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Diffusion of a Dusty Gas*

(Part 1, The Velocity Distribution of Turbulent Jet)

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Turbulent jet of a dusty gas in a fully developed region is studied using following assumptions; an interaction force between the dust particles and the gas stream follows the Stokes law; a mean velocity of particulate phase is equal to that of gas stream, but fluctuating components of particulate phase are negligible as compared with those of gas stream; and the settling velocity of particles is small compared with the characteristic velocity of gas stream. A similar solution of velocity is obtained under the uniform concentration of dust particles. It is shown that eddy viscosity is reduced due to the presence of dust particles, and that rates of the decrement of axial velocity and of the spread of jet of dusty gas are small compared with those of gas jet. An applicable range of the present analysis is determined by the assumptions. Results of the analysis well explain experimental results by M. K. Laats.

1. Introduction

A dusty gas suspending small solid particles shows, unlike a pure gas, some interesting phenomena. M. K. Laats⁽¹⁾ carried out an experimental study on the circular jet and showed that in the dusty gas compared with a pure gas the spread of jet and the decrement of axial velocity are small and that the jet moves farther distance.

One explanation for this phenomenon is that the inertia of the dusty gas is increased because of the solid particles. If so, the particles have the same velocity as the jet stream and when the jet stream slows down as it moves away from an orifice, the velocity of particles also slows down. A force which decelerates the solid particles, however, is caused only by the relative motion of the particles to the jet stream. Then, if the particles move at the same velocity as that of the jet stream, this force never occurs nor the particles decelerate. In this way P. R. Owen⁽²⁾ pointed out that from the view point of the inertia increase alone it is impossible to explain this phenomenon, and showed that when the particles are small enough to follow turbulent fluctuation of the gas stream and there are a concentration distribution of the particles and a mean flow rate of the gas stream, a lag of the velocity of particles occurs, and therefrom he suggested a reduction of eddy viscosity of the dusty gas.

A relaxation time of the particles, however, is not so small as Owen assumed. And further, it is controversial that he

derives the velocity lag of particles relative to the gas stream, assuming that the particles should follow the gas stream.

In this paper it is assumed that the solid particles do not follow turbulent fluctuation of the gas stream by considering coarse particles of which relaxation time is long enough compared with that of energy-containing eddy of turbulence. And it is shown that an analysis using this assumption can explain the experimental results by M. K. Laats.

2. Equations of motion

The jet issuing from a circular orifice is assumed to be an axisymmetric turbulent flow. When the velocity of particle relative to the gas stream is small, the Stokes interaction force can be assumed to occur between particulate phase and gas flow. Thus, for a steady incompressible flow of the gas we obtain the following equations of motion and continuity,

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = - \frac{\partial p}{\partial x} + \rho \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right) - Kn(u - u_p) \quad \dots\dots\dots (1)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} - \frac{w^2}{r} \right) = - \frac{\partial p}{\partial r} + \rho \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (rv) \right) \right) - Kn(v - v_p) \quad \dots\dots\dots (2)$$

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (vr) = 0 \quad \dots\dots\dots (3)$$

where x is the longitudinal co-ordinate of jet, r is the radial, u , v and w are the instantaneous velocity components of gas

Fig. 1 Description of symbols.

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stream in the x , r and θ directions respectively, n is the number density of particulate phase, p is the pressure, ρ and ν are the density and the kinematic viscosity of the gas, and the suffix p refers to the particulate phase (see Fig.1). On the other hand, the equations of motion and continuity for the particulate phase are

$$mn\left(u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial r}\right) = Kn(u - u_p) \dots (4)$$

$$mn\left(u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial r} - \frac{w_p^2}{r}\right) = Kn(v - v_p) \dots (5)$$

$$\frac{\partial}{\partial x}(nu_p) + \frac{1}{r} \frac{\partial}{\partial r}(nv_p r) = 0 \dots (6)$$

where m is the mass of a particle, K is a

constant equal to $K=3\pi\rho\nu d$ according to the Stokes law, provided that the particulate phase consists of spheres of uniform size of d . In Eqs. (4) and (5), pressure of particulate phase is omitted because of its smallness compared with that of gas.⁽³⁾ Assuming the settling velocity of the particles to be small enough compared with the characteristic velocity of the jet, the action of gravity is also negligible. Under these assumptions the axisymmetric flow is brought about. It is seen from Eqs. (1), (2), (4) and (5) that the Stokes interaction force of the right hand side is negative for the gas and positive for the particulate phase.

