha \cos \eta} d\eta \\ & \dots \dots (48) \end{aligned}$$

Equation(48) is expressed in the following form (i.e. asymptotic expansion with respect to $Re(+0)$ and power series expansion with respect to ε):

$$\begin{aligned} c_D \approx & \frac{8\pi}{R_*} \frac{1}{0.5 - \gamma + \ln 8 - \ln R_*} \\ & \times \left(1 + \frac{R_*^2}{4} \varepsilon \frac{0.125 - \gamma + \ln 8 - \ln R_*}{0.5 - \gamma + \ln 8 - \ln R_*} \right) \dots (49) \end{aligned}$$

For temperature field, transformation of Eq.(20) into (ξ, η) -coordinates gives

$$\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) \theta = \frac{R_* P_r}{2} \left(\frac{\partial \theta}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right) \quad (50)$$

Since solving this linear partial differential equation analytically is rather difficult, numerical method is applied to it. Derivatives are approximated by difference method (mainly based on intermediate difference) and h denotes mesh-interval. If we select the closing error as $O(h^2)$, then at the mesh point (i, j) ($\xi_i = (i-1)h$, $\eta_j = (j-1)h$)

$$\begin{aligned} \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) \theta \approx & \frac{1}{h^2} \{ \theta(i+1, j) + \theta(i-1, j) \\ & + \theta(i, j+1) + \theta(i, j-1) - 4\theta(i, j) \} \\ \frac{\partial \theta}{\partial \xi} \approx & \frac{1}{2h} \{ \theta(i+1, j) - \theta(i-1, j) \} \end{aligned}$$

and so on. Boundary conditions are

$$\xi = 0 (r=1): \theta = 1; \xi = \xi_\infty (r=r_\infty): \theta = \theta_\infty \quad (\text{Eq. (24)})$$

$$\eta = 0: \frac{\partial \theta}{\partial \eta} = 0 \quad (\text{symmetry})$$

$$\eta = \pi: \frac{\partial \theta}{\partial \eta} = 0 \quad (\text{symmetry})$$

In this section $h=\pi/40$ and $\xi_{\infty}=29h$. However since it is almost impossible due to the capacity and computation time of the computer to solve the simultaneous equations derived from Eq.(50) by difference method, we consider the following equations instead:

$$\begin{aligned} & \bar{\theta}(i,j+1)-4\bar{\theta}(i,j)+\bar{\theta}(i,j-1) \\ & = \frac{PrR_s}{4} h \left[\frac{\partial \psi}{\partial \eta} \{ \theta(i+1,j)-\theta(i-1,j) \} \right. \\ & \quad \left. - \frac{\partial \psi}{\partial \xi} \{ \theta(i,j+1)-\theta(i,j-1) \} \right] \\ & \quad - \theta(i+1,j)-\theta(i-1,j) \dots\dots\dots (51) \end{aligned}$$

where $2 \leq i \leq 29$, $2 \leq j \leq 40$. Similar equations are used at $j=1$ and $j=41$ under boundary conditions. Equation(51) is solved by the accelerated line Liebmann method. The process is as follows. First assume suitable value for $\bar{N}_s$ and calculate θ_{∞} according to Eq.(24). Secondly suitable values for θ are assumed at all mesh points. Next for fixed i , we calculate the right hand side of Eq.(51), solve them for $\bar{\theta}(i,j)$ ($j=1, \dots, 41$) and replace the value $\theta(i,j)$ with the value

$$\{ \theta(i,j) + \omega \{ \bar{\theta}(i,j) - \theta(i,j) \} \}$$

where ω is an acceleration factor and in this section $\omega=1$. After one cycle of iteration ($i=2,3, \dots, 28, 29, 28, \dots, 2$), $\bar{N}_u$ is computed and θ_{∞} is modified according to Eq.(24). This cycle $i=2 \sim 2$ is iterated until values of θ converge. Criteria for convergence are as follows:

necessary condition:

at all mesh points $|\bar{\theta}(i,j) - \theta(i,j)| < 0.01$

convergence condition:

relative variation of $\bar{N}_u < 0.02$.

