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On the Elasto-Plastic Plane Theory in a Stress Space*

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In this paper, a new analytical method in a stress space is suggested to solve the elasto-plastic plane problem that has not been treated analytically even with the total strain theory of plasticity. At first, a general formulation for the elasto-plastic plane theory is established for a compressible isotropic work-hardening material obeying a nonlinear stress-strain law. Then, the basic nonlinear differential equation in ordinary Cartesian coordinate system can be linearized in the system of the stress space, and a complex displacement function, which gives us the solution, can be written in the closed form. Provided that Ramberg-Osgood's law holds as the nonlinear stress-strain relation, the strain component and the corresponding coordinate can be obtained easily and the stress component can be evaluated by the numerical calculation.

1. Introduction

An elasto-plastic stress analysis either for Saint-Venant's problem or for a plane one, is difficult even with the total strain theory of plasticity. Previous solutions to these problems have been given for a non-hardening rigid-plastic material and only in the author's previous paper⁽¹⁾, an analytical method for solving Saint-Venant's problem has been formulated for a compressible isotropic work-hardening material with a new concept on a stress space.

In this paper, an analytical method for solving the plane problem is suggested for an isotropic work-hardening material obeying a nonlinear stress-strain law $\sigma_{ij}' = 2G_s \varepsilon_{ij}'$, where σ_{ij}' , ε_{ij}' and G_s are deviatoric stress, strain and secant modulus related to deviatoric stress σ_{ij}' . Evolving a theory such as to satisfy the equilibrium and compatibility conditions, the governing equation, presented as a system of nonlinear partial differential equation in ordinary Cartesian coordinate can be linearized, adopting the new parameters in the system of the stress space and a complex displacement function, which gives us the solution, can be written in the closed form.

Provided Ramberg-Osgood's law holds as the nonlinear stress-strain relation, the components of strain and corresponding coordinate can be also written in the closed form. Furthermore, the components of stress can be obtained by solving numerically the algebraic equation of higher order.

2. Construction of plane theory of elasto-plasticity

The equilibrium equation for this problem can be represented as follows, with the complex variable $z = x + iy$:

$$\left. \begin{aligned} \frac{\partial}{\partial \bar{z}}(\sigma_x + \sigma_y) + \frac{\partial}{\partial z}(\sigma_x - \sigma_y + 2i\tau_{xy}) &= 0 \\ \frac{\partial}{\partial z}(\sigma_x + \sigma_y) + \frac{\partial}{\partial \bar{z}}(\sigma_x - \sigma_y - 2i\tau_{xy}) &= 0 \end{aligned} \right\} \dots (1)$$

The components of strain can be written as follows, with those of displacement U_x and U_y :

$$\left. \begin{aligned} \varepsilon_x + \varepsilon_y &= \frac{\partial}{\partial z}(U_x + iU_y) + \frac{\partial}{\partial \bar{z}}(U_x - iU_y) \\ \varepsilon_x - \varepsilon_y + 2i\gamma_{xy} &= 2 \frac{\partial}{\partial \bar{z}}(U_x + iU_y) \\ \varepsilon_x - \varepsilon_y - 2i\gamma_{xy} &= 2 \frac{\partial}{\partial z}(U_x - iU_y) \end{aligned} \right\} \dots (2)$$

The nonvanishing rotation for this problem is one about the z_s -axis, which is perpendicular to the x - and y axes, and can be written as follows:

$$\omega_z = -\frac{i}{2} \left\{ \frac{\partial}{\partial z}(U_x + iU_y) - \frac{\partial}{\partial \bar{z}}(U_x - iU_y) \right\} \dots (3)$$

Eliminating the components of displacement U_x and U_y from Eq.(2), the following compatibility condition can be obtained:

$$\begin{aligned} \frac{\partial^2}{\partial z \partial \bar{z}}(\varepsilon_x + \varepsilon_y) &= \frac{1}{2} \left\{ \frac{\partial^2}{\partial z^2}(\varepsilon_x - \varepsilon_y + 2i\gamma_{xy}) \right. \\ &\quad \left. + \frac{\partial^2}{\partial \bar{z}^2}(\varepsilon_x - \varepsilon_y - 2i\gamma_{xy}) \right\} \dots (4) \end{aligned}$$

