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Oscillation Phenomena of Pseudo-Shock Waves*

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In the previous paper, we presented the diffusion model of pseudo-shock waves, by which the time-mean static pressure distributions in pseudo-shocks could be explained. In the actual flow, however, the pseudo-shock wave is not steady, but oscillates about a mean position and causes the fluctuations of the local static pressures.

In this paper the oscillation phenomena of pseudo-shocks in a straight pipe were experimentally investigated and the causes and the frequencies of oscillations were discussed. As the results, it was found that the oscillation of the pseudo-shock became stronger with the increasing Mach number and the maximum wall static pressure fluctuation induced by the oscillation amounted to about 60 per cent of the difference of static pressures before and behind the shock. Moreover, it was showed that this oscillating phenomena could be explained by the interaction of the shock with the small disturbances existing in the supersonic flow upstream the shock.

1. Introduction

In the previous paper⁽¹⁾, the authors presented the diffusion model of pseudo-shock waves, by which the pseudo-shock was explained as the dissipative phenomenon of two flows, that is, the high speed flow in the central region and low speed flow near the wall of a duct. The length and time-mean static pressure distribution on the duct wall in the pseudo-shock predicted by the diffusion model agree well with the experimental results. In the actual flow, however, the pseudo-shock is not steady, but oscillates about a mean position and causes the flow fluctuations.

In regard to the oscillation phenomena of shock waves, the oscillatory instabilities⁽²⁾⁽³⁾ of shock waves over aerofoils in transonic flow and the intake buzz of supersonic inlet diffusers⁽⁴⁾ are well known and many studies have been reported so far on such flow instabilities. On the other hand, the oscillation of shock waves in a duct has not been reported yet, though it is important in engineering.

In this paper the oscillation phenomena of the pseudo-shocks in a straight pipe

and the resulting flow fluctuations were experimentally investigated, and the causes and frequencies of the oscillations were discussed.

Nomenclature

a	: sound speed
c_p	: specific heat at constant pressure
D	: equivalent diameter of duct (=60mm)
f	: frequency
M_1	: flow Mach number just upstream pseudo-shock
P_0	: pressure at plenum chamber
p	: static pressure
Re_D	: Reynolds number based on equivalent diameter of duct
R_{ij}	: correlation function between ξ_i and ξ_j
r_{ij}	: coefficient of correlation function R_{ij}
S	: spectral density function
s	: smoothed spectral density function
T	: temperature
t	: time
w	: dimensionless velocity (Crocco Number) [$\equiv u/\sqrt{2c_p T}$]
X	: distance along duct
x	: dimensionless distance ($\equiv X/D$)
ξ	: deviation ($\equiv x - \bar{x}$)
γ	: ratio of specific heats

Subscript

$1, 2, \dots$	: the first shock, the second shock, ...
$1, 2$	: right before and behind pseudo-shock
o, i, e	: plenum chamber, duct entrance, duct exit, respectively
H	: Helmholtz resonance
o	: pipe resonance

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Fig.1 Schematic diagram of experimental arrangement

2. Experimental procedures

The experimental apparatus, the working gas and the working conditions in this experiment are all the same as those in the previous paper.⁽¹⁾ The system is shown schematically in Fig.1. The high pressure dry air in a reservoir enters the plenum chamber and then it is accelerated by a laval nozzle to supersonic speed and led to the test section, which is the straight duct with a cross section of 60mm square and a length of 830mm. Then the flow is decelerated to subsonic speed by a pseudo-shock and discharged into the atmosphere through a silencer.

The oscillation of the pseudo-shock was observed by high-speed schlieren photography. The frame rate was 2000~8000FPS. The fluctuation of the wall static pressure induced by the oscillation of the pseudo-shock was measured at seven measuring points on the wall as shown in Fig. 1 by semiconductor pressure gauges with sensitivity of 67.6 mV/kg/cm^2 and natural frequency of about 20 KHz.

