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Behavior of a Bingham Solid in Hydrodynamic Lubrication* (Part 3, Application to Journal Bearing)

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In the first paper, we already presented the equivalent Raynolds equation for a Bingham solid. In the second paper we applied this theory to a step bearing and carried out an experimental study on it using grease as a lubricant. The core profile and the performances in the step bearing were made clear.

In this paper, we apply the equivalent Raynolds equation for a Bingham solid to a journal bearing and obtain theoretically the core profile formed in the bearing and the bearing performances such as the pressure distribution. On the other hand, we observe the core formation and measure the bearing performances, such as the pressure distribution, in the journal bearing lubricated with grease which is a sort of a Bingham solid. The experimental results agree well with the theoretical ones and it is confirmed that the core is formed at the middle of width of inlet side and adheres to the bearing surface when the eccentricity ratio becomes large. As the dimensionless yield stress increases, the range of core and the load capacity increase.

1. Introduction

We have investigated the behavior of a Bingham solid in hydrodynamic lubrication. In the first paper⁽¹⁾, we already derived the equivalent Raynolds equation for finite bearings and established a general theory on the core formation. In the previous paper⁽²⁾, we applied this theory to a step bearing and obtained the core profile and the bearing performances. We carried out as experimental study on the step bearings lubricated with grease, which is a sort of a Bingham solid. We observed the core distribution and measured the pressure distribution in the bearing, and compared the results with the theory.

In this paper, we apply the equivalent Raynolds equation for a Bingham solid to a journal bearing and clarify the core profile formed in the bearing and the bearing performances such as the pressure distribution.

On the other hand, we observe experimentally the core formation and measure the bearing performances, such as the pressure distribution, in the journal bearing lubricated

with grease. By comparing the experimental results with the theoretical ones, we show how the yield value of shear stress affects the core formation and the bearing performances. The nomenclatures in this paper are as follows.

D : diameter of journal
F : frictional force
 F_j : frictional force on journal surface
 F_b : frictional force on bearing surface
L : bearing width
R : radius of journal
S : Sommerfeld number
U : circumferential velocity of journal
W : load capacity
c : radial clearance
e : eccentricity
f : coefficient of friction
 f_j : coefficient of friction on journal surface
 f_b : coefficient of friction on bearing surface
h : film thickness
p : film pressure
 θ, y, z : co-ordinates
u : velocity of lubricant in θ -direction
w : velocity of lubricant in z-direction
 $\dot{\gamma}$: shear rate
 ϵ : eccentricity ratio
 μ : plastic viscosity of Bingham solid
 τ : shear stress
 τ_0 : yield stress of Bingham solid
 ϕ : attitude angle

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2. Theoretical analysis and discussions

By applying the equivalent Raynolds equation for a Bingham solid presented in the first paper to a journal bearing shown in Fig.1, the core formed in the bearing and the bearing performances such as the film pressure are theoretically studied. In the figure, the origin of the bearing angle is taken at the point of the maximum film thickness. The film thickness h can be expressed by $c(1+\varepsilon\cos\theta)$. In the theoretical analysis, we use the half Sommerfeld boundary conditions in which $p=0$ at $\theta=0, \pi$ and $\pi \leq \theta \leq 2\pi$. The non-dimensional quantities used in this analysis are shown in Table 1.

2.1 Pressure distribution and core formation

The shear stress acting on the bearing surface and the core profile are shown in Fig.2 and Fig.3. The dark parts in the figures indicate the range of core adhering to the bearing surface and the hatched parts the range of core floating in the film. The arrows in the figure show the magnitude and the direction of shear stress $\bar{\tau}$ acting on the bearing surface. The unit length of shear stress $\bar{\tau}$ is shown in the upper side of the figure. The width-to-diameter ratio of the bearing is unity. In the case of $\bar{\tau}_0=0.4$, no core is formed when $\varepsilon=0.60$. When $\varepsilon=0.65$, a core is formed and adheres to the bearing surface nearly at $\theta=70^\circ$ in the middle

of width where the shear stress becomes very small. The range of adherent core increases as the eccentricity ratio increases. A reverse flow appears near the adherent core and the core floats in the regions when $\varepsilon=0.75$. When the eccentricity ratio increases further, the region of adherent core separates into two parts and the range of floating core increases gradually. In the case of $\bar{\tau}_0=0.80$, a core is formed when $\varepsilon=0.60$ and the range of adherent core becomes larger with an increase of the eccentricity ratio. In this case, a reverse flow is present when $\varepsilon=0.80$ and the region of adherent core separates into two parts when $\varepsilon=0.85$. It may be considered that when the yield stress $\bar{\tau}_0$ becomes larger, the shear stress acting on the bearing surface in the circumferential direction increases and the reverse flow is reduced. The range of adherent core increases with an increase of the yield stress.