For a compressible isotropic work-hardening material, without any distinct yielding point, as observed in the behaviour of deformation of aluminum and copper, the following nonlinear stress-strain relation can be used;

$$\sigma_{ij}' = 2G_s \varepsilon_{ij}' \dots (5)$$

where σ_{ij}' and ε_{ij}' are the deviatoric stress and strain, respectively, and G_s the secant modulus which is a function of the deviatoric stress. According to an extended Michell's theorem⁽²⁾, the solution for the

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stresses is independent of Poisson's ratio ν . Therefore in Eq.(5), it is assumed that Poisson's ratio ν is constant even in plastic deformation and the relation $E_s = 2G_s(1+\nu)$ holds. In a special case, where Poisson's ratio is 1/2, Eq.(5) coincides with Hencky's stress-strain equation for the total strain theory. The mean normal strain ε_0 is related to the mean normal stress σ_0 as follows:

$$\varepsilon_0 = \frac{1}{E_s}(1-2\nu)\sigma_0 \quad (6)$$

From Eqs.(5) and (6), the following relation can be obtained:

$$\left. \begin{aligned} \sigma_x + \sigma_y &= G_s \kappa_2 (\varepsilon_x + \varepsilon_y) \\ \sigma_x - \sigma_y + 2i\tau_{xy} &= 2G_s (\varepsilon_x - \varepsilon_y + 2i\gamma_{xy}) \end{aligned} \right\} \quad (7)$$

where $\kappa_2 = 2(1+\nu)/(1-\nu)$ for a plane stress problem and $\kappa_2 = 2/(1-2\nu)$ for a plane strain one.

Now, let us introduce a complex displacement function W defined as follows:

$$W = U_x + iU_y \quad (8)$$

Substituting Eq.(8) into (2), the following relation can be obtained:

$$\left. \begin{aligned} \varepsilon_x + \varepsilon_y &= \frac{\partial W}{\partial z} + \frac{\partial \bar{W}}{\partial \bar{z}} \\ \varepsilon_x - \varepsilon_y + 2i\gamma_{xy} &= 2 \frac{\partial W}{\partial \bar{z}} \\ \varepsilon_x - \varepsilon_y - 2i\gamma_{xy} &= 2 \frac{\partial \bar{W}}{\partial z} \end{aligned} \right\} \quad (9)$$

Substituting Eq.(9) into (4), the compatibility condition is satisfied, as might be expected.

Hereafter, a restriction that the rotation ω , ought to be zero, is imposed on this theory. Under this restriction, that is $\partial W/\partial z = \partial \bar{W}/\partial \bar{z}$ from Eq.(3), Eq.(9) can be written as follows:

$$\left. \begin{aligned} \varepsilon_x + \varepsilon_y &= 2 \frac{\partial W}{\partial z} = 2 \frac{\partial \bar{W}}{\partial \bar{z}} \\ \varepsilon_x - \varepsilon_y + 2i\gamma_{xy} &= 2 \frac{\partial W}{\partial \bar{z}} \\ \varepsilon_x - \varepsilon_y - 2i\gamma_{xy} &= 2 \frac{\partial \bar{W}}{\partial z} \end{aligned} \right\} \quad (10)$$

Moreover, the complex displacement function W is divided into two parts.

$$W = W_0 + W_1 \quad (11)$$

in which W_0 is a displacement function, from which only the shear deformation can occur and W_1 is one for the uniform deformation defined as

$$W_1 = 2cz \quad (12)$$

Substituting Eq.(12) into (2), the uniform strain can be provided as follows:

$$\varepsilon_x + \varepsilon_y = 4c \quad (13)$$

On the other hand, for the shear deformation the corresponding parts of the stress components expressed by σ'_x , σ'_y and τ'_{xy} satisfy the relation $\sigma'_x + \sigma'_y = 0$ and are given by

$$\left. \begin{aligned} \sigma'_x + \sigma'_y &= G'_s \kappa_2 (\varepsilon_x + \varepsilon_y) = 2G'_s \kappa_2 \frac{\partial W_0}{\partial z} \\ &= 2G'_s \kappa_2 \frac{\partial \bar{W}_0}{\partial \bar{z}} = 0 \\ \sigma'_x - \sigma'_y + 2i\tau'_{xy} &= 4G'_s \frac{\partial W_0}{\partial \bar{z}} \\ \sigma'_x - \sigma'_y - 2i\tau'_{xy} &= 4G'_s \frac{\partial \bar{W}_0}{\partial z} \end{aligned} \right\} \quad (14)$$

where G'_s is the secant modulus in the stress system of Eq.(14). Substituting Eq.(14) into (1) for σ'_x , σ'_y and τ'_{xy} , the following nonlinear, partial differential equation can be obtained:

$$\frac{\partial}{\partial z} \left(G'_s \frac{\partial W_0}{\partial \bar{z}} \right) = 0 \quad (15)$$