3. Experimental results

3.1 Oscillation of pseudo-shock waves

Figure 2 shows a series of typical high-speed schlieren photographs of the oscillation phenomena of the pseudo-shock. The flow Mach number M_1 just before the pseudo-shock is 2.79, and these pictures are printed every 24 frames (time interval of 3.15ms) of the original high-speed film. It is evident from Fig. 2 (a) that the pseudo-shock is composed of many oblique shocks and these shocks form the first X-type shock of the pseudo-shock, the second shock and so on. Fig.2 (a) ~ (e) shows that the positions of these shocks vary with time. Let $X_1, X_2, \dots$ be the distances from the reference point shown as the arrow in the figure to the first shock (the intersection point of X-shape), the second shock, ..., then the motion of the pseudo-shock is represented by plotting the dimensionless distance X/D against time t .

Reference point (450mm from the duct inlet)
3.15ms interval, $M_1=2.79$
 $Re_D=2.21 \times 10^6$

Fig.2 Optical observations of the oscillation of a pseudo-shock wave

Figure 3 shows the oscillation of each shock thus obtained. In the case of $M_1 = 1.44$ [Fig.3(a)], the shape of each shock composing the pseudo-shock is nearly normal, and the position fluctuates with time but the amplitude is comparatively small. In Fig.3 (b), where $M_1 = 1.93$, the pseudo-shock is composed of some λ -shape shocks, and their amplitudes are larger than those in Fig.3 (a). In Fig.3 (c), where M_1 is 2.79 which is the same as that of Fig.2, the pseudo-shock is composed of X-shape shocks, and their amplitudes are larger than the tube diameter. Also the oscillation seems regular considerably. In every case of Fig.3 (a)~(c), the amplitude of the downstream shock is larger than that of the first shock.

3.2 Correlation of oscillation

In order to clarify the degree of correlation between two shocks composing a pseudo-shock, the deviation of the position of i -th shock x_i from its time-mean value $\bar{x}_i$

$$\xi_i(t) = x_i(t) - \bar{x}_i \quad (i=1,2,\dots) \quad (1)$$

was obtained and the correlation function $R_{ij}(\tau)$ was calculated by the following equation.

$$R_{ij}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \xi_i(t) \xi_j(t+\tau) dt \\ \approx \frac{1}{N} \sum_{n=1}^N \xi_i(n\Delta t) \xi_j(n\Delta t + k\Delta t), \\ (k=0,1,2,\dots) \quad (2)$$

where Δt is time interval between two successive frames of the high speed film, and N is the number of data used in the calculation, being 100~250. Equation(2) gives an autocorrelation function in the case of

Fig.3 Oscillations of pseudo-shock waves.

$i = j$, and a cross correlation function in the case of $i \neq j$. The coefficient of correlation $\gamma_{ij}(\tau)$ is obtained from $R_{ij}(\tau)$ divided by the product of the standard deviations of x_i and x_j . That is,

$$\gamma_{ij}(\tau) = \frac{R_{ij}(\tau)}{\sqrt{R_{ii}(0)R_{jj}(0)}} \quad \dots (3)$$

Figure 4 shows the calculated results of $\gamma_{ij}(\tau)$ based on the oscillation of the first shock ($i=1$), and Fig.4(a), (b) and (c) correspond to Fig.3 (a), (b) and (c), respectively. Although the accuracy of each γ_{ij} is high near $\tau = 0$, the larger $|\tau|$ is, the larger the error is, because the number N of the data used in the calculation is finite. That is why $|\gamma_{ij}| > 1$ where $|\tau|$ is larger in Fig.4(a). In the case that N is infinity, $|\gamma_{ij}|$ should not be larger than unity theoretically.