The pressure distributions generated in the central section of the bearing ($\bar{z}=0.5$) are shown in Fig.4 and Fig.5. In these figures, the curve of $\bar{\tau}_0=0$ is identical with the film pressure of a Newtonian fluid. The film pressure of a Bingham solid is larger than that of a Newtonian fluid and also it increases as the yield stress $\bar{\tau}_0$ increases. In the case of $\varepsilon=0.5$, no core is formed in the bearing for any value of yield stress $\bar{\tau}_0$. However, the film pressure increases as the yield stress $\bar{\tau}_0$ increases. From the figure, it may be considered that the existence of yield stress affect to reduce the side flow and reverse flow, and increases the film pressure even if the core is not formed.

Now let us consider the core formation across the film comparing with the pressure distribution. For example, the pressure distributions in the circumferential and the axial direction where $\varepsilon=0.8$ and $\bar{\tau}_0=0.4$ are shown in Fig.6. The core profile under the same condition is shown in Fig.7. The core profiles across the film in sections A-A', ..., D-D' of Fig.7(a) are shown in Fig.7(b). The pressure distribution in the circumferential direction in section B-B' is similar to that in section A-A' but the thickness of core in section B-B' is very small in comparison with that in section A-A'. This is due to the increase of pressure gradient $|P'|$ in the axial direction. In the section C-C', the core adheres to the bearing surface and floats in the film in section D-D'. In both cases, the thickness of core has a maximum value at the bearing center where the pressure gradient $|P'|$ is equal to zero. The thickness decreases as $|P_z|$ increases.

The core formation and the pressure distribution under different

Table 1 Dimensionless quantities

Variables	Dimensionless quantities
θ	$\theta = x/R$
y	$\bar{y} = y/c$
z	$\bar{z} = z/L$
τ	$\bar{\tau} = \tau c / \mu U$
τ_0	$\bar{\tau}_0 = \tau_0 c / \mu U$
p	$\bar{p} = p c^2 / \mu U R$
w	$\bar{w} = w c^2 / \mu U R^2 L$
s	$\bar{s} = \mu U R^2 L / w \pi c^2$
f	$\bar{f} = F c / \mu U R L$
f	$f = \frac{R}{c} \frac{\bar{F}}{\bar{W}}$

Fig.1 Geometry of a journal bearing

width-to-diameter ratios L/D are shown in Fig.8 and Fig.9. In the case of $L/D=0.5$, no core is formed anywhere in the bearing since the pressure gradient $|p_x'|$ near the inlet edge is very small. A core is formed and adheres to the bearing surface when the width-to-diameter ratio L/D is equal to unity. As the width-to-diameter ratio becomes larger, the pressure gradient $|p_x'|$ at the inlet side increases and the core floats in the film. The region of core adhering to the journal surface, which is indicated by the dark part bounded by

the short dashed curves in Fig.8(c), appears in the outlet side. In this region, the pressure gradient $|p_x'|$ is very steep. From these figures, it is considered that the yield stress and the width-to-diameter ratio have an effect on the core formation and the pressure distribution.

2.2 Bearing performances

The relation between the load and the eccentricity ratio is shown in Fig.10. In this figure, the load capacity of a Bingham solid is larger than that of a Newtonian fluid and

Fig.2 Shear stress and core profile ($L/D=0.4$)

Fig.3 Shear stress and core profile ($L/D=0.8$)

increases together with the yield stress $\bar{\tau}_0$.

In Fig.11, the relation between the frictional force and the eccentricity ratio is presented. The thick curves indicate the frictional forces on the journal surface and the short dashed curves the frictional forces on the bearing surface. The frictional forces on both surfaces increase with the eccentricity ratio. The frictional force on the journal surface is larger than that on the bearing surface under the same condition. This may be due to the reverse flow near the inlet side of the bearing surface and the frictional force on this surface decreasing as the eccentricity ratio increases, while the shear stresses acting on the journal

surface are very large everywhere. The frictional forces become large with an increase of yield stress $\bar{\tau}_0$.

The coefficient of friction increasing with an increase of the yield stress $\bar{\tau}_0$ under the same value of Sommerfeld number, as shown in Fig.12. The coefficient of friction on the journal surface is larger than that on the bearing surface. This tendency is more pronounced as the Sommerfeld number decreases.

The relation between the eccentricity ratio and the attitude angle is shown in Fig.13. The locus of journal center in a Bingham solid is similar to that in a Newtonian fluid.

Let us consider how the width-to-diameter ratio L/D affects the bearing performances. The effect of the width-to-diameter ratio on the load capacity is shown in Fig.14 and that on the frictional force in Fig. 15. The load capacity and the frictional force on the journal surface increase but the frictional force on the bearing surface decreases, as the width-to-diameter ratio increases. This is owing to the reverse flow on the bearing surface.

