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Simulation of Elastic Modulus and Poisson's Ratio of Spheroidal Graphite Cast Iron*

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It has been well known that the elastic constants and breaking strength of the spheroidal graphite cast iron are larger than those of the flake graphite cast iron, etc. But, the quantitative explanation on that has not been given.

In this paper, the theoretical method to estimate stochastically the elastic modulus and Poisson's ratio of the spheroidal graphite cast iron is investigated by using the finite element method and the Monte Carlo method. The simulation model of the cast iron is composed of the elements (a) and (b), (a) being made of a matrix only, (b) of an element of the graphite grain enclosed by a matrix. These elements are disposed in the composed model by using the random numbers and the finite element method on two-dimensional problems is used to analyze the deformation behavior of this model

1. Introduction

As one of the materials in engineering, a cast iron is very important. But its mechanical property is very various because the composition is composed of many matrices such as ferrite, pearlite, etc. Table 1 shows the techniques to improve the mechanical property of a cast iron. As an example of the technique that changes the geometrical shape of graphite grain, a spheroidal graphite cast iron is well known. The elastic constants and breaking strength of this cast iron are larger than those of the flake graphite cast iron, etc. But the quantitative explanation has not been given theoretically.

In this paper, the theoretical method to estimate stochastically the elastic modulus and Poisson's ratio of a spheroidal graphite cast iron is investigated. The method is an application of the technique used to estimate the elastic behavior for a grinding wheel in the previous paper.⁽¹⁾ The F.E.M and the Monte Carlo method are used in this method. The model to simulate the cast iron is composed of the elements (a) and (b), and this unit model (a) is made of a matrix only, while (b) is an element of the graphite grain enclosed by a matrix. The simulation method is proposed on the spheroidal graphite cast iron which is composed of a ferritic matrix, and the results are compared with the experimental values.

2. Simulation technique

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Tab.1 Improvement methods on a mechanical property of cast iron.

Fig.1 SEM of fracture surface of specimen

Fig.2 Model of spheroidal graphite cast iron

2.1 Simulation model

Fig.1 shows the fracture surface of spheroidal graphite cast iron obtained by scanning electron microscopy. From this three-dimensional view it is recognized that the graphite cast iron is composed of the matrix and the spherical graphite grains, of which the size and the disposition are random as shown in Fig.2.

Let us consider now this cast iron as one of the grain dispersed composite materials. Then to estimate the elastic modulus and Poisson's ratio, the simplified unit models (a) and (b) as shown in Fig. 3 will be utilized. Namely the element (a) is made of a matrix only, while (b) is the element of a graphite grain enclosed by a matrix. These elements are disposed in the composed model by using the random vectors

Fig.3 Unit model

Fig.4 Simulation technique

Fig.5 Composed models

and the F.E.M is used to analyse a deformation behavior of this model. Figure 4 shows the simulation technique. By using this technique we can analyse quantitatively the effect of a graphite grain distribution or a volume percent on the elastic behavior of the cast iron.

About the composed model, it must be considered that the model simulates exactly the employed cast iron. By the degree of this consideration the composed model will be classified as follows:

- (1) Deterministic model
- (2) Probabilistic models
 - (i) on order one
 - (ii) on order two

The detailed discussion on these models will be given next.

2.2 Deterministic model

A model in which the grain size and the disposition are determined as shown in Fig.5 (a) is named the deterministic model. This model will not exactly simulate the spheroidal graphite cast iron in actual state but the fundamental consideration will be given from this model.

Now, if a volume percent V_g of the graphite grain and the number n are determined, the diameter $2a_0$ of a grain in the model is expressed as

$$(\pi/4)(n/m)(b_0/a_0)^2 = V_g \dots \dots (1)$$

where m is the total number of elements in this model.

2.3 Probabilistic models

2.3.1 On order one

In the actual materials, the grain size and the disposition are random. A model in which the disposition only is random as shown in Fig.5 (b) is named the probabilistic model of order one in here. To determine the disposition of grain in a simulation model the random numbers obtained by a computer will be used.

