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Construction of a High-Speed Liquid Flow Apparatus Operated by Compressed Air*

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By using an apparatus in which a water column is accelerated by compressed air in a tube, the column velocities up to about 100 m/sec are obtained. The liquid column motion in the tube is approximately analysed in order to determine the driving pressure and the apparatus dimensions such that the desired velocity of liquid flow can be obtained at the end of the tube. Magnitude and duration of impact pressure caused by the water column stopping at the closed end of the tube are calculated, considering the variation of compressibility of water with a change in pressure. Those results coincide with experimental data given by Rice and Walsh, and by Cook et al., and involve Heymann's formula as being the first approximation. Experimental results obtained in the constructed apparatus showed that the actual impact pressure were influenced by vacuum pressure in the tube before the impact.

1. Introduction

Experimental devices for a high-speed liquid flow of several hundred meters per second have not been fully developed in comparison with those for a high-speed gas flow. The main reason for this seems that it requires by far the higher pressure and the higher energy than a gas flow case to maintain the steady high-speed flow, with the result that a large-scale apparatus is needed. However, it is possible to make a high-speed liquid flow rather easily by accelerating a small amount of liquid, although the flow duration is very short. Based on this idea, an apparatus in which a liquid column is accelerated by compressed air in a tube and the column velocity reaches the maximum at the end of the tube was constructed. This apparatus is simple in construction and the driving pressure of air need not be very high, thus making easy and safe operation possible. These features are like those of a shock tube for gases.

It is the purpose of this report to discuss the accelerating process for a water column in a tube as well as the impact pressure of the column against the closed end of

the tube in order to design and to construct the apparatus. By using the constructed apparatus it is shown that, in case the inside diameter of the tube is 20 mm and a water column length is 0.5 m, the column velocity of over 100 m/sec can be obtained by compressed air of from 20 to 30 kg/cm² and the impact pressure caused by the column stopping at the closed end of the tube reaches about 1700 kg/cm². These results agree well with the calculated results on this apparatus.

Similar method to this apparatus has already been employed in a few devices. For example, Bowden and Brunton⁽¹⁾ has described an apparatus in which a small mass of liquid was extruded through a orifice by a slug fired from a gun. And O'Keefe et al.⁽²⁾, have improved the method to succeed in producing supersonic liquid jet. In the latter case a liquid column was accelerated by a smokeless powder gun and its acceleration mechanism resembles that of the constructed apparatus. In these devices, efforts were concentrated on making the velocity up and so the flow area is too small and the flow duration too short. In the constructed apparatus described in this report, it has been also paid to make the higher-speed liquid flow possible in future with the use of a convergent nozzle or of an explosive.

2. Basic problems in design

2.1 Water column motion in the tube

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Figure 1 shows a model of calculation for the apparatus that was constructed. The piston in contact with the water column serves to prevent deformation of the column during its movement. In this apparatus, the column velocity reaches the maximum value at a certain point of the tube since pressure of the driving gas drops and friction force increases as the column moves. By an simple analysis it will be shown how the pressure of the driving gas and length of the tube are necessary for obtaining the desired flow velocity.

In Fig.1, suppose that the driving gas will uniformly make adiabatic expansion through the gas reservoir and the acceleration tube, that the wall friction acting on the column is proportionate to the square of the column velocity, and that the gas pressure ahead of the column and the friction force acting on the piston are both negligible. Then, an energy equation of the water column is expressed as follows.

$$\rho l_e \frac{U^2}{2} = \int_0^X \left[P_0 \left(1 + \frac{\pi D^2 x}{4 V_r} \right)^{-\kappa} - \lambda_0 \frac{l_0 \rho U^2}{2} \right] dx \quad (1)$$

where X : column position m.
 U : column velocity m/sec.
 ρ : density of water $\text{kg} \cdot \text{s}^2 \text{m}^{-4}$.
 l_0 : water column length m.
 $l_e = (1 + \beta) l_0$: equivalent length of water column and piston m, $\beta = (\text{piston weight}) / (\text{water column weight})$.
 λ_0 : friction coefficient
 P_0 : initial pressure of driving gas kg/m^2 .
 κ : specific heat ratio of driving gas.
 D : inside diameter of tube m.
 V_r : volume of gas reservoir m^3 .