Local Nusselt number N_s becomes according to a forward difference formula

$$\begin{aligned} N_s &= -\frac{1}{2h} \{ 4\theta(2,j) - \theta(3,j) - 3\theta(1,j) \} \\ &= -\frac{1}{2h} \{ 4\theta(2,j) - \theta(3,j) - 3\theta(1,j) \} \dots\dots\dots (52) \end{aligned}$$

3.2 Case of moderate Reynolds number

When $1 \leq Re \leq 100$, application of Oseen's approximation is not valid and application of boundary-layer approximation near the surface of the cylinder is not appropriate either. Hence the velocity field is obtained through numerical solution of Eqs. (17) ~ (19) by difference approximation. Equations used are from Eqs.(17) and (18)

$$\Delta \zeta = \frac{R_s}{2} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \zeta + \frac{R_s \varepsilon}{2} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \Delta \zeta \dots\dots\dots (53)$$

$$\Delta \psi_0 = -\zeta, \quad \psi_0 = \psi - y \dots\dots\dots (54)$$

Boundary conditions are

$$r=1: \psi_0 = -\sin \eta, \quad (\partial \psi_0 / \partial \xi) = -\sin \eta$$

$$r=r_{\infty}: \psi = \psi_{\infty} \text{ (Eq. (22))}, \quad \zeta = \zeta_{\infty} \text{ (Eq. (23))}$$

$$\eta=0: \psi_0 = \zeta = 0 \text{ (symmetry of flow)}$$

$$\eta=\pi: \psi_0 = \zeta = 0 \text{ (symmetry of flow)}$$

Solving procedure by the accelerated line Liebmann method is the same as in 3.1, that is, we regard the left-hand sides of Eqs.(53) and (54) as unknown (partly known) and the right-hand sides as known. Here $h=\pi/40$ and $\xi_{\infty}=36h$. The order of computation is reverse to that of temperature field,

that is, the calculation is iterated with the order $i=36, \dots, 4, 3$ and after $i=3$, $\psi_0(2, j)$ is computed under the boundary condition, then the calculation is continued with the order $i=4, \dots, 36$. After this one cycle we calculate vorticity over the surface with Eq.(53), next calculate c_D and modify ψ_{∞} and ζ_{∞} . This process is a little different from that of Takami et al. (15) The acceleration factors ω_{ψ} , ω_{ζ} for ψ_0, ζ lie in the range

$$0 < \omega_{\zeta} < 0.5, \quad 1.0 \leq \omega_{\psi} \leq 1.5$$

Criteria for convergence are as follows:

necessary condition:

$|\zeta - \zeta_{\infty}| < 0.01$, $|\psi_0 - \psi_{\infty}| < 0.005$ (at all mesh points);

convergence condition:

relative variation of c_D in 20 cycles < 0.002 or | variation of c_D in 5 cycles | < 0.0011 ,

where these values correspond to those in the case $Re=20$.

Determination of temperature field:

In this case through inference from experimental results on heat transfer in air or water it is predicted that $N_s \geq 2$ as in water when $Re \geq 10$. This means that the variation of the velocity field due to the existence of a circular cylinder in a uniform flow extends more than ten diameters whereas the variation of temperature field is limited to the vicinity of the cylinder. For this reason the temperature at the place far from the cylinder, where an asymptotic expansion Eq.(24) holds, i.e. where heat transfer is mainly due to heat convection and partly due to heat conduction, is expected to be nearly coincident with that of a uniform flow. Hence it is not so useful to select a large temperature field to analyze because of truncation errors and so on. Since the temperature variation is severe near the cylinder, refinement of the mesh distance is necessary lest errors due to truncation errors should become large. In the case $Re=20$, the following are set:

$$\xi_{\infty} = 14\pi/40, \quad h = \pi/80.$$

And also the assumption $\theta_{\infty}=0$ is introduced with a little error. Numerical computation is the same as in 3.1. A criterion for convergence is as follows:

relative variation of mean Nusselt number < 0.01 .