Introducing the variables (τ', α) in the stress space

$$2\tau' e^{i\alpha} = (\sigma'_x - \sigma'_y) + 2i\tau'_{xy} \quad (16)$$

and substituting Eq.(16) into (14), the following relation, concerning the function W_0 , can be obtained:

$$\left. \begin{aligned} \frac{\partial W_0}{\partial z} &= 0 \\ \frac{\partial W_0}{\partial \bar{z}} &= \frac{1}{2G'_s} \tau' e^{i\alpha} \end{aligned} \right\} \quad (17)$$

2.1 Treatment in a stress space

From Eq.(17), the following relation can be obtained:

$$d\bar{z} = 2G'_s \frac{e^{-i\alpha}}{\tau'} dW_0 \quad (18)$$

By integrating Eq.(18), we have the relation:

$$\bar{z} = 2 \int G'_s \frac{e^{-i\alpha}}{\tau'} dW_0 \quad (19)$$

carrying out differentiations $\partial^2 \bar{z} / \partial \tau' \partial \alpha$ and $\partial^2 \bar{z} / \partial \alpha \partial \tau'$, and comparing the results with each other, we obtain the following relation;

$$\frac{\partial W_0}{\partial \tau'} - iK \frac{\partial W_0}{\partial \alpha} = 0 \quad (20)$$

in which K is a function of the parameter τ' only, defined as follows;

$$K = \frac{G}{G'_s} \tau' \frac{\partial}{\partial \tau'} \left(\frac{G'_s}{G} \frac{1}{\tau'} \right) \quad (21)$$

where G'_s is a function of the variable τ' only. Since Eq.(20) is a linear differential equation, its solution can be, in general, written in the form;

$$W_0 = \sum_n a_n \exp \left(- \int n K d\tau' \right) e^{in\alpha} \quad (22)$$

in which n and a_n are constants to be determined by the boundary condition.

From the previous discussion, we can conclude that the nonlinear differential equation (15) of variables x and y can be transformed into the linear differential equation (20) of variables τ' and α , the solution of which is given by Eq.(22).

2.2 Corresponding coordinate system

Since the complex displacement function W_0 is expressed in the form of a function of variables (τ', α) , the following relation can be obtained:

$$dW_0 = \frac{\partial W_0}{\partial \tau'} d\tau' + \frac{\partial W_0}{\partial \alpha} d\alpha \quad (23)$$

Substituting Eq.(23) into (18), we find the following relation:

$$dz = 2 \left\{ G' \frac{e^{-i\alpha}}{\tau'} \frac{\partial W_0}{\partial \tau'} d\tau' + G' \frac{e^{-i\alpha}}{\tau'} \frac{\partial W_0}{\partial \alpha} d\alpha \right\} \quad (24)$$

Using Eq.(20), we can easily find that Eq.(24) is a complete differential equation, and so, can be integrated quite easily as follows:

$$\bar{z} = 2 \left\{ \int G' \frac{e^{-i\alpha}}{\tau'} \frac{\partial W_0}{\partial \alpha} d\alpha + \int \left(G' \frac{e^{-i\alpha}}{\tau'} \frac{\partial W_0}{\partial \tau'} - \frac{\partial}{\partial \tau'} \int G' \frac{e^{-i\alpha}}{\tau'} \frac{\partial W_0}{\partial \alpha} d\alpha \right) d\tau' \right\} \quad (25)$$

Since the function W_0 is, in general, represented by Eq.(22), we can obtain the corresponding coordinate z in terms of variables τ' and α .

2.3 Solution with Ramberg-Osgood's law

We proceed to the analysis with an actual nonlinear stress-strain law. In this paper, we use Ramberg-Osgood's law⁽³⁾ that is adopted for the numerical calculation in many approaches to the elasto-plastic problem. The dependence of the shape of the stress-strain curve on the parameter m is exhibited in Fig.1, which presents plots of $(E\epsilon_t/(\sqrt{3}k))$ versus $(\sigma_t/(\sqrt{3}k))$ for several values of m . For large values of the exponent m , the shape of the curve approaches that of an elasto-perfectly-plastic material.