If we use $\lambda_a = \lambda_0 / (1 + \beta)$ as the apparent tube friction coefficient and if we make the position, velocity and parameter dimensionless, respectively, like

$$\chi = \lambda_a X / D, \quad \psi = U \sqrt{(\lambda_a l_e / D) / (2 P_0 / \rho)}$$

and $q = (\pi D^3 / (4 V_r)) / \lambda_a$.

Equation (1) will be

$$\frac{d\psi^2}{d\chi} + \psi^2 = (1 + q\chi)^{-\kappa} \quad (2)$$

Therefore, changes in the column velocity can be represented by the following equation.

$$\psi(\chi) = e^{-\chi/2} \left\{ \int_0^\chi (1 + q\chi)^{-\kappa} e^{\chi/2} d\chi \right\}^{1/2} \quad (3)$$

Fig.1 Calculation model for apparatus

In case the driving gas is air, namely the ratio $\kappa = 1.4$, the calculated results are as shown in Fig. 2. The right side of Eq.(3) is integrated numerically by the Simpson's method. From Fig.2, it is seen that the water column velocity reaches the maximum value at a certain point of the tube (shown in the dashed line). This is due to a pressure drop of driving gas and a resistance increase of wall friction. In Fig.2 the dimensionless velocity $\psi = 1$ represents the final velocity of the water column in case the driving pressure is kept constant. The maximum velocity ψ_{max} and the corresponding distance χ_m to the ψ_{max} are shown against the parameter q as in Fig.3. The initial pressure P_0 of the driving gas and the length L of the tube which are necessary for obtaining the desired velocity U_0 can be calculated by the following equations.

$$P_0 = (\lambda_a l_e / D) (\rho U_0^2 / 2) / (\psi_{max}(q))^2 \quad (4)$$

$$L = D \chi_m(q) / \lambda_a \quad (5)$$

Equation (5) shows that the tube length L is little influenced by the driving pressure and by the column length.

In this apparatus, the inside diameter of the tube was set at 20 mm and the length of the water column was set at 0.5 m, with the velocity of the column aimed at 100 m/sec. By evaluating the friction coefficient from the Moody diagram and by making the volume of the gas reservoir large enough, it is derived that the initial pressure of the driving gas and the length of the

Fig.2 Velocity changes of liquid column along acceleration tube

Fig.3 Maximum velocity ψ_{max} and tube length χ_m against parameter q

tube are about 20 kg/cm² and 5 m, respectively (see Fig.3).

2.2 Impact pressure of high-speed water column in the tube

The maximum pressure produced in this apparatus is the impact pressure at the time of impact of the water column against the closed end of the tube. We will determine the magnitude and its duration in such a case. As mentioned earlier, the maximum velocity of water column in this apparatus is only about 100 m/sec, but the range of calculation includes the velocity of up to several thousand meters per second.

Assuming that pressure waves produced by impact of the column against the closed end are plane waves perpendicular to the tube axis, that the tube wall is a rigid body and that pressure inside the column before the impact is negligibly small compared to the impact pressure, we have for conservation of mass and momentum across the plane waves,

$$\rho_1(c_s - U_0) = \rho_0 c_s \quad (6)$$

$$P_1 = \rho_0 c_s U_0 \quad (7)$$

where U_0 : impact velocity of water column, m/sec.

P_1 : impact pressure, kg/m².

ρ_0 : density of undisturbed water, kg·sec²·m⁻⁴.

c_0 : sound velocity of undisturbed water, m/sec.

ρ_1 : density of water at impact, kg·sec²·m⁻⁴.

c_s : propagation velocity of impact pressure into undisturbed water, m/sec.

Furthermore, the following approximate formula is used as equation of state of water(3).

$$P = B[(\rho/\rho_0)^n - 1] \quad (8)$$

where B and n are constant parameters (in general, the coefficient B is regarded as a function of entropy). The relation between the coefficient B and the bulk modulus of water K_0 can be expressed by the following equation.

$$B = K_0/n \quad (9)$$

From Eqs.(6)~(9), the following equation can be derived for the relation between impact velocity and propagation velocity of compression waves.

$$(1 + n M_0 \zeta_s) \left(1 - \frac{M_0}{\zeta_s}\right)^n = 1 \quad (10)$$

where $M_0 = U_0/c_0$ and $\zeta_s = c_s/c_0$. The values of ρ_1 and P_1 can be also obtained from equation (6) and (7). Assuming that M_0 is relatively small, an approximation for ζ_s is derived from equation (10), as follows.

$$\zeta_s = 1 + \frac{n+1}{4} M_0 \quad (11)$$

This equation coincides with Heymann's empirical formula(4) putting the exponent $n = 7$. From Eqs. (6) and (7), the following approximations for the impact pressure and density variation can be also derived.

$$\frac{P_1}{K_0} = M_0 + \frac{n+1}{4} M_0^2 \quad (12)$$

$$\frac{\rho_1}{\rho_0} = \frac{1 + (n+1) M_0/4}{1 + (n-3) M_0/4} \quad (13)$$

The first term of the right side in equation (12) corresponds to the water-hammer equation and the second term results from the non-linear effect of water compressibility.