The coefficient of autocorrelation $\gamma_{11}(\tau)$ of the first shock in Fig.4 shows that, in every case of M , the oscillation of the shock is composed of the component of regular periodical oscillation and that of random oscillation of which frequency ranges widely.⁽⁵⁾ The high coefficient of correlation with a period of $\tau = 10 \sim 20$ ms

shows the existence of the component of the periodical oscillation, and the existence of random oscillation is evident from the fast decrease of $\gamma_{11}(\tau)$ with the increase of τ from $\tau = 0$. The curve of γ_{11} in Fig.4 (a) and (c) is comparatively smooth and shows the periodic component clearly. On the other hand the curve of γ_{11} in Fig.4(b) is not so smooth as those of Figs.(a) and (c), the periodicity is not so clear, and γ_{11} decreases rapidly near $\tau = 0$. This means that the first shock at $M_1 = 1.93$ [in this case the shock has a λ -shape] oscillates at random more than in the cases of $M_1 = 1.44$ [the shape of the shock is nearly normal] and 2.79 [X-shape shock]. From the curves of the coefficient of cross correlation $\gamma_{12} \sim \gamma_{17}$ of the second and the following shocks to the first shock, the correlation at smaller M_1 [Fig. (a)] is smaller as the shock locates downstream the first shock. However at larger M_1 [Fig.(c)] the maximum value of γ_{ij} ($j = 2 \sim 6$) is nearly

equal to 1.0 and so it can be said that each shock oscillates almost uniformly.

The value of γ_{11} is maximum and equal to 1.0 at $\tau = 0$. The maximum value of γ_{ij} is delayed to the positive time (right side of $\tau = 0$) as j becomes larger. This means that the oscillation of the first shock has influence on the oscillation of one downstream shock after another. Considering also the fact that in Fig. 3 the amplitude of oscillation of the downstream shock is larger than that of the first shock, it may be said that the oscillation of the first shock is amplified and propagated through the following shocks.

3.3 Spectral density of oscillation

The spectral density function $S(f)$ may be obtained from the Fourier transform of the double of the coefficient of autocorrelation. That is,

$$S_i(f) = \int_{-\infty}^{\infty} 2\gamma_{ii}(\tau) e^{-i2\pi f\tau} d\tau \quad \dots (4)$$

As γ_{ii} is an even function,

$$S_i(f) = 4 \int_0^{\infty} \gamma_{ii}(\tau) \cos(2\pi f\tau) d\tau \quad \dots (5)$$

and it can be calculated approximately by the following equation

$$S_i(f) \approx 4 \sum_{k=0}^N \gamma_{ii}(k\Delta t) \cos(2\pi f_k k\Delta t) \Delta t. \quad (6)$$

(a) $M_1=1.44$, $P_0=2.12 \text{ ata}$, $Re_0=1.93 \times 10^6$

(b) $M_1=1.93$, $P_0=2.68 \text{ ata}$, $Re_0=2.04 \times 10^6$

(c) $M_1=2.79$, $P_0=4.21 \text{ ata}$, $Re_0=2.21 \times 10^6$

Fig.4 Coefficients of correlation of pseudo-shock oscillations

where the frequency f_n is given as follows, taking a proper number M smaller than the number of data N used in the calculation.

$$f_n = \frac{n}{2M\Delta t} \quad (n = 0, 1, 2, \dots) \quad (7)$$

In order to correct the errors due to number of data being finite, S_i is transformed into s by the following equation⁽⁶⁾

$$\left. \begin{aligned} S_i(f_0) &= \frac{1}{2} \{S_i(f_0) + S_i(f_1)\} \\ S_i(f_n) &= \frac{1}{4} \{S_i(f_{n-1}) + 2S_i(f_n) + S_i(f_{n+1})\} \end{aligned} \right\} \quad (8)$$

(a) $M=1.44$, $P_0=2.12 \text{ ata}$, $Re_0=1.93 \times 10^6$

(b) $M=1.93$, $P_0=2.68 \text{ ata}$, $Re_0=2.04 \times 10^6$

(c) $M=2.79$, $P_0=4.21 \text{ ata}$, $Re_0=2.21 \times 10^6$

Fig.5 Spectral densities of pseudo-shock oscillations

An example of the spectral density function $S_x(f)$ of each shock thus obtained is shown in Fig. 5, where (a), (b) and (c) correspond to (a), (b) and (c) in Figs. 3 and 4, respectively. The spectral density function represents the ratio of a component of oscillation with a certain frequency to the total oscillation, and it is a dimensionless quantity.