3.3 Case of high Reynolds number

Even if we adopt the difference method for equations of motion, difficulties arise from the consideration for wake flow, and so the velocity field is obtained through boundary-layer approximation. Only in this section let the forward stagnation point be at the origin, x -axis being along the surface of the cylinder, y -axis being perpendicular to the x -axis. When u and v denote physical components of velocity in x - and y -directions respectively and an effect of curvature is neglected, the boundary-layer equation becomes

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial x} (T_{xx} - T_{yy}) + \frac{\partial}{\partial y} T_{xy} - \frac{d}{dx} P_{\infty} \quad (55)$$

With setting $y=\infty$ in Eq.(55), we obtain

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial x} (T_{xx} - T_{yy}) + \frac{\partial}{\partial y} T_{xy} + \rho U_0 \frac{dU_0}{dx} \quad (56)$$

where $U_0(x)$ denotes the velocity outside the boundary-layer. In a two-dimensional flow, from the equation of continuity, the following holds:

$$B_{(1)}^{xx} - B_{(1)}^{yy} = 2 \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) = -4 \frac{\partial v}{\partial y} \quad (57)$$

When boundary-layer approximation is applied to $B_{(2)}$, we obtain

$$B_{(2)}^{xx} - B_{(2)}^{yy} \approx -2 \left(\frac{\partial u}{\partial y} \right)^2 \quad (58)$$

Hence laminar boundary-layer approximation gives

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -4 \mu \frac{\partial^2 v}{\partial x \partial y} + 2 \lambda \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} T_{xy} + \rho U_0 \frac{dU_0}{dx} \quad (59)$$

Integration of Eq.(59) from 0 to h ($dh/dx=0$) with respect to y gives

$$\frac{d}{dx} \int_0^h u(U_0 - u) dy + \frac{dU_0}{dx} \int_0^h (U_0 - u) dy = \frac{\tau_0}{\rho} - \frac{2\lambda}{\rho} \frac{d}{dx} \int_0^h \left(\frac{\partial u}{\partial y} \right)^2 dy \quad (60)$$

where τ_0 means the shear stress on the surface:

$$\tau_0 = \mu \left[\frac{\partial u}{\partial y} \right]_{y=0} \quad (61)$$

When elastic behaviour of the solution is not so great as in a dilute polymer solution, the second term of the right-hand side of Eq.(60) may be estimated from the analogy of Newtonian fluid flow. Hence the relation

$$\int_0^h \left(\frac{\partial u}{\partial y} \right)^2 dy \approx \frac{1}{2\nu} \frac{d}{dx} \int_0^h u(U_0^2 - u^2) dy \quad (62)$$

may hold⁽¹⁰⁾. With this approximation (e. g. when $\epsilon^* \leq 0.01$) we obtain

$$\frac{\tau_0}{\rho} = \frac{d}{dx} \int_0^h u(U_0 - u) dy + \frac{dU_0}{dx} \int_0^h (U_0 - u) dy + \frac{\lambda}{\mu} \frac{d^2}{dx^2} \int_0^h u(U_0^2 - u^2) dy \quad (63)$$

Velocity u is assumed to be of the form

$$u = U_0 f(y/\delta, x) \equiv U_0 f(\eta, x), \quad \delta = \delta(x) \quad (64)$$

Boundary conditions used here are

$$f(0, x) = 0, \quad f(1, x) = 1, \quad \frac{\partial f}{\partial \eta}(1, x) = \frac{\partial^2 f}{\partial \eta^2}(1, x) = 0 \quad (65)$$

Since elastic behaviour of a dilute polymer solution is relatively weak, the following is assumed:

$$\mu \left(\frac{\partial^2 u}{\partial y^2} \right)_{y=0} \approx -\rho U_0 \frac{dU_0}{dx} \quad (66)$$

Hence we obtain

$$\frac{\partial^2}{\partial \eta^2} f(0, x) = -\frac{\delta^2}{\nu} \frac{dU_0}{dx} \equiv \varphi \quad (67)$$

$$\tau_0 = \mu U_0 \frac{1}{\delta} \frac{\partial}{\partial \eta} f(0, x) \quad (68)$$

From the form of the boundary conditions, the following is assumed:

$$\begin{aligned} f(\eta, x) &= F(\eta) + \varphi G(\eta) \quad (69) \\ F(0) &= G(0) = 0, \quad F''(0) = 0, \quad G''(0) = -1 \\ F(1) &= 1 \\ F'(1) &= F''(1) = G(1) = G'(1) = G''(1) = 0 \end{aligned}$$