Substituting Eqs. (4) and (5) into Eqs. (1) and (2), we obtain

$$\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r}\right) + mn\left(u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial r}\right) = -\frac{\partial p}{\partial x} + \rho\nu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right)\right\} \dots (7)$$

$$\rho\left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} - \frac{w^2}{r}\right) + mn\left(u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial r} - \frac{w_p^2}{r}\right) = -\frac{\partial p}{\partial r} + \rho\nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial r}\left\{\frac{1}{r} \frac{\partial}{\partial r}(rv)\right\}\right] \dots (8)$$

In turbulent flow the instantaneous quantities of flow can be separated into the time average component and the fluctuating one, that is,

$$u = U + u', \quad v = V + v', \quad w = W + w', \quad p = P + p', \quad n = N + n' \dots (9)$$

where the capital letters refer to the time average and the small letters with a dash to the turbulent fluctuation. Now we assume that the average velocity of the particulate phase is equal to that of the gas, namely,

$$U_p = U, \quad V_p = V, \quad W_p = W \dots (10)$$

Substituting Eqs. (9) and (10) into Eqs. (7) and (8), and taking a time average, we obtain

$$\left(\rho + mN\right)\left(U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r}\right) + \rho\left\{\frac{\partial(u'^2)}{\partial x} + \frac{1}{r} \frac{\partial(u'v'r)}{\partial r}\right\} + mN\left(u_p' \frac{\partial u_p'}{\partial x} + v_p' \frac{\partial u_p'}{\partial r}\right) + m\left(n'u_p' \frac{\partial u_p'}{\partial x} + n'v_p' \frac{\partial u_p'}{\partial r}\right) = -\frac{\partial P}{\partial x} + \rho\nu\left\{\frac{\partial^2 U}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial U}{\partial r}\right)\right\} \dots (11)$$

$$\left(\rho + mN\right)\left(U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial r}\right) + \rho\left\{\frac{\partial(u'v')}{\partial x} + \frac{1}{r} \frac{\partial(v'^2 r)}{\partial r} - \frac{w'^2}{r}\right\} + mN\left(u_p' \frac{\partial v_p'}{\partial x} + v_p' \frac{\partial v_p'}{\partial r} - \frac{w_p'^2}{r}\right) + m\left(n'u_p' \frac{\partial v_p'}{\partial x} + n'v_p' \frac{\partial v_p'}{\partial r} - \frac{n'w_p'^2}{r}\right) = -\frac{\partial P}{\partial r} + \rho\nu\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial}{\partial r}\left\{\frac{1}{r} \frac{\partial}{\partial r}(rV)\right\}\right] \dots (12)$$

From Eqs. (3) and (9) we obtain

$$\left.\begin{aligned} \frac{\partial U}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r}(Vr) &= 0 \\ \frac{\partial u'}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r}(v'r) &= 0 \end{aligned}\right\} \dots (13)$$

From Eqs. (6), (9) and (13) we obtain

$$U \frac{\partial N}{\partial x} + \frac{V}{r} \frac{\partial}{\partial r}(Nr) + \frac{\partial}{\partial x}(n'u_p') + \frac{1}{r} \frac{\partial}{\partial r}(n'v_p' r) = 0 \dots (14)$$

$$U \frac{\partial n'}{\partial x} + \frac{V}{r} \frac{\partial}{\partial r}(n'r) + \frac{\partial}{\partial x}(Nu_p') + \frac{1}{r} \frac{\partial}{\partial r}(Nv_p' r) = 0 \dots (15)$$