Figure 4 and 5 illustrate the relation of ζ_s and P_1/K_0 against M_0 , respectively, where the exponent $n = 7$. It is noted from Fig.4 that propagation velocity of compression waves given by Eq.(10) agrees rather well with experimental data by Rice (5) and Cook(6). Fig.4 also suggests that the range of application of approximation (11) shown in the dashed line can be regarded as $M_0 < 1.2$ as pointed out by Heymann(4). It is a well known fact that as impact velocity of a water column increases, its impact pressure grows larger than the value derived from the water-hammer equation. The approximate formula (12), on the other hand, gives somewhat excessive value. The comparison of these is shown in Fig.5.

Fig.4 Propagation velocity of compression waves in water caused by impact

Fig.5 Impact pressure versus impact velocity of water column

Now, we will obtain the duration of the impact pressure at the end of the tube. When compression waves inside the water column caused by impact are reflected by the piston behind the column, propagation velocity of its reflection waves drops gradually because of a decreasing motion of the piston. However, we will consider here two extreme cases, a case where there is no piston and a case where the decreasing motion of the piston can be disregarded because the piston is sufficiently heavy. In the former case, the reflection waves become expansion waves and propagation velocity of their wave front equals the speed of sound $c_0(\rho_1/\rho_0)^{(n-1)/2}$ in a state of compression at impact. On the other hand, in the latter case, reflection waves become stronger compression waves than the former and its propagation velocity can be approximately expressed by

$$c_0(\rho_0/\rho_1)[1+(3/4)(n+1)M_0]$$

Therefore, if we use the dimensionless quantity $\zeta_r = (\rho_1/\rho_0)(c_r/c_0)$, the following inequality can be derived,

$$(\rho_1/\rho_0)^{(n+1)/2} \leq \zeta_r < 1 + (3/4)(n+1)M_0 \dots (14)$$

where c_r is the mean propagation velocity of the reflection wave from the piston surface to the end of the tube. Then, the duration t_p of the impact pressure at the end of the tube can be expressed as follows,

$$t_p = (l_0/c_0)((1/\zeta_r) + (1/\zeta_r)) \dots (15)$$

Fig.6 Duration of impact pressure at closed end of tube

Fig.7 General view of constructed apparatus

where l_0 is the water column length before impact. In case the exponent $n = 7$ for the equation of state, the relation between the duration time and the impact velocity is shown in Fig.6. From this diagram, it is seen that the duration of the impact pressure will decrease rapidly as the impact velocity of the column increases, and that the duration will be little influenced by the piston.

3. Experimental results

3.1 Outline of the apparatus

Figure 7 presents a general view of the high-speed liquid flow apparatus that was constructed. The operation of this apparatus is carried out as follows.

At first, a part of the tube in the D' section is partitioned with thin diaphragms (Teflon, 0.1 mm in thickness) and a water column inside the section is formed by pouring water therein. At this time, a Teflon piston is placed behind the water column from deformation during acceleration. Next, air inside the tube D is exhausted in order to decrease the resistance in acceleration of the water column. Compressed air is supplied to the air reservoir A and the water column is accelerated by opening the ball valve B.

The size of the air reservoir A was determined such (the inside diameter: 150 mm, length: 1000 mm, see Fig.3) that reservoir pressure can be kept almost constant during acceleration. The tube D measures 20 mm in inside diameter and 5500 mm in whole length. It is composed of several tubes of 500 mm or 1000 mm length which are connected with union nuts H. Each tube was smoothly bored by special drill. The column length can be varied by changing the combination of 500 mm tubes. The tube wall can stand pressure up to 2000 kg/cm² in stopping a water column with some 100 m/sec velocity. In accelerating the column, it took about 50 ~ 80 milliseconds to fully open the ball valve manually. We had used an aluminum diaphragm (0.4 mm thick) in place of

the ball valve to discharge compressed air instantly into the tube. However, the column velocity was little different from that with use of the ball valve. Since the pressure in the downstream of the ball valve does not rise until the valve is almost fully open, the effective time of

opening the valve seems considerably shorter than the actual opening time.