Hence Eq.(63) becomes

$$\begin{aligned} &\varphi'(0.5 A_3 + 1.5 A_4 \varphi + 2.5 A_5 \varphi^2) \\ &+ (-F'(0) - \varphi G'(0) + \varphi(A_1 + \varphi A_2) \\ &+ 2\varphi(A_3 + A_4 \varphi + A_5 \varphi^2)) U_0' U_0^{-1} \\ &- \frac{1}{2} \left(\frac{U_0''}{\nu} \delta^2 \right) (A_3 + A_4 \varphi + A_5 \varphi^2) \\ &+ \frac{\rho \lambda}{\mu^2} \frac{\partial U_0'}{\partial x^2} \frac{d^2}{dx^2} (U_0^3 \delta (A_0 + \varphi A_7 + \varphi^2 A_3 + \varphi^3 A_9)) \\ &= 0 \quad (70) \end{aligned}$$

At $x=0$, $\varphi'=0$ and

$$\begin{aligned} &-F'(0) - \varphi G'(0) + \varphi(A_1 + \varphi A_2) \\ &+ 2\varphi(A_3 + \varphi A_4 + \varphi^2 A_5) + 6\lambda U_0' \mu^{-1} \varphi(A_3 \\ &+ \varphi A_7 + \varphi^2 A_9 + \varphi^3 A_9) = 0 \quad (71) \end{aligned}$$

where

$$\begin{aligned} A_1 &= \int_0^1 (1-F) d\eta, \quad A_2 = -\int_0^1 G d\eta \\ A_3 &= \int_0^1 F(1-F) d\eta, \quad A_4 = \int_0^1 G(1-2F) d\eta \\ A_5 &= -\int_0^1 G^2 d\eta, \quad A_6 = \int_0^1 F(1-F^2) d\eta \\ A_7 &= \int_0^1 G(1-3F^2) d\eta, \quad A_8 = -3 \int_0^1 F G^2 d\eta \\ A_9 &= -\int_0^1 G^3 d\eta \end{aligned}$$

Moreover if relatively minute terms (having a coefficient λ) in Eq.(70) are omitted, Eq.(70) becomes

$$\begin{aligned} &\varphi'(0.5 A_3 + 1.5 A_4 \varphi + 2.5 A_5 \varphi^2) \\ &+ (-F'(0) - \varphi G'(0) + \varphi(A_1 + \varphi A_2) \\ &+ 2\varphi(A_3 + A_4 \varphi + A_5 \varphi^2)) U_0' U_0^{-1} \\ &- \frac{1}{2} \left(\frac{U_0''}{\nu} \delta^2 \right) (A_3 + A_4 \varphi + A_5 \varphi^2) \\ &+ \frac{6\lambda U_0'}{\mu} \frac{U_0'}{U_0} \varphi(A_0 + \varphi A_7 + \varphi^2 A_3 + \varphi^3 A_9) = 0 \quad (72) \end{aligned}$$

Temperature field: Since it is expected that the temperature field will show an aspect of a boundary-layer type, dimensionless temperature θ becomes in boundary-layer approximation

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{\kappa}{\rho c} \frac{\partial^2 \theta}{\partial y^2} \quad (73)$$

Integrating Eq.(73) from 0 to ∞ with respect to y , we obtain

$$\frac{d}{dx} \int_0^\infty (u\theta) dy = -\frac{\kappa}{\rho c} \left[\frac{\partial \theta}{\partial y} \right]_{y=0} \quad (74)$$

The stream function ψ and dimensionless temperature θ are near $\xi=0$ expanded in the following forms:

$$\begin{aligned} \psi &= \xi^2 F_2(\eta) + \xi^3 F_3(\eta) + \dots \\ \theta &= 1 + \xi T(\eta) - \frac{1}{6} \xi^3 T''(\eta) + \frac{1}{24} P_7 R_7 \xi^4 \\ &+ [T(\eta) F_2'(\eta) - 2T'(\eta) F_2(\eta)] + \dots \end{aligned}$$

(The preceding two equations depend on the coordinate system Eq.(21).) Therefore on the analogy of a thermal boundary-layer along a flat plate, θ is assumed to be

$$\theta = 1 - 2\eta_T + 2\eta_T^3 - \eta_T^4 \quad (0 \leq \eta_T \leq 1, \quad \eta_T = y/\delta_T) \quad (75)$$