3. The motion of coarse particles

When the relaxation time of particle is sufficiently large compared with the characteristic time of energy-containing eddy, turbulent fluctuations of the particulate phase are negligible,

$$u_p' \approx 0, \quad v_p' \approx 0, \quad w_p' \approx 0 \dots (16)$$

The relaxation time of particle t^* is given as follows,

$$t^* = \frac{1}{18} \frac{\rho_p}{\rho} \frac{d^2}{\nu} \dots (17)$$

since the Stokes law is assumed. According to Laats' experiment t^* is calculated as 2.22×10^{-2} s with $d = 50 \mu\text{m}$, $\rho_p/\rho = 2 \times 10^3$ and $\nu = 1.5 \times 10^{-3} \text{ m}^2/\text{s}$. Since a longitudinal compo-

nent of turbulent velocity level of gas jet is approximately of the same order in any parts of any cross section, we assume this fluctuation velocity as the characteristic velocity of turbulence, which is also regarded as that of energy-containing eddy of turbulence, and we obtain $\sqrt{u'^2} \approx 0.02U_m(x)$. Using this velocity and the

kinematic viscosity of gas, the characteristic time of turbulent fluctuation of the gas stream is given as $\nu/\bar{u'^2}$. About $30D$ downstream from the orifice we obtain $\nu/\bar{u'^2} \approx 5.85 \times 10^4$ s, since $U_m \approx 0.2U_1^{(4)}$, where D is the orifice diameter and U_m is the axial velocity and U_1 is assumed to be equal to 40 m/s . This value is much smaller than that of t^* , then the assumption (16) seems to be satisfactory. However, it is noted that $\nu/\bar{u'^2}$ increases with an increase in distance from the orifice, since U_m decreases. Substituting Eq. (16) into Eqs. (11) (12) and (14), we obtain,

$$\left(1 + \frac{mN}{\rho}\right)\left(U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r}\right) + \frac{\partial(u'^2)}{\partial x} + \frac{1}{r} \frac{\partial(u'v'r)}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu\left\{\frac{\partial^2 U}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial U}{\partial r}\right)\right\} \dots (18)$$

$$\left(1 + \frac{mN}{\rho}\right)\left(U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial r}\right) + \frac{\partial(u'v')}{\partial x} + \frac{1}{r} \frac{\partial(v'^2 r)}{\partial r} - \frac{w'^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \nu\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial}{\partial r}\left\{\frac{1}{r} \frac{\partial}{\partial r}(rV)\right\}\right] \dots (19)$$

$$U \frac{\partial N}{\partial x} + V \frac{\partial N}{\partial r} = 0 \quad (20)$$

These equations can be solved in the same way as for the gas jet.⁽³⁾ Taking account of the significant terms alone using a boundary layer approximation and the continuity equation, Eq. (19) becomes,

$$\frac{1}{r} \frac{\partial(\bar{v}^2 r)}{\partial r} - \frac{\bar{w}^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \nu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) \quad (21)$$

When the Reynolds number is large enough to omit the viscous term, Eq. (21) is integrated as $P_0 \cong P + \rho \bar{v}^2$ provided that $\bar{v}^2 \cong \bar{w}^2$ over the cross-section of jet, where P_0 is the pressure in the undisturbed gas. Since $\partial P_0 / \partial x = 0$ we obtain

$$\frac{1}{\rho} \frac{\partial P}{\partial x} = -\frac{\partial \bar{v}^2}{\partial x} \quad (22)$$

Simplifying Eq. (18) in the same way and using Eq. (22), we obtain

$$\left(1 + \frac{mN}{\rho}\right) \left(U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r} \right) + \frac{\partial(\bar{w}^2 - \bar{v}^2)}{\partial x} + \frac{1}{r} \frac{\partial(\bar{w}^2 r)}{\partial r} = \nu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) \quad (23)$$