It is evident from Fig. 5 that in every case of M_1 each shock has a large component of oscillation in the range of low frequencies. When $M_1=1.44$ [Fig. (a)] there are two prominent peaks in the range lower than 100Hz and between 100 and 200Hz, and the peak between 100 and 200Hz is more prominent as the shock comes near the first shock. In the case of $M_1=1.93$ [Fig. (b)] the seventh shock has also two prominent peaks in the range lower than 100Hz and between 100 and 200Hz, and the peak in the range lower than 100Hz is more prominent as the shock comes near the first shock. In Fig. (c), where $M_1=2.79$, each shock does not have a prominent component of oscillation with frequency larger than 100Hz and a peak occurs at about 40Hz. On the other hand the component of oscillation with a frequency higher than 400Hz hardly exists in the cases of $M_1=1.44$ and 2.79, but in the case of $M_1=1.93$ it exists in a wide range. This agrees with the fact that the oscillation at $M_1=1.93$ in Fig. 4 is more random than those at $M_1=1.44$ and 2.79.

3.4 Fluctuation of wall static pressure

Figure 6 shows an example of the fluctuation of pressure on the wall. The above figure is a schlieren photograph, where the head of the pseudo-shock is observed. The pulse mark on the output of the photoelectric unit of the oscillogram denotes the moment of taking the above photograph. The measured point I of the wall static pressure is at the position of the foot of the first shock and the point II is $5D$ downstream from the point I. The point II is assumed to be at the position near the end of the pseudo-shock. From the oscillogram it is observed that there are strong pressure fluctuations in the pseudo-shock and in particular, the fluctuation at the head of the pseudo-shock amounts to about 0.2Kg/cm^2 . This large fluctuation of pressure is caused by the oscillation of the first shock.

Figure 7 shows sketches of the oscillograms showing the fluctuation of wall static pressure, and sketches of schlieren photographs of the pseudo-shock are also shown in each figure above. The measured points are denoted as the signals 1, 2, a, ..., e. It is known from these figures how the static pressure in the pseudo-shock fluctuates.

As is evident from the curve p_1 , there is a small static pressure fluctuation upstream the pseudo-shock. This means that the supersonic flow upstream the pseudo-shock is not necessarily uniform and, as described later, this is supposed to be the

Fig. 6 An example of wall pressure fluctuations

cause of the oscillation of the pseudo-shock. Next, consider the pressure in the pseudo-shock. In fig. 7 (a), where M_1 is small and the pseudo-shock is nearly normal, the fluctuation of static pressure p_a near the first shock is the largest of all. In the case of λ -shape pseudo-shock in Fig. 7(b), in addition to p_a , p_b also fluctuates prominently. In the χ -shape pseudo-shock [Fig. 7 (d)], the degrees of fluctuations of p_a , p_b and p_c are nearly the same. This phenomenon is explained as follows. As was clarified in the previous paper⁽¹⁾ the static pressure distribution on the duct wall in the pseudo-shock is steep near the first shock at lower M_1 and it becomes more gradual as M_1 increases. Therefore, at low M_1 the fluctuation of static pressure is prominent near the first shock, and it also becomes prominent downstream the first shock as M_1 increases.

In the case of $M_1=1.93$ [Fig. 7 (c)] the fluctuations of p_a and p_b are extremely large. The same phenomenon was observed at $M_1=2.01$. This may be something like resonance, but at present the detail is not clear.