From Eq.(75), Eq.(74) becomes

$$\left(\varphi + \frac{1}{2} \varphi' \frac{U_0}{U_0'} - \frac{\varphi}{2} \frac{U_0 U_0'}{U_0'^2} \right) \{ P_1(\chi) + \varphi P_2(\chi) \} + \frac{U_0}{U_0'} \varphi \frac{d}{d\chi} \{ P_1(\chi) + \varphi P_2(\chi) \} = \frac{2}{P_r} \frac{1}{\chi} \quad (76)$$

where $\chi = \delta_T/\delta$,

$$P_1(\chi) = \chi \int_0^\infty F(\eta) \theta(\eta_T) d\eta_T$$

$$P_2(\chi) = \chi \int_0^\infty G(\eta) \theta(\eta_T) d\eta_T$$

Initial conditions are (when $\chi = \chi_0$ and $\varphi = \varphi_0$ at $x=0$)

$$\varphi_0 \{ P_1(\chi_0) + \varphi_0 P_2(\chi_0) \} P_r \chi_0 = 2 \quad (77)$$

$$\left[\frac{d}{d\chi} \chi \right]_{\chi=\chi_0} = 0 \quad (78)$$

The local Nusselt number Nu (based on a radius a) is given by

$$Nu = \frac{2}{\chi} \frac{a}{\delta} = \frac{2}{\chi} \sqrt{\frac{U_0' a^2}{\nu \varphi}} \quad (79)$$

A numerical example is given by the following conditions:

$$U_0 = 2U_\infty \sin(x/a), \quad \varepsilon^* \equiv 6\varepsilon R,$$

$$F = 2\eta - 2\eta^3 + \eta^4, \quad G = \eta(1-\eta)^{3/6}$$

And the function $P_1(\chi)$ etc. become

$$P_1(\chi) = \begin{cases} \frac{2}{15}\chi^2 - \frac{3}{140}\chi^4 + \frac{1}{180}\chi^6 & (0 < \chi \leq 1) \\ \frac{3}{10}\chi - \frac{3}{10} + \frac{2}{15}\chi^{-1} - \frac{3}{140}\chi^{-3} + \frac{1}{180}\chi^{-5} & (1 \leq \chi) \end{cases}$$

$$P_2(\chi) = \begin{cases} \frac{1}{90}\chi^2 - \frac{1}{84}\chi^4 + \frac{3}{560}\chi^6 - \frac{1}{1080}\chi^8 & (0 < \chi \leq 1) \\ \frac{1}{120} - \frac{1}{180}\chi^{-1} + \frac{1}{840}\chi^{-3} - \frac{1}{3024}\chi^{-5} & (1 \leq \chi) \end{cases}$$

$$Nu R_a^{-0.5} = 2 \chi^{-1} (\varphi^{-1} \cos(x/a))^{0.5} \quad (80)$$

4. Results of the analysis

Stream lines and equivorticity lines ($Re=20, \varepsilon=0$) are shown in Fig.2. And relative stream lines are shown in Fig.3. Vorticity distributions along the surface of the cylinder, which contribute greatly

Fig.2 Stream lines and equi-vorticity lines ($Re=20, \varepsilon=0$)

to the drag coefficient, are shown in Fig. 4 (in the case $Re=20$) and in Fig.5 (in the case $R \leq 1$). In these cases due to the viscoelastic behaviour of the solution the absolute value of vorticity near the front portion of the cylinder increases compared with that of a solvent. In the case $Re=20$, the separation point is moved backward due to the viscoelastic behaviour (at the separation point $\zeta=0$). In Fig.6 velocity distribution at the section ($\eta=\text{const.}$) are shown ($Re=20$). From these it is found that near the cylinder the circumferential component of velocity increases and a little decreases far away compared with that in Newtonian fluid flow. In Fig.7 ratios of drag coefficients to those in Newtonian fluid (solvent) flow are shown in the case $R \leq 1$ and in Fig.8 drag coefficients are shown in the case $Re=20$.