We seek a similar solution for Eq. (23). When the Reynolds number is large enough, the viscous term is negligible. Then, the motions of mean flow and of energy-containing eddy of turbulence are determined only by the boundary condition of the flow. Thus, if there is a similar solution, the velocity profile non-dimensionalized on u_0 becomes a function only of x/l_0 , where u_0 and l_0 are the scales of velocity and length respectively, and are functions only of x . Also as to the concentration we assume the similarity and put

$$\left. \begin{aligned} U &= u_0(x) f(\eta), \quad \bar{w}^2 = u_0^2(x) g_1(\eta) \\ \bar{v}^2 &= u_0^2(x) g_2(\eta), \quad \bar{w} \bar{v} = u_0^2(x) g_{12}(\eta) \\ N &= n_0(x) h(\eta), \quad \eta = r/l_0 \end{aligned} \right\} \quad (24)$$

Using Eqs. (13) and (24) Eq. (23) becomes as follows, where $(\quad)' = d/d\eta$

$$\begin{aligned} & u_0 \frac{du_0}{dx} \left\{ f^2 + 2(g_1 - g_2) - \frac{f'}{\eta} \int \eta f d\eta \right\} \\ & - \frac{u_0^2}{l_0} \frac{dl_0}{dx} \left\{ \eta f f' - \frac{f'}{\eta} \int \eta^2 f' d\eta \right. \\ & \left. + \eta(g_1 - g_2) \right\} + \frac{u_0^2}{l_0} \eta(g_{12}' \eta + g_{12}) \\ & + n_0 u_0 \frac{du_0}{dx} \frac{m}{\rho} \left(f^2 h - \frac{f'}{\eta} \int \eta f d\eta \right) \\ & - \frac{n_0 u_0^2}{l_0} \frac{dl_0}{dx} \frac{m}{\rho} h \left(\eta f f' - \frac{f'}{\eta} \int \eta^2 f' d\eta \right) \\ & = \frac{u_0}{l_0^2} \nu (\eta f'' + f') \quad (25) \end{aligned}$$

Therefore, when f , g_1 , g_2 , g_{12} and h have similar solutions, it is necessary that $l_0 \propto (x - x_0)$, $u_0 \propto (x - x_0)^{-1}$ and $n_0 = \text{constant}$.

Now the origin of the co-ordinate is shifted to the virtual origin $(x_0, 0, 0)$ and u_0 , l_0 and n_0 are newly set as follows,

$$u_0 = U_1 D/x, \quad l_0 = x, \quad n_0 = N_0, \quad \eta = r/x \quad (26)$$

where N_0 is the number density of particles at the orifice exit. Using Eq. (26), Eq. (20) becomes

$$\frac{U_1 D}{x^2} \frac{h'}{\eta} \int \eta f d\eta = 0$$

Thus, we obtain $h(\eta) = \text{constant}$, hence

$$N = N_0 = \text{constant} \quad (27)$$

Then Eq. (23) becomes as follows using Eqs. (24), (26) and (27),

$$\begin{aligned} & -\left(1 + \frac{mN_0}{\rho}\right) \left(\eta f^2 + \frac{df}{d\eta} \int_0^\eta f \eta d\eta \right) \\ & - \frac{d}{d\eta} (\eta^2 (g_1 - g_2)) + (g_{12} + \eta \frac{dg_{12}}{d\eta}) \\ & = \frac{\nu}{U_1 D} \frac{d}{d\eta} \left(\eta \frac{df}{d\eta} \right) \quad (28) \end{aligned}$$

In Eq. (28) the viscous term is also similar, which is due to the local Reynolds number $u_0 l_0 / \nu = U_1 D / \nu$ being independent of x .⁽³⁾ When the normal Reynolds stresses $(g_1 - g_2)$ are omitted and the shear Reynolds stress $\bar{w} \bar{v}$ is assumed as follows using eddy viscosity ν_T ,

$$\bar{w} \bar{v} = \nu_T \frac{\partial U}{\partial r}, \quad \nu_T = \text{constant} \quad (29)$$

then we obtain $g_{12} = (\nu_T / U_1 D) (df/d\eta)$, and Eq. (28) becomes

$$\begin{aligned} & \left(1 + \frac{mN_0}{\rho}\right) \left(\eta f^2 + \frac{df}{d\eta} \int_0^\eta f \eta d\eta \right) \\ & = \frac{1}{U_1 D} (\nu_T - \nu) \frac{d}{d\eta} \left(\eta \frac{df}{d\eta} \right) \quad (30) \end{aligned}$$