3.2 Water column velocity in the tube

The arrival time of the front surface of the water column was measured at several points of the tube to obtain the column velocity. The measurement was made by using copper electrodes of 3 mm in diameter fitted in the joints H. The resistance in the circuit changes stepwise when the front section of the column arrives at each electrode. The conditions under which the measurement was made were as follows.

The length of the water column: 0.50 m, 1.00 m.

The initial pressure of compressed air: $10 \sim 30$ kg/cm².

The initial pressure in the tube: approx. 10 mmHg.

The piston: diameter 19.9 mm, length 20 mm and weight 13.2 g.

Since the dimensionless quantities used in section 2.1 included the unknown value λ_a , dimensionless quantities such as the following were used here instead of the former:

$$x = X/D, \quad \varphi = (U/\sqrt{2P_0/\rho})\sqrt{I_a/D}$$

$$\tau = t\sqrt{2P_0/\rho}/\sqrt{I_a/D}$$

$$\text{and} \quad \varepsilon = \pi D^3/(4V_r)$$

where t denotes the arrival time of the column. The results are shown in Fig. 8. The value τ shown by a full line in this figure is calculated in the following manner. In the case of this apparatus, $\varepsilon = 3.55 \times 10^{-4}$ and $\lambda_a = 1.25 \times 10^{-2}$. Then ψ is derived from equation (3), where $q = \lambda_a/\varepsilon = 2.84 \times 10^{-2}$. After transforming ψ into φ by the relation $\varphi = \sqrt{\lambda_a}\psi$, the dimensionless arrival time τ is calculated from the relation $\tau = \int_0^x dx/\varphi(x)$. The value of τ is integrated numerically by the Simpson's method. The measured value τ agrees well with the calculated result.

Changes in the column velocity φ are illustrated in Fig. 9. This figure also gives, for comparison, the calculated curves based on such

Fig. 8 Arrival time of water column at each point of acceleration tube

assumptions as (1) the volume of air reservoir is infinite (i.e. $\varepsilon = 0$) and (2) the volume of air reservoir is infinite and there is no tube wall friction (i.e. $\varepsilon = 0$, $\lambda_a = 0$). This figure shows that, at the end of the tube, the column velocity is almost maximum and is close to a state of constant velocity. The initial pressure of the driving gas and the column length influence very little on this tube length. The energy loss by the time when the column reaches the end of the tube is about 70 % of the work done by compressed air.

The relation between the column velocity U_0 at the end of the tube and the initial acceleration $P_0/(\rho l_e)$ is plotted in Fig. 10. The measured value in this figure is the average speed of the column in the last one-meter section of the tube. This figure shows that the column velocity at the tube's end is proportional to the square root of the initial acceleration. The friction coefficient ($\lambda_0 \div \lambda_a = 1.25 \times 10^{-2}$) derived from this measurement agrees fairly well with the value that is estimated from the Moody diagram at the time of designing (the Reynolds number used in this case is: $U_0 D/\nu = 1 \sim 2 \times 10^6$).

Fig. 9 Velocity changes of water column along acceleration tube

Fig. 10 Water column velocity at end of acceleration tube

3.3 Turbulence at the front section of the column

Since the front section of the water column moving in the tube has a free surface, its shape is considered unstable and irregular. This phenomenon is examined by preliminary experiment as follows.

Figure 11 presents pressure change and the arrival signal of the water column during the column passes a certain point of the tube. The pressure is measured by using a quartz piezoelectric gauge (Kistler, 603A), which is fixed at the wall of the tube. The measurement shows that pressure along the axis of the column does not change linearly, and has a spike-shaped wave that pressure changes irregularly in the front section of the column and that the arrival signal of the column changes incompletely stepwise and its transient part presents fluctuation. It is therefore suggested that the front section of the column is not kept cylindrical shape and that air has been mixed into the section. We will use the following quantity as an estimation of its length.

$$\Delta l = U \times \Delta t_d$$

where Δt_d is time interval in the region where the pressure changes irregularly (see Fig.11) and U is the water column velocity at the measuring point.