Figure 8 shows the experimental results on the maximum amplitudes δp_1 and δp_2 of the static pressure fluctuations just before and behind the pseudo-shock, the maximum amplitude in the pseudo-shock δp_{max} , and the difference of the time-mean values of p_1 and p_2 . From this the following points are obvious: (1) δp_1 decreases slightly with an increasing M_1 ; (2) δp_2 is nearly equal to δp_1 at lower M_1 , and δp_2 increases with M_1 except in the region of resonance near $M_1=2.0$; (3) δp_{max} is about 0.2Kg/cm^2 at $M=1.5$, and, increasing with M_1 , it becomes nearly constant in the range of $M=2.0\sim 2.79$; and (4) the maximum value of δp_{max} is 0.5Kg/cm^2 at $M_1=2.01$, and the value amounts to 60 per cent of the static pressure difference $\bar{p}_2 - \bar{p}_1$. These facts mean that the duct containing

(a) $M = 1.44$, $P_0 = 2.12 \text{ ata}$, $Re_D = 1.93 \times 10^6$ (b) $M_1 = 1.71$, $P_0 = 2.06 \text{ ata}$, $Re_D = 1.72 \times 10^6$ (c) $M = 1.93$, $P_0 = 2.68 \text{ ata}$, $Re_D = 2.04 \times 10^6$ (d) $M_1 = 2.60$, $P_0 = 3.75 \text{ ata}$, $Re_D = 1.97 \times 10^6$

Fig.7 Wall pressure fluctuations in regions of pseudo-shock waves

the pseudo-shock receives locally a large fluctuating force caused by the fluctuation of pressure. This phenomenon cannot be overlooked in design of ducts.

4. discussion

Fig.8 Amplitudes of wall static pressure fluctuations

It is self-evident that when the pseudo-shock oscillates, the local static pressure in the pseudo-shock fluctuates. Therefore in this section we do not discuss the static pressure fluctuation in the pseudo-shock, but we examine the relation between the static pressure fluctuation upstream the shock and the oscillation of the pseudo-shock and its frequency. In the following, the pseudo-shock is regarded as a plane normal shock and the flow is assumed to be one-dimensional. Further, for simplicity, the heat transfer of the duct is neglected and the friction coefficient λ is assumed to be constant along the duct.

Let $w \equiv u/\sqrt{2c_p T_0}$ be a dimensionless velocity, then the following equations hold for the flow between the duct inlet i and a point 1 just before the shock.

momentum equation:

$$\frac{r+1}{r} \ln \frac{w}{w_i} + \frac{r-1}{2r} \left(\frac{1}{w_i^2} - \frac{1}{w^2} \right) = -\lambda (x - x_i) \dots (9)$$

equation of continuity:

$$\frac{pw}{1-w^2} = \frac{p_i w_i}{1-w_i^2} \dots (10)$$

Across the shock the following two equations hold.

Prandtl's equation:

$$w_1 w_2 = \frac{r-1}{r+1} \dots (11)$$

Equation of continuity:

$$\frac{p_2 w_2}{1-w_2^2} = \frac{p_1 w_1}{1-w_1^2} \dots (12)$$

Further, for the flow between a point 2 just behind the shock and the duct exit

Momentum equation:

$$\frac{r+1}{r} \ln \frac{w}{w_2} + \frac{r-1}{2r} \left(\frac{1}{w_2^2} - \frac{1}{w^2} \right) = -\lambda (x - x_2) \dots (13)$$

equation of continuity:

$$\frac{pw}{1-w^2} = \frac{p_2 w_2}{1-w_2^2} \dots (14)$$

Equations (9) ~ (14) will determine the flow, provided that the values p_i and w_i at the duct inlet and the pressure p_E at the duct exit are given.

Figure 9 shows an example of the calculated pressure distribution where p_i and w_i are fixed and p_E varies. It is obvious from this figure that, when p_E decreases,

Fig.9 Pressure distributions along the duct

the shock moves downstream and the pressure ratio across the shock decreases.