Fig. 1

A generalized Ramberg-Osgood's law can be written as follows;

$$2G\epsilon_{ij} = \left[1 + A \left(\frac{J_2}{k^2} \right)^m \right] \sigma_{ij} \quad (26)$$

in which k is the yield shearing stress, J_2 the second invariant of the stress tensor, and A is a constant. The results for $A=1$, which is the stress-strain relation defined by Greenberg et al.⁽⁴⁾, are shown in Fig.1. Replacing m in Eq.(26) by $(m-1)/2$, and putting $A=3/7$, we can obtain the stress strain relation which is adopted by Budiansky⁽⁵⁾ and Mangasarian⁽⁶⁾ in their papers.

The second invariant of the stress deviatoric tensor is, in general, written

as follows for the plane problem;

$$J_2 = \frac{1}{4} [(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2] + \kappa_1 (\sigma_x + \sigma_y)^2 \quad (27)$$

where κ_1 is equal to $1/12$ for a plane stress problem and $\kappa_1 = (1-2\nu)^2/12$ for a plane strain one. The relation $J_2 = \tau'^2$ is obtained for the stress system given by Eq.(14), and then, the secant modulus G' can be given as follows with Eqs.(26) and (5)

$$\frac{G}{G'} = \left[1 + A \left(\frac{\tau'}{k} \right)^{2m} \right] \quad (28)$$

The above equation (28) shows that the secant modulus G' is independent of the variable α , as mentioned previously. Using the value of

$$K = - \frac{[1 + A(2m+1)(\tau'/k)^{2m}]}{\tau'(1 + A(\tau'/k)^{2m})}$$

calculated from Eqs.(21) and (28), the complex displacement function W_0 is finally represented in the following closed form:

$$W_0 = \sum_n a_n \tau'^n \left(\frac{G}{G'} \right)^n e^{ni\alpha} = \sum_n a_n \tau'^n \left\{ 1 + A \left(\frac{\tau'}{k} \right)^{2m} \right\}^n e^{ni\alpha} \quad (29)$$

Substituting Eqs.(28) and (29) into (25), we can obtain the corresponding coordinate z in the following closed form:

$$\begin{aligned} \bar{z} &= 2G \sum_n \frac{a_n}{n-1} \left(\frac{G}{G'} \right)^{n-1} \tau'^{n-1} e^{(n-1)i\alpha} \\ &= 2G \sum_n \frac{a_n}{n-1} \tau'^{n-1} \left\{ 1 + A \left(\frac{\tau'}{k} \right)^{2m} \right\}^{n-1} e^{(n-1)i\alpha} \quad (30) \end{aligned}$$

To the complex displacement function W_0 shown in Eq.(29), we may add the displacement function given by Eq.(12), which provides the uniform strain distributions described by Eq.(13). Therefore, with Eqs. (13), (14), (16) and (28), the components of strain can be represented as follows:

$$\begin{aligned} \epsilon_x + \epsilon_y &= 4c \\ \epsilon_x - \epsilon_y + 2i\tau_{xy} &= \frac{1}{G} \tau' \left\{ 1 + A \left(\frac{\tau'}{k} \right)^{2m} \right\} e^{i\alpha} \quad (31) \end{aligned}$$

2.4 The components of stress

The second invariant of the deviatoric tensor J_2 for the stress system given by Eq.(7), which gives the components of stress of the problem considered, is represented in Eq.(27). Consequently, the secant modulus G' in this system can be obtained as follows with Eqs.(5) and (26):

$$\frac{G}{G'} = \left[1 + A \left(\frac{J_2}{k^2} \right)^m \right] \quad (32)$$