Equation (30) is solved using the boundary condition, $f \rightarrow 0$ at $\eta \rightarrow \infty$ as

$$f(\eta) = \frac{A}{(1 + \eta^2/8\alpha)^2} \quad (31)$$

where the molecular viscous force is omitted as negligibly small, and

$$\left. \begin{aligned} A &= \frac{3}{32} \frac{U_1 D}{\nu_T} \left(1 + \frac{mN_0}{\rho}\right) \\ \alpha &= \frac{32}{3} \left\{ \frac{\nu_T}{U_1 D} \frac{1}{(1 + mN_0/\rho)} \right\}^2 \end{aligned} \right\} \quad (32)$$

It is seen from Eq. (32) that the eddy viscosity decreases by $(1 + mN_0/\rho)^{-1}$ times owing to the presence of solid particles. It means that the velocity field becomes narrower with an increasing N_0 . Substituting Eq. (31) and $U_m = U(x, 0)$ for U , we obtain,

$$U/U_m = 1 / \left\{ 1 + \frac{1}{8\alpha} \left(\frac{r}{x} \right)^2 \right\}^2 \quad (33)$$

The radius $r_{0.5U_m}$ where the local velocity is equal to one half of the axial velocity is given by Eq. (33) as,

$$r_{0.5U_m} = x \sqrt{8(\sqrt{2} - 1)\alpha} \quad (34)$$

From Eqs. (33) and (34), we obtain

$$U/U_m = 1 / \left\{ 1 + (\sqrt{2} - 1) \left(\frac{r}{r_{0.5U_m}} \right)^2 \right\}^2 \quad (35)$$

Thus, when the velocity profile is non-dimensionalized on U_m and $r_{0.5U_m}$, the profile becomes independent of the concentration. Indeed, Laats indicated that the experimental results are represented independently of the concentration of particles when non-dimensionalized on U_m and $r_{0.5U_m}$ and

Fig. 2 Velocity profile of jet.

that they lie on the Schlichting profile, $U/U_m = (1 - (r/R)^{1.5})^2$, in the $2/3$ of non-dimensional radius of the jet (see Fig. 2), where R is the jet radius. The rate of decrement of the axial velocity is given by Eq. (31) as

$$U_m/U_1 = \frac{3}{16} \frac{U_1 D}{\nu_T} \left(1 + \frac{m N_0}{\rho}\right) \frac{r_0}{x}, \quad r_0 = D/2 \quad (36)$$

which shows the rate decreasing with an increase in N_0 . The rate of spread of the jet is given by Eq. (34) as

$$r_{0.5} U_m / r_0 = \sqrt{\frac{256(\sqrt{2}-1)}{3}} \frac{\nu_T}{U_1 D} \times \frac{1}{(1 + m N_0 / \rho)} \frac{x}{r_0} \quad (37)$$

which also shows the rate of spread decreasing with an increase in N_0 .

4. Discussion

We discuss the above-mentioned analysis comparing with Laats' experimental results. Figure 2 shows a comparison of the experimental values, which are obtained by varying N_0 , U_1 and D , with Eq. (35). It is seen that Eq. (35) fits the experimental values in the most part. The decrement of the axial velocity is shown in Fig. 3, where the constant in Eq. (36), $R_T = U_1 D / \nu_T$, is determined such that the experimental values of the pure gas jet fit the case of $N_0 = 0$ of Eq. (36). The experimental results seem to be explained by the present analysis on the whole. With an increasing N_0 the deviations are seen, however, for small values of x/r_0 . According to the present analysis, this means that the larger N_0 is, the longer the mixing layer

Fig. 3 Decrement of axial velocity of jet.