2.3.2 On order two

A model in which the variations of both the grain size and the disposition are considered is called the probabilistic model of order two and one of its examples is shown in Fig.5 (c). Especially, on the grain size distribution the measurement result on a cast iron, which is very similar to the log-normal distribution⁽²⁾, must be used for the simulation model.

3. Example of simulation

As an example to examine the simulation technique described in chapter 2, an estimation of E and ν of the spheroidal graphite cast iron of which the matrix is almost made of only a ferrite will be carried out.

3.1 Experiment

3.1.1 Specimens

The experiment is carried out to obtain the fundamental data (the elastic constants and the grain size distribution) for the simulation and to know the elastic behavior of the cast iron itself. The

Table 2 Chemical composition of specimen (%)

C	Si	Mn	S	Mg	P
3.28	2.55	0.25	0.015	0.045	0.008

Table 3 Chemical composition of matrix (%)

C	Si	Mn	S	P
0.03	2.55	0.25	0.015	0.008

Fig. 6 Heat treatment condition

Fig. 7 Microstructure of the Specimen

test-piece is made under the heat condition as shown in Fig. 6 and the chemical composition is given in Tab. 2. A microstructure of the specimen is shown Fig. 7 and the composition of matrix tabulated in Tab. 3. From these data, it is known that the matrix of the specimen is almost composed of only the ferrite.

3.1.2 Tensile test

A tensile test specimen to obtain E and ν of the cast iron is shown in Fig. 8 and Figure 9 shows one of the stress-strain curves measured by the strain gages. In this experiment, eighteen test pieces are used and all the measurements of E or ν are carried out within the bounds of the permanent strain 0.03%.

3.2 Calculation

3.2.1 Fundamental data

In a calculation of the simulation technique in chapter 2, E_0 and ν_0 of a matrix, E'_0 and ν'_0 of graphite grain, the grain volume percent, and the grain size distribution must be known. These data will be given next.

(1) E_0 , ν_0 , E'_0 , ν'_0

As the experiment to obtain E_0 and ν_0 is very difficult, these values are determined by applying many data on the irons showing that their microstructure and their chemical composition are nearly similar to

Fig. 8 Tensile test piece

Fig. 9 Stress-strain curve

Fig. 7 and Tab. 3. That is, comparing Table 3 with the values on the pure irons tabulated in Tables 4^(a) and 5⁽⁴⁾, these compositions are nearly equal except that S_i percent in the pure irons is smaller than it of the matrix. From the above mentioned results we consider the following values as E_0 and ν_0 .

$$E_0 = 21000 \text{ kg/mm}^2$$

$$\nu_0 = 0.29$$

Next, let us consider E'_0 and ν'_0 of the graphite grain. The crystal composition of this grain formed in a cast iron is nearly radial and so by considering as an isotropic material the following values are determined from the report of Huntigton^{(3), (6)}:

$$E'_0 = 500 \text{ kg/mm}^2$$

$$\nu'_0 = 0.16$$

(2) Grain volume percent V_g and grain size distribution

On the technique of quantitative microscopy we know that the volume fraction of a constituent in a cast iron can be estimated by measurement of either areal, lineal or point fractions on the plane of polish. That is, to estimate V_g the point counting method, the linear analysis and the areal analysis are utilized. But here, as the grain size distribution must be given at the same time as the measurement of V_g , the technique of measuring immediately the grain distributions in the microstructure of specimen is utilized. In Fig. 10 (a) the grain size distribution is shown and V_g is tabulated in Tab. 6. Figure 10 (b) is the three-dimensional distribution of Fig. 10 (a) transformed by the method of stereology^(a) that will be described

Table 4 Chemical composition of pure iron (%)

Impurities kind	C	Si	Mn	S	P
Electrolytic iron	0.002	0.001	0.002	—	0.0001
Armco iron	0.023	0.007	0.025	0.02	0.007
Carbonyl iron	0.01	0.02	0.02	0.02	0.075

Table 5 Mechanical property of pure iron

Kind	Yield strength kg/mm ²	Ultimate strength kg/mm ²	Elastic modulus kg/mm ²	Poisson's ratio
Electrolytic iron	10~14	18~25	21 000	0.29
Armco iron	—	—	21 000	0.29
Carbonyl iron	9~17	20~28	20 700	—