(a) p - x

(b) t - x

Thick solid line: Shock wave
Thin solid line: Pressure wave
Broken line: Entropy discontinuity

Fig.10 Interaction between a shock wave and a small perturbation in the supersonic duct flow

Based on the present experimental results, consider the case where p_E is kept constant and the pressure upstream the shock fluctuates slightly. Figures 10 (a) and (b) show $p-x$ and $t-x$ diagrams in this case, respectively. When the pressure p_1 just upstream the shock changes to $p_1 + \delta p_1$, the shock varies its strength and starts to move as shown in Fig.10 (b). As the shock strength varies, a pressure wave and a discontinuity surface of entropy propagate downward. The ratio of the strength of the pressure wave δp_2 to δp_1 is given by the following equation (8).

$$\frac{\delta p_2}{\delta p_1} = \frac{M_1 + 1}{M_2 + 1} \frac{2(r-1)M_1 M_2^2 (M_1^2 - 1) - (M_1 + 1)(r-1)M_1^2 + 2}{2(r-1)M_1^2 (M_1^2 - 1) - (M_1 + 1)(r-1)M_1^2 + 2} \quad (15)$$

This pressure wave is reflected at the duct exit and interacts with the shock again. After some period of interaction the shock will be stabilized at a position where the compatibility of pressure holds.

Let δx be the displacement of the shock as shown in Fig.10(a), and consider the relation between δx and δp_1 . Assuming that the flow rate is kept constant, Eq. (10) gives the relation $p_1 w_1 / (1 - w_1^2) = \text{constant}$. Differentiating this relation yields

$$\frac{\delta w_1}{w_1} = - \frac{1 - w_1^2}{1 + w_1^2} \frac{\delta p_1}{p_1} \quad \dots (16)$$

Similarly from Eq. (11), we obtain

$$\frac{\delta w_2}{w_2} = - \frac{\delta w_1}{w_1} \quad \dots (17)$$

The flow between the shock and the duct exit is the same before and after the transient period and so Eqs.(13) and (14) hold. Therefore the displacement of the shock δx due to the variation of velocity δw_2 just behind the shock is represented as follows by differentiating Eq. (13).

$$\delta x = - \frac{2}{\lambda} \left\{ 1 - \frac{r-1}{2r} \left(1 + \frac{1}{w_1^2} \right) \right\} \frac{\delta w_1}{w_1} \quad \dots (18)$$

Substitution of Eqs. (16) and (17) into Eq. (18) yields

$$\delta x = - \frac{2}{\lambda} \left\{ 1 - \frac{r-1}{2r} \left(1 + \frac{1}{w_1^2} \right) \right\} \frac{1 - w_1^2}{1 + w_1^2} \frac{\delta p_1}{p_1} \quad \dots (19)$$

The above equation gives the displacement of the shock when the pressure upstream the shock varies a little in a one-dimensional adiabatic flow with friction and a constant flow rate in ducts.

Figure 11 shows the comparison between the experimental pressure ratio $\delta p_2 / \delta p_1$ obtained from Fig. 8 and the theoretical value of Eq. (15). The theory seems to explain the tendency of the experimental results except in the region of resonance near $M_1 = 2.0$.

Fig.11 Ratios of pressure fluctuations across pseudo-shock waves

The maximum amplitude δx of each shock composing the pseudo-shock is plotted against M_1 in Fig. 12, where the solid line was calculated from Eq. (19) with $\lambda = 0.012$ and $\delta p_1 / p_1 = 0.01$. It can be said that the tendency of the experimental results agrees with Eq. (19).

Fig.12 Amplitudes of pseudo-shock oscillations

From the discussion described above, it may be concluded that the oscillation phenomenon of the flow including the pseudo-shock is caused by the interaction of the shock with small perturbation in the supersonic flow upstream the shock. The flow model presented here may be improved if the mechanism of the pseudo-shock, the variation of δp , with time, etc. are made more clear.