Fig. 4 Pressure distribution

Fig. 5 Pressure distribution

(a) Pressure in the circumferential direction

(b) Pressure in the axial direction

Fig. 6 Pressure distribution

($\varepsilon=0.8, \bar{\tau}_0=0.4$)

(a) Core formation on the bearing surface

(b) Core profile in any section

Fig. 7 Core profile ($\varepsilon=0.8, \bar{\tau}_0=0.4$)

3. Experimental results and discussions

The flow characteristics of grease used in this experiment are shown in Fig.12. From the figure, it is confirmed that grease has the behavior of a Bingham solid. We carry out an experiment with the journal bearing using grease as a lubricant and clarify how the yield value of

shear stress affects the core formation and the bearing performances.

3.1 Bearing performances

The equipment used for testing the journal bearing is illustrated schematically in Fig.17. There are ten holes, each 1 mm in diameter, on the bearing surface in the circumferential direction at the central cross-section of the journal bearing

Fig.8 Shear stress and core profile ($\varepsilon=0.7$, $\bar{\tau}_0=0.4$)

Fig.9 Pressure distribution

Fig.10 Load capacity versus eccentricity ratio

Fig.11 Frictional force versus eccentricity ratio

Fig.12 Coefficient of friction versus Sommerfeld number

Fig.13 Locus of journal center

to measure the film pressure. The film pressure is detected by a small pressure transducer inserted into each hole. The bearing is lubricated with grease which is made to fill an oil groove at the unloaded side by a grease pump. The displacement of the bearing is measured by two dial gauges with an accuracy of $1\text{ }\mu\text{m}$, set at right angles to each other. The diameter and the width of the bearing are about 50 mm and the radial clearance is $235\text{ }\mu\text{m}$. The loads are from 0.5 to 30 kg and the rotational speeds are from 40 to 120 rpm. The change of the temperature is within 2°C .

The pressure distributions in the circumferential direction generated in the central section of the bearing are shown in Figs. 18, 19, 20 and 21. In these figures, the plotted points represent the experimental

values and the curves the theoretical ones. In the theoretical analysis, the half Sommerfeld boundary conditions are used. In every case, the experimental results agree well with the theoretical ones. Figures 18 and 19 show the pressure distribution when the yield stress $\bar{\tau}_0$ are 0.4 and 0.8 respectively. A negative pressure is measured slightly in diverging films but its value is very small as compared with the positive pressure in converging films. These results

Fig. 14 Load capacity versus width-to-diameter ratio

Fig. 15 Frictional force versus width-to-diameter ratio

Fig. 16 Flow characteristics of grease

Fig. 17 Journal bearing testing equipment

Fig. 18 Pressure distribution

Fig. 19 Pressure distribution

may satisfy the half Sommerfeld boundary conditions.

The pressure distribution under the same eccentricity ratio are shown in Figs.20 and 21. In both cases, the film pressure becomes larger as the yield stress $\bar{\tau}_0$ increases. From these figures, it will be considered that the existence of yield stress has an effect to reduce the side flow and reverse flow, and makes the film pressure large.

The relation of the load capacity and the eccentricity ratio is shown in Fig.22, and the locus of

Fig.20 Pressure distribution

Fig.21 Pressure distribution

Fig.22 Load capacity versus eccentricity ratio

journal center is shown in Fig.23. In these figures, the plotted points represent the experimental values and the curves the theoretical ones.

The experimental values agree well with the theoretical ones, and the load capacity becomes large as the yield stress $\bar{\tau}_0$ increases, as shown in Fig.22.

A continuous film exists over the whole range of bearing clearance and the attitude angle is nearly equal to 90° when the eccentricity ratio is small, as shown in Fig.23. As the eccentricity ratio becomes large, a film breakdown occurs in diverging films and the attitude angle decreases. After the film breaks down, the locus of journal center is similar to a semicircle and these results agree well with the theoretical ones. There is scarcely a change in the locus of journal center for any value of yield stress $\bar{\tau}_0$.

3.2 Observation of core formation

In order to observe the shape of a core formed in the journal bearing, a bearing made of a transparent material is used in observational experiment, as shown in Fig.24. To

$\bar{\tau}_0$	Full film	Half film
0.4	○	●
0.6	□	■
0.8	△	▲

Fig.23 Locus of journal center

Fig.24 Photograph of a bearing

confirm easily the range of core adhered to the bearing surface, we spread grease colored with black oil-paint thinly over the bearing surface before operating, and we are able to observe the behavior of colored grease.