Using Eq.(32), the following relation between the components of strain and those of stress, is obtained from Eq.(7);

$$\frac{\sigma_x - \sigma_y + 2i\tau_{xy}}{\sigma_x + \sigma_y} = \frac{2}{\kappa_2} \frac{\epsilon_x - \epsilon_y + 2i\tau_{xy}}{\epsilon_x + \epsilon_y} = \delta_1 + i\delta_2 \quad (33)$$

where the values of δ_1 and δ_2 are known from Eq.(31). Substituting Eq.(33) into (27), the invariant J_2 is written as follows;

$$J_2 = \delta(\sigma_x + \sigma_y)^2 \quad (34)$$

where δ is equal to $(\delta_1^2 + \delta_2^2)/4 + \kappa_1$. Substituting Eq.(34) into (32) and using the first equation of (7) and Eq.(13), we finally obtain the following algebraic equation of higher order:

$$A\delta^m \left(\frac{\sigma_x + \sigma_y}{k} \right)^{2m+1} + \frac{\sigma_x + \sigma_y}{k} - \frac{4G\kappa_2}{k} c = 0 \quad (35)$$

As the exponent m of Ramberg-Osgood's law is a positive integer, e.g. $m=9$ for aluminum and copper, only one real root of the solution $((\sigma_x + \sigma_y)/k)$ can be obtained from Eq.(35). Therefore, the component of stress can be evaluated finally from Eq.(33).

The application of this theory for an elasto-plastic problem will be shown in other papers. In this paper, the results for a special case $G_r = G_r' = G = \text{const.}$, i.e. an elastic problem are shown in the following section.

2.5 Some application of this theory for the elastic problems.

For the extreme case $\tau' \ll k$, i.e. for $G_r = G_r' = G$, equations (7), (29) and (30) are reduced to

$$\left. \begin{aligned} \sigma_x + \sigma_y &= G\kappa_2(\varepsilon_x + \varepsilon_y) \\ \sigma_x - \sigma_y + 2i\tau_{xy} &= 2G(\varepsilon_x - \varepsilon_y + 2i\gamma_{xy}) \end{aligned} \right\} \quad (36)$$

$$\bar{z} = 2G \sum_n a_n \frac{n}{n-1} \tau'^{n-1} e^{(n-1)i\alpha} \quad (37)$$

$$\begin{aligned} W &= \sum_n a_n \tau'^n e^{ni\alpha} \\ &+ 4Gc \sum_n \bar{a}_n \frac{n}{n-1} \tau'^{(n-1)} e^{-(n-1)i\alpha} \end{aligned} \quad (38)$$

Equations (36)~(38) give the elastic solutions. Since the restriction that rotation ought to be zero is imposed on this theory, we apply this theory to some axially symmetric problems in this section. Thus, it is convenient to express stresses and displacements in polar coordinates (r, θ) . From Eqs.(8), (37) and (38), we can express the components of displacement in polar coordinates as follows with well known formulae of analytical geometry:

$$\begin{aligned} U_r + iU_\theta &= (U_r + iU_\theta)e^{-i\theta} \\ &= \left(\sum_n a_n \tau'^n e^{ni\alpha} + 2c r e^{i\theta} \right) e^{-i\theta} \end{aligned} \quad (39)$$

Replacing z on left handside of Eq.(37) by $r e^{i\theta}$, the following equation can be obtained:

$$r e^{-i\theta} = 2G \sum_n a_n \frac{n}{n-1} \tau'^{n-1} e^{(n-1)i\alpha} \quad (40)$$

The components of stress in polar coordinates are defined as follows from Eqs.(31) and (36), and using the condition $\tau' \ll k$.

$$\left. \begin{aligned} \sigma_r + \sigma_\theta &= \sigma_x + \sigma_y = 4Gc\kappa_2 \\ \sigma_r - \sigma_\theta + 2i\tau_{r\theta} &= (\sigma_x - \sigma_y + 2i\tau_{xy}) e^{-2i\theta} \\ &= 2\tau' e^{i\alpha} e^{-2i\theta} \end{aligned} \right\} \quad (41)$$

Since the shear stress $\tau_{r\theta}$ should be zero for an axially symmetric problem, the relation that variable α is equal to 2θ , can be obtained from the second equation of (41), and the unknown constant n must be $1/2$ from Eq.(40). Therefore, the relation between τ' and r can be obtained as follows from Eq.(40):

$$\tau'^{1/2} = -2Ga_n \frac{1}{r} \quad (42)$$

Substituting Eq.(42) into (41), the components of stress can be obtained as follows:

(1) for a plane stress problem;

$$\left. \begin{aligned} \sigma_r &= \frac{2E}{1-\nu} c + \left(2Ga_n \frac{1}{r} \right)^2 \\ \sigma_\theta &= \frac{2E}{1-\nu} c - \left(2Ga_n \frac{1}{r} \right)^2 \end{aligned} \right\} \quad (43)$$