Fig.3 Relative stream lines ($Re=20, \varepsilon=0$)

Fig.4 Vorticity distribution along the surface

Fig.5 Vorticity distribution along the surface ($R \leq 1$)

Fig.6 Velocity distribution ($Re=20$)

These show the increment of drag due to the viscoelastic behaviour and are consistent with experimental results by Acosta et al.⁽¹⁶⁾ In Fig.9 values of C_D are shown under the assumption $\epsilon=0$. The

Fig.7 Drag coefficient ($Re \leq 1$)

Fig.8 Drag coefficient ($Re=20$)

Fig.9 Drag coefficient

Fig.10 Isotherms ($Re=20, \epsilon=0, Pr=7.01$)

Fig.11 Isotherms ($Re=1, \epsilon=0, Pr=7.01$)

difference between Eqs.(48) and (49) is hardly remarkable. The temperature fields in Newtonian fluid (i.e. $\epsilon=0$) are shown in Figs.10~12. In Fig.13 isotherms photographed by Eckert et al. are shown (in air, $Re=1260$). In the flows with separation (i.e. in Figs.10 and 13) analytical and experimental results may have the same trend. It is also shown that heat transfer by convection is relatively violent outside the separating stream line and that heat transfer by conduction is violent in the front portion of the cylinder. In order to show the effect of viscoelas-

Fig.12 Isotherms ($Re=0.5, \epsilon=0, Pr=7.01$)

Fig.13 Isotherms (from an interference photograph by E. Eckert and E. Soehngen, $Re=1260$)

Fig.14 Local Nusselt number ($Re \leq 1, Pr=7.01$)

Fig.15 Local Nusselt number ($Re=20, Pr=7.01$)

Fig.16 Local Nusselt number ($Re \geq 1, Pr=7.01$)

ticity on heat transmission, distributions of local Nusselt number in the cases $R_e \leq 1$, $R_e = 20$, $R_e \gg 1$ ($Pr = 7.01$) are shown in Figs. 14, 15, 16 respectively. Fig. 14 shows the reduction of heat transmission due to viscoelastic behaviour, which has the same trend as in the experimental results by Acosta et al.⁽¹⁶⁾ When $Re = 20$, the variation of heat transmission is not necessarily clear, but heat transmission will vary to some extent when ϵ increases. The circumstance $R_e \gg 1$ will appear when $a \gg \sqrt{\lambda/\rho}$ and in this case heat transmission does not vary a great deal compared with that in Newtonian fluid flow, which is coincident with the prediction by Metzner et al.⁽¹⁴⁾ Next in order to show the effect of characteristic values on heat transmission, the aspects of Nu ($\epsilon = 0$) with variation of Pr are shown in Figs. 17, 18, 19. The variation of characteristic values will produce comparable effects with those due to viscoelasticity. In these circumstances, when $R_e \leq 1$, characteristic values will be converted at the temperature $\theta_0 = 0.5(\theta_\infty + \theta_w)$ and when $10 \leq R_e$, $\nu = \nu_{\theta_0}$ for the velocity field and $P_r = \nu_{\theta_0}/a_{\theta_0}$ (a_{θ_0} : thermal diffusivity) for the temperature field and when $R_e \gg 1$, they will be converted at $\theta = \theta_0$.

Fig. 17 Local Nusselt number ($Re=1, \epsilon=0$) Fig. 18 Local Nusselt number ($Re=20, \epsilon=0$) Fig. 19 Effect of Pr on Nu at the forward stagnation point ($R_e \gg 1, \epsilon = 0$)

Fig. 20 Mean Nusselt number

From a viewpoint of these, experimental and analytical results for water are shown in Fig. 20. In the case $R_e \gg 1$, boundary-layer thickness and flow measure at the forward stagnation point are shown in Figs. 21, 22 respectively.

5. Conclusions

The problem of a flow field around and heat transfer from a circular cylinder located perpendicularly in a two-dimensional steady laminar flow of a dilute polymer solution is treated analytically. As the result it is found that due to the viscoelastic behaviour of the solution the drag coefficient increases compared with that in a solvent and that a separation point is moved a little backward if it exists. And it is predicted that in the case of relatively low Reynolds number such as in using a hot-wire velocimeter heat transmission decreases, whereas in the case of high Reynolds number it does not vary a great deal.

Since in a dilute polymer solution the dependence of relative heat transmission rate (to that in a solvent) on the kind and concentration of polymers varies according to Reynolds number, in the case of using a hot-wire velocimeter it is necessary to correct the relation between standard velocity and output at many standard values in each solution.

Numerical computation is executed with HITAC 8500/8400 computer at Tokyo Institute of Technology.

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Fig. 21 Boundary-layer thickness ($R_e \gg 1$) Fig. 22 Characteristic values at the forward stagnation point ($R_e \gg 1$)

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