Changes in Δl in relation to the point of measurement, to the initial pressure of the driving gas and to the initial pressure in the acceleration tube are illustrated in Fig.12, Fig.13 and Fig.14. Each measurement was made several times under the same conditions and the measured values showed considerably wide variations. These results, however, indicate at least the following tendencies (1) disturbance at the front section of the column grows as the column moves (Fig.12), (2) the amount of scatter in the measured values increases as the initial pressure of the driving gas increases, namely, as the column ve-

Fig.11 Pressure change and arrival signal in water column being accelerated (column length: 1.00 m, position: $X = 3.53$ m)

locity increases (Fig.13) and (3) the disturbance is restrained somewhat by the increase of initial pressure in the acceleration tube (Fig.14). The fact that the length of disturbance exceeds 20 cm at the end of the tube leads us to presume that the disturbance section decreases the frictional force acting on the column.

3.4 Impact pressure of the water column at the tube end

Impact pressure of the water column at the closed end of the tube was measured by using a quartz piezoelectric gauge (Kistler, 607A) as shown in Fig.15. From this photograph, it is seen that reflected waves from the piston become strong compression waves. This indicates that the piston can be considered as a rigid body in reflection of pres-

Fig.12 Changes in Δl with measuring point

Fig.13 Effect of initial pressure of driving air on Δl

Fig.14 Effect of initial pressure in acceleration tube on Δl

sure waves. The conditions of measurements are the same as those described in section 3.2. Hence, the range of measurements is limited to a part of the calculations represented in section 2.2 ($M_0 < 0.07$).

Impact pressure P_i and its duration t_p are plotted against the impact velocity U_0 of the column in Fig.16 and Fig.17. In evaluating the accuracy of measurement, it is difficult to clearly distinguish the water hammer equation from Heymann's approximate formula (12) within the range of this impact velocity, and the measured value comes between the two curves as shown in Fig.16. The measured value for the duration t_p is considerably less than the calculated one. The actual pressure of impact does not rise completely stepwise. This is presumably affected by disturbance at the front section of the water column mentioned in section 3.3.

Then, the form of pressure waves due to the initial pressure in the acceleration tube was examined, while setting the column length and the initial pressure of the driving gas at constant. Figure 18 illustrates P_i and t_p thus measured. This measurement revealed that disturbance in the rising section of pressure wave becomes large when the initial pressure in the acceleration tube is less than 20 mmHg (almost pressure of aqueous vapor) and that the duration t_p is shortened. On the other hand, the initial pressure of above 50 mmHg causes compression resistance of air to take effect and pressure waves are increasingly rounded, with P_i and t_p diminishing gradually. The effect of the initial pressure in the acceleration tube on such pressure waves is similar to that on disturbance at the front section of the column (Fig.14).

Fig.15 Impact pressure of water column at closed end of tube (column length: 1.00 m)

4. Conclusions

The main results derived from the construction of the high-speed liquid flow apparatus and its experiments are as follows.

(1) The water column velocity reaches the maximum value at a certain point of the acceleration tube because of a drop in pressure of the driving gas and because of an increase in friction force of tube wall. This is shown by a simple approximate calculation.

Fig.16 Impact pressure versus impact velocity of water column

Fig.17 Duration of impact pressure versus impact velocity of water column

Fig.18 Effect of initial pressure in acceleration tube on impact pressure and its duration

(2) A diagram is given for obtaining the pressure of the driving gas necessary to obtain the desired column velocity and the length of the tube in which the column velocity reaches the maximum at the tube end.

(3) In this apparatus, water columns measuring 0.5 m and 1.00 m in length were accelerated by compressed air from 5 to 30 kg/cm². As a result, the column velocity of up to 115 m/sec was obtained and validity of the approximate calculation was confirmed in that range.

(4) Pressure change in the water column during acceleration is measured to examine the disturbance at the front section of the column. It is shown experimentally that the disturbance increases as the column travels but that it is restrained by a increase of gas pressure at the front of the column.

(5) Impact pressure and its duration at the time of impact of a high-speed water column at the closed end of the tube are shown by calculation and measurement. Dependence of bulk modulus of water on pressure is taken into account in calculation and the results are compared with the water hammer equation, the approximate formula and the experimental data.

(6) From the measurement, pressure of the water column at the end of the tube is obtained between the water hammer equation and Heymann's

approximate formula. It is noted, however, that the duration of the impact pressure shortens, with pressure change showing disturbance, if air pressure of the tube before impact of the column is exhausted lower than pressure of saturated aqueous vapor.

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