(2) for a plane strain problem;

$$\left. \begin{aligned} \sigma_r &= \frac{4G}{1-2\nu} c + \left(2Ga_n \frac{1}{r} \right)^2 \\ \sigma_\theta &= \frac{4G}{1-2\nu} c - \left(2Ga_n \frac{1}{r} \right)^2 \end{aligned} \right\} \quad (44)$$

(a) Bi-axial tension of a plate weakened by a circular hole

Let $\sigma_r = 0$ at $r = a$, where a is the radius of this circular hole, and

$$\sigma_r = \sigma_\theta = p, \quad \text{at} \quad r = \infty$$

where p is a constant tension. Then, we can determine unknown constants c and a_n as follows from Eq.(43):

$$c = \frac{1-\nu}{2E} p, \quad a_n = \pm i \frac{\sqrt{p}}{2G} a \quad (45)$$

Substituting Eq.(45) into (43), the following components of stress can be obtained:

$$\sigma_r = p \left(1 - \frac{a^2}{r^2} \right), \quad \sigma_\theta = p \left(1 + \frac{a^2}{r^2} \right) \quad (46)$$

Substituting Eqs.(42) and (45) into (39), the components of displacement can be expressed as follows:

$$U_r = \frac{p}{2Gr} \left\{ \frac{1-\nu}{1+\nu} r^2 + a^2 \right\}, \quad U_\theta = 0 \quad (47)$$

(b) Uniform normal pressure applied to the edge of a circular hole

Consider now the case when the edge of the hole is subjected to uniform normal pressure p and the stresses vanish at infinity. Then, $\sigma_r = -p$ at $r = a$, $\sigma_r = \sigma_\theta = 0$ at $r = \infty$. Then, we can determine unknown constants c and a_n as follows:

$$c = 0, \quad a_n = \pm i \frac{\sqrt{p}}{2G} a \quad (48)$$

Substituting Eq.(48) into (43), we can obtain the components of stress as follows:

$$\sigma_r = -p \frac{a^2}{r^2}, \quad \sigma_\theta = p \frac{a^2}{r^2} \quad (49)$$

Substituting Eqs.(42) and (48) into (39), the components of displacement can be expressed as follows:

$$U_r = \frac{p a^2}{2Gr}, \quad U_\theta = 0 \quad (50)$$

(c) Hollow cylinder submitted to uniform pressure p on the inner surface

Let a and b denote the inner and outer radii of the cylinder, and p the uniform internal pressure. Then, $\sigma_r = 0$ at $r = b$, $\sigma_r = -p$ at $r = a$. Therefore, we can obtain unknown constants c and a_n as follows;

$$c = \frac{1-2\nu}{4G} p c_0, \quad a_n = \pm i \frac{b}{2G} \sqrt{p c_0} \dots\dots\dots (51)$$

where $c_0 = 1/(b^2/a^2 - 1)$. Substituting Eq.(51) into (44), we can obtain the components of stress as follows:

$$\sigma_r = p c_0 \left(1 - \frac{b^2}{r^2}\right), \quad \sigma_\theta = p c_0 \left(1 + \frac{b^2}{r^2}\right) \dots\dots\dots (52)$$

Substituting Eqs.(42) and (51) into (39), the components of displacement can be obtained as follows:

$$U_r = \frac{p c_0}{2Gr} \{(1-2\nu)r^2 + b^2\}, \quad U_\theta = 0 \dots\dots\dots (53)$$

The results in (a), (b) and (c), are in good agreement with those obtained from the ordinary theory of elasticity⁽⁷⁾, as might be expected.

3. Conclusions

The following conclusions are made as a result of this study;

(1) The governing equation for an elasto-plastic plane problem, presented as a system of nonlinear, partial differential equation in usual Cartesian coordinate, is linearized in the system of the stress space, by adopting new parameters.

(2) Provided that Ramberg-Osgood's law holds as the nonlinear stress-strain relation, the components of displacement, strain and corresponding coordinates can be described in the closed form.

(3) The components of stress can be obtained from numerical solutions of the algebraic equation of higher order.

(4) For the extreme case with the secant modulus of the constant one of elasticity, the solutions obtained by author's method are in good agreement with those by the ordinary theory of elasticity.

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