Table 6 Data for simulation

Elastic modulus and Poisson's ratio of matrix	$E_0 = 21000 \text{ kg/mm}^2$ $\nu_0 = 0.29$
Elastic modulus and Poisson's ratio of graphite	$E_g = 500 \text{ kg/mm}^2$ $\nu_g = 0.16$
Mean value of graphite grain diameter	$2b_0 = 28.57 \mu$
Volume percent of graphite	11.92 %
Volume percent of graphite in element (b)	29.49 %
Elastic modulus and Poisson's ratio of element (b)	$E_b = 10490 \text{ kg/mm}^2$ $\nu_b = 0.2994$

Fig.10 Distribution of graphite grain size

Fig.11 Example of probabilistic model of order one

Fig.12 Deformation behavior of unit model (b)

Fig.13 Deformation of model in Fig.11

in appendix.

3.2.2 Probabilistic model of order one

Let us consider a composed element of $m = 13 \times 13$ as the model that simulates the microstructure of a cast iron in Fig.6. The number of the elements (b) in this model is given as $n = 66$ because V_g and the mean value of grain size are determined by the measurement of microstructure. About

this grain numbers, we know from the previous paper that if n in the model of $V_g = 20\%$ is larger than 36 the elastic constants of this model have nearly a constant value. A disposition of the elements (b) is determined by the random vectors from the computer and one of the examples is shown in Fig.11.

Next, on the elastic constants of the unit model (a), E_i and ν_i are equal to E_0 and ν_0 of a matrix, respectively.

But to obtain E_i' and ν_i' of the unit model (b) the F.E.M must be used. Figure 12 shows the analytical model restricted by the displacement δ and E' will be given by a ratio of δ to the reaction-force. Also ν_i' will be determined as a ratio of the y-strain to x. These elastic constant values are tabulated in Tab.6.

Now, the probabilistic model of order one as shown in Fig.11 is used to obtain E and ν of cast iron. The deformation behavior of this model is shown in Fig.13 and the results of E and ν are shown in Figs.16 and 17 together with the experimental values.

3.2.3 Probabilistic model of order two

4	3	4	0	0	2	3	5	0	3	2	0
3	0	3	3	0	4	2	0	0	3	4	2
0	3	1	5	5	0	0	2	3	6	0	4
3	3	0	0	0	4	3	5	4	4	5	3
4	5	0	3	4	2	3	0	0	0	5	0
0	2	4	0	2	0	4	0	0	3	4	0
0	0	5	1	0	0	3	0	4	4	0	4
3	0	4	4	0	3	2	2	4	0	2	1
0	1	2	4	0	0	6	0	1	2	0	0
0	4	3	6	0	0	3	3	0	0	4	2
0	2	5	4	0	0	4	3	1	3	0	0
3	0	2	0	2	0	0	4	3	4	3	6
5	3	1	2	4	6	0	3	0	3	0	1

Fig.14 Example of probabilistic model of order one

Fig.15 Deformation of model in Fig.14

By using the three-dimensional distribution of grain sizes as shown in Fig. 10 (b), the probabilistic model of order two is made. That is, seven unit models that simulate the distribution of Fig.10 (b) constitute a composed model as shown in Fig.14. The deformation of this composed model is shown in Fig.15 and the elastic constants are shown in Figs.16 and 17.

3.3 Consideration

Let us consider the spheroidal graphite cast iron as one of the grain dispersed composite materials. Paul's formula⁽⁹⁾, that is derived from the law of energy, can be utilized to estimate E and ν . The formula is

$$\frac{1-\nu_0+2\lambda(\lambda-2\nu_0)}{1-\nu_0-2(\nu_0)^2} E_0' V_g + \frac{1-\nu_0+2\lambda(\lambda-2\nu_0)}{1-\nu_0-2\nu_0^2} E_0(1-V_g) \\ \cong E \cong \frac{1}{(V_g/E_0') + [(1-V_g)/E_0]} \quad (2)$$

where

$$\lambda = \frac{\nu_0(1+\nu_0)(1-2\nu_0)E_0' V_g + \nu_0(1+\nu_0)(1-2\nu_0)E_0(1-V_g)}{(1+\nu_0)(1-2\nu_0)E_0' V_g + (1+\nu_0)(1-2\nu_0)E_0(1-V_g)}$$

The value of E by Eq.(2) is compared with the experimental or the simulation results in Tab.7. From this table, Figs.16 and 17, the following considerations will be obtained.