Next, consider the frequency of the oscillation. The perturbation upstream the pseudo-shock is considered to be something like a white noise. Nevertheless some prominent frequencies are observed in the oscillation of the pseudo-shock as shown in Fig.5. These frequencies may depend on the geometry of the duct as described as follows. As shown in Fig.1, the test section in the present work is followed by a large settling chamber just like the Helmholtz resonator. Let L , S and V be the length of the straight duct between the first shock and the duct exit, the cross-sectional area of the duct and the volume of the settling chamber, respectively, then the resonance frequency is given as follows⁽⁴⁾.

$$f_H = \frac{a_2}{2\pi} \sqrt{\frac{S}{LV}} \quad \dots(20)$$

where a_2 is sonic velocity downstream the shock. Further, based on two characteristic lines which propagate with the velocities $a_2 u_2$ and $a_1 u_1$ between the shock and the duct exit as shown in Fig.10(b), the frequency of pipe resonance f_0 is given as follows.

$$f_0 = \frac{a_2^2 - u_1^2}{2a_2 L} \quad \dots(21)$$

Fig.13
Characteristic frequencies of the duct geometry.

Figure 13 shows f_H and f_0 , where the plenum temperature is assumed to be 283°K and the values of $S = (0.06)^2 \text{ m}^2$, $L = 0.5\text{m}$ and $V = 0.19\text{m}^3$ are used. From this, f_H is almost independent of M_1 and the value is about 10Hz. f_0 increases with M_1 , but in the range of the present experiment ($M_1 = 1.44 \sim 2.79$) it is nearly equal to 200Hz. Taking the fact into account that in the real flow u_1 and u_2 vary along the duct and L cannot be determined because the shock is not a plane normal shock but a pseudo-shock, the two prominent frequencies of a few 10Hz and 100~200Hz in Fig. 5 may be considered to correspond with f_H and f_0 , respectively. Besides these frequencies determined from the duct geometry, the interaction of the bifurcated foot of the shock with the boundary layer also causes the fluctuation of the pressure⁽⁹⁾, but in general its frequency is very high and the energy of high frequency is much smaller than that of low

frequency as is obvious from Fig.5. Therefore it may be unimportant in the oscillation phenomena of the pseudo-shock.

5. Conclusions

The oscillation phenomena of the pseudo-shock in a straight pipe were observed. The conclusions are summarized as follows: (1) The amplitude of oscillation of the pseudo-shock increases with the flow Mach number M_1 just before the pseudo-shock, and it becomes larger than the duct diameter when M_1 approaches 3; (2) Because of the oscillation of the pseudo-shock, the wall static pressure fluctuates, and its maximum amplitude amounts to 60 per cent of the pressure difference across the pseudo-shock; (3) Assuming a one-dimensional adiabatic flow with friction in a straight duct containing a normal shock and giving a small perturbation in pressure upstream the shock, the displacement of the shock and the ratio of pressure variations across the shock were calculated theoretically. The results agree qualitatively with the amplitude of oscillation of the pseudo-shock and the amplitude of pressure fluctuation across the pseudo-shock. From this the oscillation of the pseudo-shock maybe caused by the interaction of the shock with small perturbation in the supersonic flow upstream the shock; (4) The frequency of oscillation of the pseudo-shock depends on the geometry of the duct. In most of the present experiment, the spectral density of oscillation of the pseudo-shock has two prominent peaks of a several 10Hz and 100~200Hz. These frequencies are nearly equal to those of Helmholtz resonance and pipe resonance.

The oscillation phenomenon of the pseudo-shock is so complicated that there are many uncertain points at present. This is mainly because the mechanism of the pseudo-shock has not been clarified in detail. The mechanism of the pseudo-shock, however, is affected by the flows upstream and downstream the shock. Therefore, in order to clarify the oscillation phenomena of the pseudo-shock further, it must be studied systematically how the oscillation is influenced by the duct geometry, the small disturbance in the supersonic flow upstream the shock, the boundary layer, etc.

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