Fig. 4 Spread of jet radius.

of the jet is; namely, the potential core remains longer than in the case of pure gas jet. Thus, it is said that the larger N_0 is, the farther the jet of dusty gas advances than that of the pure gas. The rate of spread of the jet is shown in Fig. 4, where the constant in Eq. (37) is also determined from the experimental values of the pure gas jet. In this case the degree of agreement between experiment and analysis is not so good as in the case of the axial velocity. At the present stage, however, it is difficult to say more, since the experimental values in the range of large values of x/r_0 are not given.

Now we think of a range of applicability of this analysis. The assumptions we use are; the Stokes interaction force works; the relaxation time of particles is longer than the characteristic time of turbulence; and the settling velocity of particles is larger than the characteristic velocity of the flow. Since the axial velocity and the absolute value of turbulent intensity in the flow of jet decrease in inverse proportion to the distance from the orifice, these assumptions do not hold over all the fields of the jet. The first assumption is expressed by $\sqrt{u'^2}/\nu < 1$ and Eq. (36) as

$$\frac{U_1(1 + m N_0 / \rho)}{x/r_0} d < \frac{800}{3 R_T} \nu \quad (38)$$

where the longitudinal component of turbulent intensity $\sqrt{u'^2}$ is used as the relative velocity and is assumed to be equal to $0.02 U_m$. From the second assumption, $t^* > \nu/u'^2$, we obtain

$$\frac{U_1(1 + m N_0 / \rho)}{x/r_0} d > \frac{800 \sqrt{2} \nu}{R_T \sqrt{\rho_p / \rho}} \quad (39)$$

When $0.5 U_m$ is used as the characteristic velocity of the flow, the third assumption, $t^* g < 0.5 U_m$ is expressed as

$$\frac{U_1(1 + m N_0 / \rho)}{x/r_0} \frac{1}{d^2} > \frac{16}{27 R_T} \frac{g (\rho_p / \rho)}{\nu} \quad (40)$$

where g is the acceleration of gravity. These assumptions are illustrated in Fig. 5 with density ratio ρ_p / ρ as a parameter. It is found out from Fig. 5 that the maxi-

Fig. 5 Applicable range for present analysis.

imum particle diameter is $(10/\sqrt[3]{20})(\nu^2/g)^{1/3}$ and the minimum value of $U_1(1+mN_0/\rho)/(x/r_0)$ is $(80\sqrt[3]{20}/3Rr)\sqrt[3]{g\nu}$, regardless of the density ratio. In Fig. 5 we put ρ_s/ρ equal to 2×10^3 according to Laats' experimental condition and Rr equal to 55.9 from the result of Fig. 3. The applicable range of this analysis is the area surrounded by Eqs. (38)(39) and (40). Laats' experimental condition is also drawn presumably in this figure for reference. A discrepancy between experiment and analysis is seen for large value of $U_1(1+mN_0/\rho)/(x/r_0)$. It may be caused by the following; x/r_0 is taken as 10 and the Stokes law is limited up to the unit for $\sqrt{u'^2}d/\nu$. If x/r_0 is taken from the large value and the limit of applicability of the Stokes law is slightly extended, the discrepancy will be decreased.

As to the concentration distribution of particles under the uniform distribution given by Eq. (27) a similar solution of velocity distribution is obtained, which is in accord with the experimental values. Indeed, it is thought; when the jet velocity reduces with an increase in distance from the orifice the concentration may increase, but at the same time the spread of the jet increases, thus the uniform concentration seems to be applicable without serious error.

5. Conclusions

Turbulent jet of a dusty gas in a fully developed region is studied, and it is found out that compared with a pure gas jet the decrement of the axial velocity and the spread of the jet are small and then the jet of dusty gas advances farther downstream. These results well explain Laats' experimental work, and in addition the applicable range of this analysis is given, which correlates to the properties of dusty particle, to the condition at the orifice exit and to the length of jet stream. As to the concentration distribution of particles, it becomes constant from the assumptions and the approximations used in this analysis, but we shall study its validity experimentally.

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