The mean values of E and ν obtained by the simulation are nearly equal to the experimental values, but on the standard deviation the experimental results are larger than the simulation. This means that a more complicated model must be used to simulate the actual behavior of the cast iron. Also the probabilistic model of order two gives more exact values to the

Tab.7 Comparison between experiment and simulation

Elastic Modulus (Kg/mm ²)	Experiment	Mean value = 16909.2 Deviation = 838.1
	Probabilistic model (order one)	Mean value = 16191.3 Deviation = 59.7
	Probabilistic model (order two)	Mean value = 16212.7 Deviation = 212.7
	Upper and lower limit by Paul's theory	Upper limit = 18650 Lower limit = 3680
Poisson's Ratio	Experiment	Mean value = 0.2863 Deviation = 0.0368
	Probabilistic model (order one)	Mean value = 0.2867 Deviation = 0.0011
	Probabilistic model (order two)	Mean value = 0.2866 Deviation = 0.0063

experimental results than the model of order one. About the upper or lower limit value obtained by Paul's formula it is obvious that these values should not be used in the practical estimation.

4. Conclusions

In this paper, the spheroidal graphite cast iron is considered as one of the grain dispersed composite materials and its elastic behavior is analysed by applying the F.E.M and the Monte Carlo method. In the simulation models the size distribution of graphite grain and the disposition are considered to simulate exactly the actual materials. The obtained results on E and ν are nearly equal to the experimental values except that the deviation of the former is smaller than the latter.

After this, by applying the simulation technique here it is desired that the elastic behavior of many materials improve theoretically.

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Appendix

Let's consider a polydispersed system of spherical grain in a matrix as shown in Fig.2. If one of the spheroidal grains is intersected by a random plane, the probability that the plane intersects from $r = r_{i-1}$ of grain to $r = r_i$ is

$$P(i-1, i) = \frac{1}{r_{\max}} \left[\sqrt{(r_{\max})^2 - (r_{i-1})^2} - \sqrt{(r_{\max})^2 - (r_i)^2} \right] \dots (3)$$

Now, if the number of grains of radius $r = r_i$ in a matrix is x_i , the number of grains of which the radius in a intersected plane is from r_{i-1} to r_i is given

Fig.16 Comparison between experiment and simulation (Elastic modulus)

Fig.17 Comparison between experiment and simulation (Poisson's ratio)

Fig.18 Graphite grain

Fig.19 Schematic illustration of the contribution of grains in a cast iron

as follows :

$$N(i-1, i)_j(x) = P(i-1, i)x_j \dots\dots\dots(4)$$

As an example, we consider a model of material which has five classes of grain diameters and whose numbers of these grains are from x_1 to x_5 as shown in Fig.19. In this figure the sections from grain in the vertical columns have the same diameter, whereas the grain diameter vary systematically. The total number of sections of each size can be expressed as

$$\left. \begin{aligned} g_1 &= P(0.8, 1.0)x_1 + P(0.6, 0.8)x_2 \\ &\quad + P(0.4, 0.6)x_3 + P(0.2, 0.4)x_4 \\ &\quad + P(0, 0.2)x_5 \\ g_2 &= P(0.8, 1.0)x_2 + P(0.6, 0.8)x_3 \\ &\quad + P(0.4, 0.6)x_4 + P(0.2, 0.4)x_5 \\ g_3 &= P(0.8, 1.0)x_3 + P(0.6, 0.8)x_4 \\ &\quad + P(0.4, 0.6)x_5 \\ g_4 &= P(0.8, 1.0)x_4 + P(0.6, 0.8)x_5 \\ g_5 &= P(0.8, 1.0)x_5 \end{aligned} \right\} \dots\dots\dots(5)$$

In this paper, as the distribution of

$g_1, \dots, g_5$ is given from Fig.10, $x_1, \dots, x_5$ will be determined by using Eq. (5). But the accuracy of this solution is very much affected by the order of Eq.(5).

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