Photographs of the core formation are shown in Figs. 25 and 26. The dark parts show colored grease remaining on the bearing surface. The core adheres to the surface in this region. The dark streaks elongated in the axial direction from this region shows traces of the flow of colored grease. It seems that the shear stress acts in this direction.

Figure 25 shows photographs for the observation of the core formation when the yield stress $\bar{\tau}_0$ is 0.4. In this figure, a core is formed and adheres to the bearing surface at the inlet side of the middle of width

where the pressure gradient $|P'|$ and the film thickness are large x while $|P'|$ is small. The range of adherent core increases with an increase of the eccentricity ratio. A reverse flow is present on the bearing surface and the region of adherent core separates into two parts when $\bar{\epsilon}=0.78$. In this experiment, an adherent core in the inlet side can be observed. But the core in the outlet side can not be observed, because the core adhering in this region is considerably thin.

Photographs of the core formation are shown in Fig. 26 when $\bar{\tau}_0=0.8$. In this case, the yield stress $\bar{\tau}_0$ becomes larger as the rotational speed is lower. The range of adherent core increases with an increase of the eccentricity ratio similar to the case of $\bar{\tau}_0=0.4$. However, since the yield stress is large, a core is

(a) $\bar{\epsilon}=0.66, \phi=43^{\circ}40'$

(b) $\bar{\epsilon}=0.68, \phi=35^{\circ}20'$

(c) $\bar{\epsilon}=0.71, \phi=41^{\circ}20'$

(d) $\bar{\epsilon}=0.76, \phi=34^{\circ}50'$

(e) $\bar{\epsilon}=0.78, \phi=33^{\circ}00'$

(f) $\bar{\epsilon}=0.80, \phi=33^{\circ}50'$

Fig. 25 Photograph of core formation ($\bar{\tau}_0=0.4$)

formed at a small eccentricity ratio and a reverse flow is present on the bearing surface and the region of adherent core separates into two parts at larger eccentricity ratio. In this case, no core is observed in the region of thinner film thickness. The range of adherent core becomes larger than that with $\bar{\epsilon}=0.4$.

The results of observation are compared with the theory in Fig. 27. The dark part indicates the range of adherent core and the hatched part the range of floating core in the theoretical result. The arrows show the magnitude and the direction of shear stress acting on the bearing surface. The observed results for the profile and the position of core adhering to the bearing surface agree well with the theory. The observed range of adherent core is slightly smaller than the theoretical one. It

may be considered that the edge of adherent core can not be observed because the core is considerably thin.

4. Conclusions

We applied the equivalent Reynolds equation for a Bingham solid obtained in the first paper to a journal bearings, and compared the analysis with the experiment. The following results are obtained.

(1) When the eccentricity ratio is large, a core is formed and adheres to the bearing surface at $\theta=70^\circ$ where the magnitude of shear stress becomes very small. The range of adherent core increases as the eccentricity ratio increases. When the eccentricity ratio becomes very large, a reverse flow is present on the bearing surface and the core floats

(a) $\epsilon = 0.63, \phi = 49^\circ 30'$

(b) $\epsilon = 0.66, \phi = 44^\circ 30'$

(c) $\epsilon = 0.71, \phi = 41^\circ 40'$

(d) $\epsilon = 0.74, \phi = 41^\circ 10'$

(e) $\epsilon = 0.79, \phi = 33^\circ 40'$

(f) $\epsilon = 0.86, \phi = 30^\circ 10'$

Fig. 26 Photograph of core formation ($\bar{\epsilon}=0.8$)

(a) $\varepsilon = 0.63, \phi = 41^\circ 20'$ (b) $\varepsilon = 0.68, \phi = 35^\circ 20'$ (c) $\varepsilon = 0.81, \phi = 32^\circ 50'$

(i) Observational

(a) $\varepsilon = 0.65$ (b) $\varepsilon = 0.70$ (c) $\varepsilon = 0.80$

(ii) Analytical

Fig.27 Comparison of observational and analytical ($\bar{\tau}_0 = 0.6$)

in the film, and then the region of adherent core separates into two parts.

(2) When the yield stress $\bar{\tau}_0$ is larger, a core is formed at smaller eccentricity ratio and a reverse flow is present on the bearing surface and the region of adherent core separates into two parts at a larger eccentricity ratio. The range of adherent core increases as the yield stress increases.

(3) The observed results for the profile and the position of core adhering to the bearing surface agree relatively well with the theoretical ones.

(4) The film pressure, the load capacity and the frictional force of a Bingham solid are greater than those of a Newtonian fluid, and they

increase as the yield stress $\bar{\tau}_0$ increases.

(5) The experimental results about the pressure distribution, the load capacity and the locus of journal center agree well with the theoretical ones.

(6) It is seen from the analysis that the film pressure and the range of core in the bearing increase as the width-to-diameter ratio of bearing becomes large.

References

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